VECTORS AND 3-D GEOMETRY

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1 A

11(?). $a - 2 + 4 = 0 \Rightarrow a = -2$. $1 + b + 7 = 0 \Rightarrow b = -8$. $3 - 5 + c = 0 \Rightarrow c = 2$.

Ans $\rightarrow$ c

12) $\sqrt{(2-0)^2 + (1+1)^2 + (3-2)^2} = AB = 3$.

$BC = \sqrt{1^2 + 2^2 + 2^2} = 3$.

$AC = \sqrt{1^2 + 4^2 + 1^2} = \sqrt{18}$.

$AB^2 + BC^2 = AC^2$, $AB = BC$ $\boxed{\text{Ans } b}$

13) $\frac{5+4}{5+1} = \frac{5-2}{5-3} = \frac{\lambda+2}{\lambda-2}$

$\Rightarrow \frac{\lambda+2}{\lambda-2} = \frac{3}{2} \Rightarrow \lambda = 10$ $\boxed{\text{Ans} \rightarrow d}$

14) x, y, z: $\frac{(3\times1) - (5\times3)}{3-5} = x \Rightarrow \frac{-12}{-2} = x = 6$.

No option has $x = 6$. $\boxed{\text{Ans} \rightarrow D}$

15) Ratio $= (1 : -2)$ or $(-1 : 2)$ Pt be x, y, z.

$\frac{(1)(1) + (-2)(-1)}{-1} = -3$. or $\frac{(-1)(1) + (2)(-1)}{-1} = 3$.

$x = 3$. Similarly by section formula, $y = 4$, $z = -3$

$\boxed{\text{Ans} \rightarrow B}$

16) $\vec{A} \equiv i + 2j + 3k$, $\vec{B} \equiv 7i + 8j + 7k$

$\vec{AB} \equiv 6i + 6j + 4\hat{k}$.

So $6, 6, 4$ $\boxed{\text{Ans} \rightarrow a}$

17) $[a\ b\ c] = |a||b||c|$ means rectangular parallelopiped

18) $\vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})=0$

$\Rightarrow |a|^2 + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} = 0$

Also, $|b|^2 + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} = 0$.

and $|c|^2 + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c} = 0 \rightarrow (3)$

$\Rightarrow \cancel{|a|^2 + |b|^2 + |c|^2 + 2\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + 2\vec{c}\cdot\vec{a} = 0}$.

We have, from (3),

$49 + \vec{c}\,(\vec{a}+\vec{b}) = 0$

$\Rightarrow |c|\,|\vec{a}+\vec{b}| = 49$

$\Rightarrow |\vec{a}+\vec{b}| = 7$.

$\Rightarrow |a|^2 + |b|^2 + 2|a||b|\cos\theta = 49$,

$\Rightarrow 2\times3\times5\cos\theta = 15 \Rightarrow \cos\theta = \frac{1}{2}$.

$\boxed{\text{Ans} \rightarrow D}$

19) $d_1 = \vec{AB} + \vec{BC}$, $d_2 = \vec{AB} - \vec{BC}$.

$= (2\hat{i} - 2\hat{j} + 4\hat{k})$ $= (4\hat{i} - 2\hat{j})$.

$d_1\cdot d_2 = |d_1||d_2|\cos\theta$

$\Rightarrow 8 + 4 = \sqrt{24}\times\sqrt{20}\times\cos\theta$.

$\Rightarrow 12 = \sqrt{480}\cos\theta$

$= 16\sqrt{30}\cos\theta$

$\Rightarrow \cos\theta = \frac{3}{4\cancel{\ }\sqrt{30}} = \frac{\sqrt{3}}{4\cancel{\ }\sqrt{10}}$

$\boxed{\text{Ans} \rightarrow D}$

20 $$|e_1 - e_2|^2 = e_1^2 + e_2^2 - 2|e_1||e_2|\cos\theta$$

Ans $\to$ B

$$= 1 + 1 - 2\left(1 - 2\sin^2\frac{\theta}{2}\right)$$

$$= 4\sin^2\frac{\theta}{2} \Rightarrow \sin\frac{\theta}{2} = \frac{1}{2}|e_1 - e_2|$$

21

$$x = \frac{2a+b}{3}, \quad y = \frac{2a-b}{1}.$$

check

$$x+y = \left(\frac{2a+b}{3}\right) + \left(\frac{2a-b}{1}\right) = \frac{4a}{3} - \frac{4b}{3} = \frac{4a-4b}{3}$$

Ans B

22,

$$(\vec{a} + 3\vec{b}) \cdot (3\vec{a} - \vec{b}) = 3|a|^2 + 9\vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{b} - 3|b|^2$$

$$= 3 - 6 + 8|a||b|\cos\frac{2\pi}{3}$$

$$= -3 + 8 \times 2 \times -\frac{1}{2}$$

$$= -11.$$

Now,

$$\left[(\vec{a} + 3\vec{b}) \cdot (3\vec{a} - \vec{b}) = 11\right]^2 + \left\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\right\}^2$$

$$= |\vec{a} + 3\vec{b}|^2 |3\vec{a} - \vec{b}|^2$$

$\Rightarrow$ Ans $\to$ D

23, Let the sides $\vec{AB} = \vec{c}$ & so on.

Let bisector be $\vec{r}$

So, $$\frac{\vec{r}\cdot\vec{b}}{|r||b|} = \frac{\vec{r}\cdot\vec{c}}{|r||c|} \text{ and } [a\ b\ r] = 0, \text{ and } [a\ c\ r] =$$

Solving $\vec{r} = \ldots$

24. $(\vec{a} \times \vec{b}) \times \vec{c}$

$= (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$

$= 7\vec{b} - 21\vec{c} = 7(\vec{b} - 3\vec{c})$

$\boxed{\text{Ans} \to B}$

25) $y' = \dfrac{-16}{x^3}$

$y'(2,2) = -2$.

Line of tangent is $\dfrac{y-2}{x-2} = -2$

or $y - 2 = -2x + 4$

$2x + y = 6 \qquad \dfrac{x}{3} + \dfrac{y}{6} = 1$.

so $\vec{OB} = 3\hat{i}$.

$\vec{AB} = \vec{OB} - \vec{OA} = 3\hat{i} - 2\hat{i} - 2\hat{j}$

$= \hat{i} - 2\hat{j}$.

$\vec{AB}\cdot\vec{OB} = 3$.

$\boxed{\text{Ans} \to a}$

26) Using method in Q 23.

Ans $\to$ D

27) $|b| = \sqrt{36 + 64 + \frac{225}{4}} = \sqrt{\frac{625}{4}} = \frac{25}{2}$.

So $4|b| = |a|$.

Also for acuteness, $a = -4\vec{b}$

28) $\left|\frac{\vec{a}+\vec{b}}{2} - c\right|$

Check $= |\hat{i} + 2\hat{j} + \hat{k} - 4\hat{i} + 2\hat{j} + 6\hat{k}| = |-3\hat{i} + 4\hat{j} + 7\hat{k}| = \sqrt{74}$.

$\boxed{\text{Ans} \to C}$ Check

29) $[a\ b\ c] = \begin{vmatrix} x & y & z \\ 1 & -1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = (y+z) - 2(x-z)$

$= y + z - 2x + 2z$

$= y - 2x + 3z$.

Also, $x - y + z = 0$ and $x + 2y = 4$.

Solving for x, y, z. $\boxed{\text{Ans} \to D}$

30) $[(1-t)\hat{i} + (2+2t)\hat{j} + (3+t)\hat{k}] \cdot [3\hat{i} + \hat{j}] = 0$

$\Rightarrow 3 - 3t + 2 + 2t = 0$

$\Rightarrow t = 5$.

Ans $\to$ C

31) $[a+b \quad b+c \quad c+a] = [b \quad b+c \quad c+a] + [a \quad b+c \quad c+a]$

$= [b \quad b \quad c+a] + [b \quad c \quad c+a] + [a \quad b+c \quad c+a]$

and so on ...

to $12[a\ b\ c]$.

So $12 \times 3 = 36 =$ ans.

33) $p \perp q$, $r \perp s$

$6|a|^2 - 5|b|^2 - 7\vec{a}\cdot\vec{b} = 0$.

$\ldots -|a|^2 \quad -3\vec{a}\cdot\vec{b} + 4|b|^2 = 0$.

11 = 34) So joining line is normal.

$drs = (3+1, -5-2, 6-3)$ or $(4, -7, 3)$.

Clearly, $\boxed{\text{ans} \to c}$

12) 8

drs of line = drs of plane = 3, 4, 5.

So clearly $\boxed{\text{ans} \to a}$

13)

drs of normal = 2, 3, -6.

$dcs = \frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$ (as $\sqrt{2^2+3^2+6^2} = 7$).

ans → b

14) = 33 ?

15) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 1 - \frac{196}{225} - \frac{1}{9} = \frac{4}{225}$.

Ans → a

16 = ?) (39)

$drs = [(-1-1), (0-2, 1-(-1)]$

$= [-2, -2, 2]$.

or $(-1, -1, 1)$

or $(1, 1, -1)$

ans → b

18) $(2a \quad -3+10) = |2,-1,2| \times |a,3,5| \times \cos 45$

$\Rightarrow (2a+7) = (3) \times (\sqrt{a^2+34}) \times \frac{1}{\sqrt{2}}$

$\Rightarrow a = 4$

ans $\rightarrow$ d

19) A pt on line 1 is $(5p+4, 2p+1, p)$

on line 2 is $(2q+1, 3q+2, 4q+3)$

Matching, we get $(-1,-1,-1)$

ans $\rightarrow$ a

20) $\frac{x}{2} + \frac{z}{3} + \frac{y}{\infty} = 1$

$\Rightarrow 3x + 2z = 6$. ans $\rightarrow$ b

21) ?

22) Clearly, $z = 3$.

23) $\frac{3x - 6y + 2z + 5}{\sqrt{49}} = \pm \frac{4x - 12y + 3z - 3}{\sqrt{169}}$

To contain origin, constant term $= 0$.

Hence (b).

24. $(3 \times 2) - (6)(1) + 2k = 0$

$\Rightarrow k = 0$. (a)

25

26) A pt on the line is $(\lambda, 2\lambda+1, 3\lambda-2)$.

Plugging into plane, $\lambda = \frac{-1}{11}$

Ans $\rightarrow$ D

27) Clearly xy plane.

28) $2:3:2 \neq 4:-2:-1$ not parallel.

But $2\times4 + 3\times-2 + 2\times-1 = 0$.

So $\perp$.

Ans 5.

29) drs are same as normal, i.e. $(1,2,-5)$

So $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{(-5)}$ Ans a

30) $\begin{vmatrix} a & f & h \\ f & b & g \\ h & g & c \end{vmatrix} = 0$. $\boxed{\text{Ans d}}$

31) Clearly, ellipse.

$\frac{x^2}{15} + \frac{y^2}{9} + \frac{z^2}{9} = 1$ (eccentricity $= \sqrt{1 - \frac{a^2}{b^2}} = \frac{3}{5}$).

32). So it always inscribes a right angle. Like in a semicircle. So sphere

Ans $\rightarrow$ c

33)

$\left[\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}\right]$ $\left(\because 7 = \sqrt{2^2+3^2+6^2}\right)$.

34) d.r.s of line are $2, -3, 5$.

any point on line is $2\lambda+3, -3\lambda+4, 5\lambda+1$.

Ans B

For $z=0$, $\lambda = -\frac{1}{5}$.

So $\left[\frac{13}{5}, \frac{23}{5}, 0\right]$ is pt.

35) $(x-0):(y-0):(z-0) :: l:m:n$. and $x = lr$.

So $x = lr, y = mr, z = nr$.

Ans → c

36). drs are $-b, -c, -a$ or b, c, a.

Ans → b

37) Clearly (d)

38) any pt is $2\lambda+1, 9\lambda, 5\lambda$.

minimizing $\sqrt{(2\lambda+1-5)^2+(9\lambda-4)^2+(5\lambda+1)^2}$

we get $\sqrt{\frac{2109}{110}}$

39) $l^2+m^2+n^2=1$

$l+m+n=0$

40. drs is 2, 3, -4

So only option (d)

Also, $2(x-1) + 3(y-2) - 4(z-3) = 0$ gives $\underline{d}$.

Ex 1-B

1) $|c| = \sqrt{2}$

also, $\vec{a} \cdot \vec{c} = 1$ and $\vec{b} \cdot \vec{c} = 1$

Thus, $\boxed{a, d}$

2) $a^2 + b^2 = c^2 + d^2 = r^2$ and $ac + bd = 0$

Solving, $a^2 + c^2 = r^2$,

$b^2 + d^2 = r^2$

and $ab + cd = 0$

Hence A, B, C

3) in the same plane as $\vec{a}$ & $\vec{b}$ but $\perp$ to $\vec{a}$

A, C, D

c) Since they are ind., $K_1 = K_2 = K_3 = K_4 = 0$

A, B, C

→ qs incl.

~~5~~ 6) $[\vec{a} - \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = 2[a\ b\ c] - [abc] - [abc]$

$= 0$

So (b).

Similarly $\boxed{c, d}$

So $\boxed{B, C, D}$

7.

$$(\vec{a}\cdot\vec{c})\vec{b} \quad - \quad (\vec{a}\cdot\vec{b})\vec{c}$$

$$= -(xy + yz + xz)\,\vec{c} + (xz + xy + yz)\,\vec{b}$$

$$\Rightarrow (xy + xz + yz)(\vec{b} - \vec{c}) = (xy + xz + yz)\left((y-z)\hat{i} + (z-x)\hat{j} + (x-y)\hat{k}\right)$$

A, B, C, D.

8.

$$\frac{\vec{A} + \vec{B} + \vec{C}}{3} = \text{centroid}.$$

Let $\hat{i} + \hat{j} + \hat{k} = \vec{r}$.

Then $\vec{r}\cdot\vec{A} = a + b + c = \vec{r}\cdot\vec{B} = \vec{r}\cdot\vec{C}$

Equally inclined too. $\boxed{\text{Ans } A, B, C, D}$

9) Since linear dependent, hence co-planar too.

$$3\vec{a} + \vec{c} = 2(\vec{b} + \vec{d})$$

10). $[p+q \quad q+r \quad r+p]$

$= [p \quad q+r \quad r+p] + [q \quad q+r \quad r+p]$ etc

$= [p \quad q \quad r+p] + [p \quad r \quad r+p] + [q \quad q+r \quad r+p]$

and so on

$= 2\,[p\,q\,r]$.

Ans B, C, D.

Also, $[p-q \quad q-r \quad r-p] = 0$.

11.

By triangle law,

$x\vec{a} - y\vec{b} = \vec{a} + \vec{b}$, or $(-\vec{a} - \vec{b})$

A, B

so $x = -1,\ y = 1$.

so $\vec{a},\ \vec{b},\ -\vec{a} - \vec{b}$ is Δ.

$\cos(a \wedge b) + |\vec{a} + \vec{b}| \cos(\vec{a} \wedge (-a - \vec{b}))$

$= \cos C + c \cos B = -1$.

12).

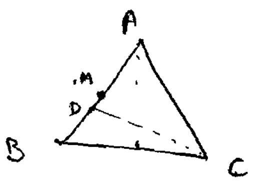
$(\vec{a} \times \vec{b}) \cdot \vec{c} = |c|^2 = [a\ b\ c]$.

$(\vec{b} \times \vec{c}) \cdot \vec{a} = |a|^2 = [a\ b\ c]$.

So $|a| = |c|$.

A, B, C.

13).



C

9

$3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{\lambda}$

$\vec{AD} = \vec{D} - \vec{A} =$

$\left(\frac{\vec{b} + \vec{d}}{2}\right) = \frac{3\vec{a} + \vec{c}}{4}$.

14) . Vertex is either $\perp$ to BC or skew $\perp$ to BC.

Only A, D satisfy.

~~also~~

15) For no value of λ or μ $r_1 = r_2$.

So don't intersect. So skew.

$\boxed{B, C, D}$

$$\frac{\vec{n_1} \cdot \vec{n_2}}{|n_1||n_2|} = \cos\theta \quad \cancel{\theta} = \frac{7}{\sqrt{58}}$$

So $\tan\theta = \frac{3}{7}$

16) drs of intersection is given by:

$\vec{n_1} \times \vec{n_2}$ which is 2, 5, 3 or -2, -5, -3

So A, C.

17. Clearly A, C, D

18. 

Clearly its $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ from figure

Thus $\pm\frac{1}{\sqrt{2}}$.

19. $(2\hat{i} + \hat{j} + 4k) \cdot (\hat{i} + 2\hat{j} - \hat{k}) = 0$

For a arbitrary value of λ,

line does not lie in plane.

20. $\dfrac{3\times 2 - 6\times 3 + 2\times 4 + 11}{\sqrt{3^2 + 6^2 + 2^2}} = \dfrac{6 - 18 + 8 + 11}{7} = \dfrac{17 - 10}{7} = \dfrac{7}{7} = 1.$

Ans $\longrightarrow$ $\underline{\underline{A}}$.

PASSAGE

1)

$(a-c)\cdot(b-c)=0$

$\Rightarrow a\cdot b + c^2 - c(a+b) = 0$

$\Rightarrow 2c(a+b) = 2(a\cdot b + c^2)$.

$|a-b|^2 = (a-b)(\cdot a-b)$

$= a^2 + b^2 - 2a\cdot b$.

C

So $LHS = a^2 + b^2 - 2a\cdot b + 2a\cdot b + 2c^2$

$= a^2 + b^2 + 2c^2$

$= 4+4+2$

$= 10$.

Ans B

2)

$(a+b-c)\cdot(a+b-c)$

C

$= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$

$= 9 + 2ab - 2c(a+b)$

$= 9 + 2ab - 2ab - 2c^2$

$= 9-2 = 7$

$\boxed{\text{Ans C}}$

3)

Max $= |a| + |b| = 4$,

Min $= |a| - |b| = 0$.

PASSAGE

$$L_1 \equiv \frac{x-7}{-3} = \frac{y-6}{2} = \frac{z-2}{4} = \lambda \quad \text{or} \quad (-3\lambda+7,\ 2\lambda+6,\ 4\lambda+2)$$

$$L_2 \equiv \frac{x-5}{2} = \frac{y-3}{1} = \frac{z-4}{3} = \mu \quad \text{or} \quad (2\mu+5,\ \mu+3,\ 3\mu+4)$$

For intersection,

$$2\mu+5 = -3\lambda+7$$

$$2\lambda+6 = \mu+3$$

$$\text{or} \quad 2\mu+3\lambda = 2.$$

$$\text{or} \quad 2\mu - 4\lambda = 6$$

$$-2\mu + 4\lambda = -6$$

$$\Rightarrow 7\lambda = -4$$

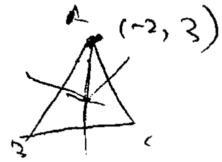
$$\lambda = -\frac{4}{7}.$$

~~Thus~~ ~~continuing~~, ~~we get C & D~~.

~~For plane $\parallel^{le}$ to L_1 & containing L_2~~,

$L_3 \equiv$

$\boxed{\text{Depends on what pt } L_3 \text{ passes by!! whole passage!}}$



PASSAGE :

Solving : $x - y = 4$ & $2x - y = 5$,

we get $x = 1$ & $y = -3$. $\equiv$ circumcentre $\equiv O$

$\vec{BC} \cdot \vec{AO} = 0$ (O = circumcentre).

Only $7x - y = 55$ satisfies.

so $AB \equiv x - y - 4 = 0$,

$AC \equiv 2x - y - 5 = 0$,

$BC \equiv 7x - y = 55$.

Given all sides, we find $\Delta = \frac{108}{5}$.

Assertion reason :

1) . True. Only way v equally inclined to all 3 ~~[illegible]~~,
is if $v \perp$ (Plane (v_1, v_2, v_3))

2) . A. ~~[illegible]~~

3) A.

4) $(\vec{a} \times \vec{b}) \times \vec{b} = (a \cdot b)\vec{b} - (b^2)\vec{a}$

$= 2\vec{b} - 6\vec{a}$

PASSAGE

$\vec{OB} = \vec{OA} + \vec{AB}$ $\qquad$ $\vec{OD} = \vec{OC} + \vec{CD}$

$B \equiv (5\hat{i} - 2\hat{j} + 26\hat{k})$ & $D \equiv (6\hat{i} + \hat{j} - 5\hat{k})$

Line AB $\equiv \vec{r} = 3\hat{i} - \hat{j} + 25\hat{k} + t(2\hat{i} - \hat{j} + \hat{k})$

Line CD $\equiv \vec{r} = 5\hat{i} + 2\hat{j} - 10k + t(\hat{i} - \hat{j} + 5\hat{k})$

Solving for r, we get

$P \equiv (13, -6, 30)$.

Now $PQ \perp AB$ and $|PQ| = 7\sqrt{2}$.

Only option satisfying is $(17, 3, 31)$.

$Q \equiv (17, 3, 31)$.

$\frac{1}{2}|\vec{AB} \times \vec{CD}| = \frac{1}{2} \times \sqrt{\left(\frac{2\times 49}{2}\right)} = \frac{7}{\sqrt{2}}$.

$\vec{AB} \times \vec{CD}$ computation : $\begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & -1 & 5 \end{vmatrix} = i(-5+1) + j(10-1) + k(-2+1)$

$= -4\hat{i} + 9\hat{j} - \hat{k}$.

$|-4\hat{i} + 9\hat{j} - \hat{k}| = |\vec{AB} \times \vec{CD}| = 98 = 2 \times 49$.

EXERCISE 1(C)

1 Let the equation of the plane containing the line $x-y-z-4=0=x+y+2z-4$ and is parallel to the line of intersection of the planes $2x+3y+z=1$ and $x+3y+2z=2$ be $x+Ay+Bz+C=0$.

Compute the value of $\frac{|A+B+C|}{11}$. [Ans. 1]

[Sol. A plane containing the line of intersection of the given planes is

$$x-y-z-4+\lambda(x+y+2z-4)=0$$

i.e. $(\lambda+1)x+(\lambda-1)y+(2\lambda-1)z-4(\lambda+1)=0$

vector normal to it

$$V=(\lambda+1)\hat{i}+(\lambda-1)\hat{j}+(2\lambda-1)\hat{k} \quad .....(1)$$

Now the vector along the line of intersection of the planes

$2x+3y+z-1=0$ and $x+3y+2z-2=0$ is given by

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ 1 & 3 & 2\end{vmatrix}=(6-3)\hat{i}-(4-1)\hat{j}+(6-3)\hat{k}=3(\hat{i}-\hat{j}+\hat{k})$$

As $\vec{n}$ is parallel to the plane (1)

Hence $\vec{n}\cdot\vec{V}=0$

$$(\lambda+1)-(\lambda-1)+(2\lambda-1)=0$$

$$2+2\lambda-1=0 \Rightarrow \quad \lambda=\frac{-1}{2}$$

Hence the required plane is

$$\frac{x}{2}-\frac{3y}{2}-2z-2=0$$

$$x-3y-4z-4=0$$

Hence $|A+B+C|=11$

2 Given $f^2(x)+g^2(x)+h^2(x)\le 9$ and $U(x)=3f(x)+4g(x)+10h(x)$. If maximum value of $U(x)$ is $\sqrt{N}$, then find $\frac{N}{1125}$. [Ans. 1]

[Sol. Let $\vec{V}_1=3\hat{i}+4\hat{j}+10\hat{k}$ and $\vec{V}_2=f(x)\hat{i}+g(x)\hat{j}+h(x)\hat{k}$

$$U(x)=\vec{V}_1\cdot\vec{V}_2=|\vec{V}_1||\vec{V}_2|\cos\theta$$

$$\le|\vec{V}_1||\vec{V}_2|$$

$$=\sqrt{9+16+100}\sqrt{f^2+g^2+h^2}=3\sqrt{125}=15\sqrt{5}$$

3 If $(x_1-x_2)^2+(y_1-y_2)^2=a^2$

$(x_2-x_3)^2+(y_2-y_3)^2=b^2$

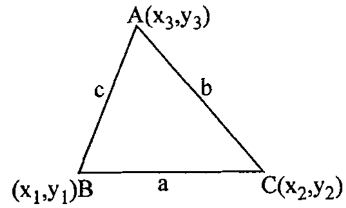
and $(x_3-x_1)^2+(y_3-y_1)^2=c^2$

then $\lambda\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}^2=(a+b+c)(b+c-a)(c+a-b)(a+b-c)$. Find the value of λ.

[Ans. 4]

[Sol. First 3 equations are suggestive that
(x_1, y_1) (x_2, y_2) and (x_3, y_3) and the vertices of a Δ BCA

Now $A = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$



$\therefore$ $4A^2 = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$

$4A^2 = 4s(s-a)(s-b)(s-c)$

$16A^2 = 2s(2s-2a)(2s-2b)(2s-2c) = \text{RHS}$

and LHS $= (16)\left(\frac{1}{4}\right)\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$

$\Rightarrow$ $4\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$ $\Rightarrow \lambda = 4$

4 Let $\vec{a} = -3\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 2\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 2\hat{j}$. If V_1 is the volume of parallelopiped whose three coterminous edges are the vectors $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ and V_2 is the volume of tetrahedron whose three coterminous edges are the vectors $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}$, then find the value of $\frac{(V_1+V_2)}{72}$. [Ans. 4]

[Sol. We have $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} -3 & 1 & 1 \\ 4 & 2 & 4 \\ 2 & 2 & 0 \end{vmatrix} = 36$

Now $V_1 = 2[\vec{a}\ \vec{b}\ \vec{c}] = 72$ and $V_2 = \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]^2 = 216$

Hence $V_1 + V_2 = 288$

5 If $\vec{V}_1 = \hat{i} + \hat{j} + \hat{k}$; $\vec{V}_2 = a\hat{i} + b\hat{j} + c\hat{k}$ where a, b, c $\in$ {–2, – 1, 0, 1, 2}, then the number of non zero vectors $\vec{V}_2$ which are perpendicular to $\frac{\vec{V}_1}{18}$·, is [Ans. 1]

[Sol. $\vec{V}_1 \cdot \vec{V}_2 = a + b + c = 0$

but a, b, c $\in$ {–2, – 1, 0, 1, 2}

now (i) if $a = 1,\ b = -1, c = 0$, number $= 3! = 6$

(ii) if $a = 2,\ b = -2, c = 0$, number $= 3! = 6$

(iii) if $a = 1,\ b = 1, c = -2$, number $= \frac{3!}{2!} = 3$

(iv) if $a = -1,\ b = -1, c = 2$, number $= \frac{3!}{2!} = \underline{3}$

$\therefore$ Total = 18

6 Let two non-collinear vectors $\vec{a}$ and $\vec{b}$ inclined at an angle $\frac{2\pi}{3}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = 4$. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given as $\overrightarrow{OP} = (e^t + e^{-t})\,\vec{a} + (e^t - e^{-t})\,\vec{b}$. If the least distance of P from origin is $\sqrt{2}\sqrt{\sqrt{a} - b}$ where a, b $\in$ N then find the value of $(a + b)/72$. [Ans. 1]

[Sol. We have $\left(\overrightarrow{OP}\right)^2 = (e^t + e^{-t})^2\,(\vec{a})^2 + (e^t - e^{-t})^2\,(\vec{b})^2 + 2\,(e^t + e^{-t})\,(e^t - e^{-t})\,(\vec{a}\cdot\vec{b})$

$$\left((\vec{a})^2 = |\vec{a}|^2 = 9,\ (\vec{b})^2 = |\vec{b}|^2 = 16 \text{ and } \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\frac{2\pi}{3}\right)$$

$\Rightarrow \left|\overrightarrow{OP}\right|^2 = 9(e^t + e^{-t})^2 + 16(e^t - e^{-t})^2 + 2(e^{2t} - e^{-2t})\cdot 3\cdot 4\cdot\left(-\frac{1}{2}\right) = 13e^{2t} + 5e^{-2t} - 14$

Now $\frac{d}{dt}\left|\overrightarrow{OP}\right|^2 = 0 \Rightarrow 26e^{2t} - 10e^{-2t} = 0 \quad \Rightarrow e^{4t} = \frac{5}{13}$, so $e^{2t} = \frac{\sqrt{5}}{\sqrt{13}}$

$\therefore \quad \left|\overrightarrow{OP}\right|^2_{min} = 13\left(\frac{\sqrt{5}}{\sqrt{13}}\right) + 5\left(\frac{\sqrt{13}}{\sqrt{5}}\right) - 14 \Rightarrow \left|\overrightarrow{OP}\right|^2_{min} = 2\sqrt{65} - 14$

$\Rightarrow \quad \left|\overrightarrow{OP}\right|_{min} = \sqrt{2}\sqrt{\sqrt{65} - 7} = \sqrt{2}\sqrt{\sqrt{a} - b}$, so $a = 65, b = 7$

Hence $(a + b) = 72$.]

7 If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors, then find the maximum value of

$$\frac{\left|2\hat{a} - 3\hat{b}\right|^2 + \left|2\hat{b} - 3\hat{c}\right|^2 + \left|2\hat{c} - 3\hat{a}\right|^2}{57}.$$ [Ans. 1]

[Sol. Let $y = \left|2\hat{a} - 3\hat{b}\right|^2 + \left|2\hat{b} - 3\hat{c}\right|^2 + \left|2\hat{c} - 3\hat{a}\right|^2$

$\Rightarrow y = 3(4 + 9) - 12\left(\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a}\right)$

$\Rightarrow y = 39 - 12\left(\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a}\right)$ ....(1)

$$\left[\begin{array}{l}\text{As } |\hat{a} + \hat{b} + \hat{c}|^2 \ge 0 \Rightarrow 3 + 2(\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a}) \ge 0 \\ \Rightarrow \hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a} \ge \frac{-3}{2}\end{array}\right]$$

$\Rightarrow y_{max} = 39 + (12)\left(\frac{3}{2}\right) = 39 + 18 = 57$

Now y will be maximum if value of $\sum \hat{a}\cdot\hat{b}$ is minimum i.e. equal to $-\frac{3}{2}$]

8 The plane denoted by $\Pi_1 : 4x + 7y + 4z + 81 = 0$ is rotated through a right angle about its line of intersection with the plane $\Pi_2 : 5x + 3y + 10z = 25$. If the plane in its new position be denoted by Π, find the distance of the plane from the origin is $\sqrt{k}$ where k $\in$ N. Find $\frac{k}{53}$. [Ans. 4]

[Sol. Equation of the plane P is

$4x + 7y + 4z + 81 + \lambda(5x + 3y + 10z - 25) = 0$

$(5\lambda + 4)x + (3\lambda + 7)y + (10\lambda + 4)z + (81 - 25\lambda) = 4x + 7y + 4z + 81 = 0$

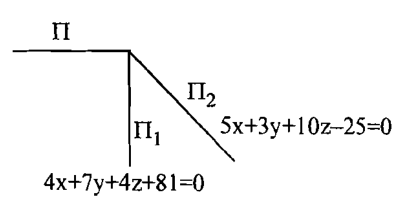
Now $4(5\lambda + 4) + 7(3\lambda + 7) + (10\lambda + 4)4 = 0$

$(20 + 21 + 40)\lambda = -(16 + 49 + 16)$

$81\lambda = -81 \quad \Rightarrow \quad \lambda = -1$

equation of the plane

$-x + 4y - 6z + 106 = 0$



$$p = \left|\frac{106}{\sqrt{1+16+36}}\right| = \left|\frac{(53)(2)}{\sqrt{53}}\right| = \sqrt{212} \qquad \Rightarrow \qquad k = 212$$

9 In a regular tetrahedron, the centres of the four faces are the vertices of a smaller tetrahedron. The ratio of the volume of the smaller tetrahedron to that of the larger is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find the value of $\frac{(m+n)}{14}$. [Ans. 2]

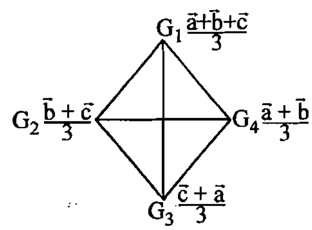
[Hint: $V_l = \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$; $V_s = \frac{1}{6}\cdot\frac{1}{27}[\vec{a}\ \vec{b}\ \vec{c}]$

Hence $\frac{V_s}{V_l} = \frac{1}{27} = \frac{m}{n}$ or $\frac{n}{27} = \frac{m}{1} = k$

$\therefore$ m and n are relatively prime $\Rightarrow$ $k = 1,\ (m+n) = 28$

further hint for

$$V_s = \frac{1}{6}\left[\frac{\vec{a}}{3}\cdot\frac{\vec{b}}{3}\cdot\frac{\vec{c}}{3}\right] = \frac{1}{6}\cdot\frac{1}{27}[\vec{a}\ \vec{b}\ \vec{c}]$$



]

10 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non zero non coplanar vectors and $\vec{p}, \vec{q}$ and $\vec{r}$ be three vectors defined as

$$\vec{p} = \vec{a}+\vec{b}-2\vec{c};\quad \vec{q} = 3\vec{a}-2\vec{b}+\vec{c} \quad \text{and} \quad \vec{r} = \vec{a}-4\vec{b}+2\vec{c}$$

If the volume of the parallelopiped determined by $\vec{a}, \vec{b}$ and $\vec{c}$ is V_1 and that of the parallelopiped determined by $\vec{p}, \vec{q}$ and $\vec{r}$ is V_2 then $V_2 = KV_1$ implies that $\frac{K}{15}$ is equal to [Ans. 1]

[Sol. Given $[\vec{a}\ \vec{b}\ \vec{c}] = V_1$

$$[\vec{p}\ \vec{q}\ \vec{r}] = \begin{vmatrix} 1 & 1 & -2 \\ 3 & -2 & +1 \\ 1 & -4 & 2 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}] = [1(-4+4)-1\,(6-1)-2(-12+2)]V_1$$

$$V_2 = (-5+20)\,V_1 = 15V_1 \quad \Rightarrow K = 15$$

11 Let a_i, $i = 1, 2, 3, \ldots\ldots, n$ denote the integers in the domain of function $f(x) = \sqrt{\log_{\frac{1}{2}}\left(\frac{4x-25}{x-21}\right)}$

where $a_i < a_{i+1}\ \forall\ i \in N$. If the line $L: \frac{2x-a_1}{4} = \frac{y+a_1}{a_2} = \frac{z-a_3}{a_5}$ meets the xy, yz and zx planes at A, B and C respectively, and if volume of the tetrahedron OABD is V, where 'O' is origin and D is the image of C in the x-axis, then find the value of $\frac{90V}{35}$. [Ans. 8]

[Sol. For domain of $f(x)$ we must have $\log_{\frac{1}{2}}\left(\frac{4x-25}{x-21}\right) \geq 0$

$$\Rightarrow \qquad 0 < \frac{4x-25}{x-21} \leq 1$$

$x \in \left[\frac{4}{3}, \frac{25}{4}\right)$

$\therefore$ Integers in the domain are 2, 3 , ..., 6.

$\Rightarrow$ $a_1 = 2, a_2 = 3, \ldots, a_5 = 6$

$\therefore$ L: $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-4}{6} = r$ (let)

At xy plane, put $z = 0$ $\Rightarrow$ $6r + 4 = 0$ $\Rightarrow$ $r = \frac{-2}{3}$

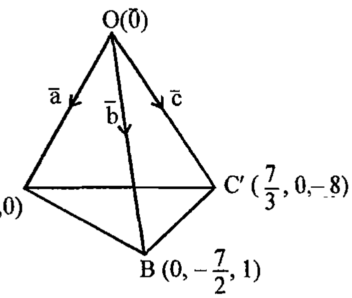
$\therefore$ $A(2r+1, 3r-2, 0) \equiv \left(-\frac{1}{3}, -4, 0\right)$

At yz plane, put $x = 0$ $\Rightarrow$ $2r + 1 = 0$ $\Rightarrow$ $r = \frac{-1}{2}$

$\therefore$ $B(0, 3r-2, 6r+4) \equiv \left(0, \frac{-7}{2}, 1\right)$

At zx plane, put $y = 0$ $\Rightarrow$ $3r - 2 = 0$ $\Rightarrow$ $r = \frac{2}{3}$

$\therefore$ $C(2r+1, 0, 6r+4) \equiv \left(\frac{7}{3}, 0, 8\right) \Rightarrow C'\left(\frac{7}{3}, 0, -8\right)$



$\therefore$ Volume of the tetrahedron OABC' = V $= \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$

$\therefore$ $V = \frac{1}{6}\begin{vmatrix} -1/3 & -4 & 0 \\ 0 & -7/2 & 1 \\ 7/3 & 0 & -8 \end{vmatrix} = \left|-\frac{28}{9}\right|$

Hence $90V = 280$

12_{0MB} If the coordinates of the point where the line $x - 2y + z - 1 = 0 = x + 2y - 2z - 5$ intersects the plane $x + y - 2z = 7$ is (α, β, γ), then find the value of $(|\alpha| + |\beta| + |\gamma|)$.

[Ans. 7]

[Sol. The required point is the point of intersection of the three planes.

$x + y - 2z = 7$ ....(1)

$x - 2y + z = 1$ ....(2)

$x + 2y - 2z = 5$ ....(3)

$\therefore$ From (3) – (1) $\Rightarrow$ $y = -2$

From 2 × (2) + (1) $\Rightarrow$ $3x - 3y = 9$ $\Rightarrow$ $x = 1$

So, from (1) , $z = -4$

Hence the point is $(1, -2, -4) = (\alpha, \beta, \gamma)$

Hence $|\alpha| + |\beta| + |\gamma| = |1| + |-2| + |-4| = 7$ **Ans.**]

13 Let $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{C} = \hat{j} + \hat{k}$

If the vector $\vec{B} \times \vec{C}$ can be expressed as a linear combination $\vec{B} \times \vec{C} = x\vec{A} + y\vec{B} + z\vec{C}$ where x, y, z are scalars, then find the value of $\frac{(100x + 10y + 8z)}{101}$.

[Ans. 1]

[Sol. We have $\vec{B} \times \vec{C} = x\vec{A} + y\vec{B} + z\vec{C}$ ....(1)

Dot with $\vec{B} \times \vec{C}$ gives $(\vec{B} \times \vec{C}) \cdot (\vec{B} \times \vec{C}) = x[\vec{A}\ \vec{B}\ \vec{C}]$ $\Rightarrow$ $x = \frac{(\vec{B} \times \vec{C})^2}{[\vec{A}\ \vec{B}\ \vec{C}]} = \frac{\vec{B}^2\,\vec{C}^2 - (\vec{B} \cdot \vec{C})^2}{[\vec{A}\ \vec{B}\ \vec{C}]}$

|||ly Dot with $\vec{C}\times\vec{A}$ gives $\quad y = \dfrac{(\vec{B}\times\vec{C})\cdot(\vec{C}\times\vec{A})}{[\vec{A}\ \vec{B}\ \vec{C}]}$

and dot with $\vec{A}\times\vec{B}$ gives $\quad z = \dfrac{(\vec{B}\times\vec{C})\cdot(\vec{A}\times\vec{B})}{[\vec{A}\ \vec{B}\ \vec{C}]}$

Now $[\vec{A}\ \vec{B}\ \vec{C}] = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 1(1+1) - 2(-2-3) = 12$

We have $\vec{B}^2\vec{C}^2 - (\vec{B}\cdot\vec{C})^2 = (6)(2) - (0) = 12$

$\therefore \quad x = \dfrac{12}{12} = 1$

$$y = \frac{\begin{vmatrix} \vec{B}\cdot\vec{C} & \vec{B}\cdot\vec{A} \\ \vec{C}\cdot\vec{C} & \vec{C}\cdot\vec{A} \end{vmatrix}}{12} = \frac{\begin{vmatrix} 0 & -3 \\ 2 & 1 \end{vmatrix}}{12} = \frac{0-(-6)}{12} = \frac{1}{2} \text{ and}$$

$$z = \frac{\begin{vmatrix} \vec{B}\cdot\vec{A} & \vec{B}\cdot\vec{B} \\ \vec{C}\cdot\vec{A} & \vec{C}\cdot\vec{B} \end{vmatrix}}{12} = \frac{\begin{vmatrix} -3 & 6 \\ +1 & 0 \end{vmatrix}}{12} = \frac{0-(6)}{12} = \frac{-1}{2}$$

Hence $100x + 10y + 8z = 100 + 5 - 4 = 101$

14 Let ABCD is any quadrilateral and P and Q are the midpoints of its diagonal.
If $\overrightarrow{AB}^2 + \overrightarrow{BC}^2 + \overrightarrow{CD}^2 + \overrightarrow{DA}^2 - \overrightarrow{AC}^2 - \overrightarrow{BD}^2 = \lambda\overrightarrow{PQ}^2$, then find the value of λ. [Ans. 4]

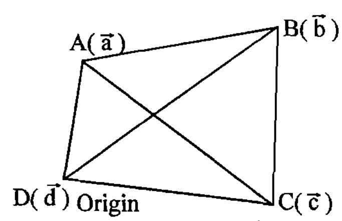
[Sol. We have $\overrightarrow{AB}^2 + \overrightarrow{BC}^2 + \overrightarrow{CD}^2 + \overrightarrow{DA}^2 - \overrightarrow{AC}^2 - \overrightarrow{BD}^2 = \lambda\overrightarrow{PQ}^2$

$$(\vec{b}-\vec{a})^2 + (\vec{c}-\vec{b})^2 + (\vec{d}-\vec{c})^2 + (\vec{a}-\vec{d})^2 - (\vec{c}-\vec{a})^2 - (\vec{d}-\vec{b})^2 = \lambda(\overrightarrow{PQ})^2$$

on simplifying gives

$$= \vec{a}^2 + \vec{b}^2 + \vec{c}^2 - 2\vec{a}\cdot\vec{b} - 2\vec{b}\cdot\vec{c} + 2\vec{a}\cdot\vec{c}$$

$$(\vec{a}+\vec{c}-\vec{b})^2 \quad .....(1)$$



again $\vec{P} = \dfrac{\vec{a}+\vec{c}}{2}$; $\vec{Q} = \dfrac{\vec{b}}{2}$;

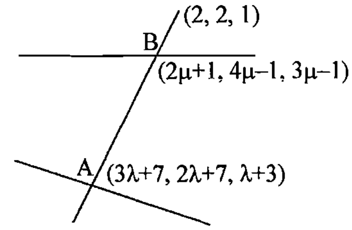
$\overrightarrow{PQ} = \dfrac{\vec{a}+\vec{c}-\vec{b}}{2} \Rightarrow 4(\overrightarrow{PQ})^2 = (\vec{a}+\vec{c}-\vec{b})^2$

$\Rightarrow \quad (\overrightarrow{PQ})^2 = \dfrac{(\vec{a}+\vec{c}-\vec{b})^2}{4} \quad .....(2)$

from (1) and (2)

$\lambda = 4$

15 Consider two lines $L_1 : \frac{x-7}{3} = \frac{y-7}{2} = \frac{z-3}{1}$ and $L_2 : \frac{x-1}{2} = \frac{y+1}{4} = \frac{z+1}{3}$. If a line L whose direction ratios are < 2, 2, 1 > intersect the lines L_1 and L_2 at A and B, then find the distance $\frac{AB}{9}$. [Ans. 2]



[Sol. $\frac{3\lambda-2\mu+6}{2} = \frac{2\lambda-4\mu+8}{2} = \frac{\lambda-3\mu+4}{1}$

On solving we get $\mu = 0$ and $\lambda = 2$

Hence A(13, 11, 5); B(1, – 1, – 1)

$AB = \sqrt{144+144+36} = \sqrt{324} = 18$

16 System of equations $x + 2y + z = 0$, $2x + 3y - z = 0$, $(\tan\theta)x + y - 3z = 0$ has non trivial solution then number of values of θ in $[-\pi, 2\pi]$ is [Ans. 3]

Sol. For non - trivial $\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ \tan\theta & 1 & -3 \end{vmatrix} = 0$

$\Rightarrow$ $\tan\theta = 6/5$

$\therefore$ number of solutions in $[-\pi, 2\pi]$ is 3

17 Let $\vec{a}, \vec{b}, \vec{c}$ be the three vectors such that

$\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{b}\cdot(\vec{c}+\vec{a}) = \vec{c}\cdot(\vec{a}+\vec{b}) = 0$ and $|\vec{a}| = 1$, $|\vec{b}| = 4$, $|\vec{c}| = 8$, then $|\vec{a}+\vec{b}+\vec{c}|$ is [Ans. 9]

Sol $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{b}\cdot(\vec{c}+\vec{a}) = \vec{c}\cdot(\vec{a}+\vec{b}) = 0$

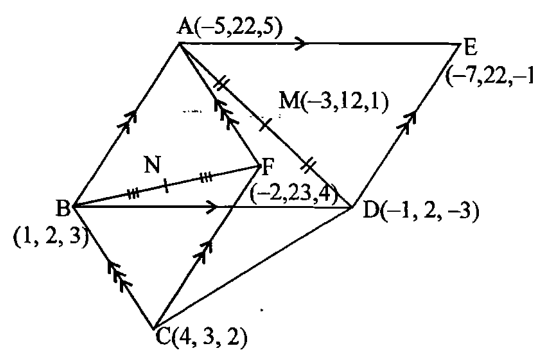
$\Rightarrow \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c}$

$= \vec{c}\cdot\vec{a} = 0$

$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})}$

$= \sqrt{1+16+64+2.0} = 9$

18 ABCD is a tetrahedron with pv's of its angular points as A(–5, 22, 5); B(1, 2, 3); C(4, 3, 2) and D(–1, 2, – 3). If the area of the triangle AEF where the quadrilaterals ABDE and ABCF are parallelograms is $\sqrt{S}$ then find the value of $\frac{S}{55}$. **[Ans. 2]**



[Sol. pv of M $= \frac{\vec{a}+\vec{d}}{2} = -3\hat{i}+12\hat{j}+\hat{k}$

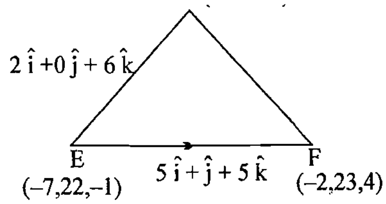
|||ly pv of N $= \frac{\vec{a}+\vec{c}}{2} = -\frac{1}{2}\hat{i}+\frac{25}{2}\hat{j}+\frac{7}{2}\hat{k}$

Now the ΔAEF is as shown

$\vec{S} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 6 \\ 5 & 1 & 5 \end{vmatrix}$

$\left|\vec{S}\right| = \left|-3\hat{i}+10\hat{j}+\hat{k}\right| = \sqrt{110}$

$\therefore \quad S = 110$



19 Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle 60°. Suppose that $\left|\vec{u}-\hat{i}\right|$ is geometric mean of $|\vec{u}|$ and $\left|\vec{u}-2\hat{i}\right|$ where $\hat{i}$ is the unit vector along x-axis then $|\vec{u}|$ has the value equal to $\sqrt{a}-\sqrt{b}$ where a, b ∈ N, find the value $\dfrac{(a+b)^3+(a-b)^3}{14}$. **[Ans. 2]**

[Sol. Let $\vec{u} = x\hat{i}+\sqrt{3}x\hat{j}$; $\quad |\vec{u}| = 2x,\ x > 0$

now $\quad |\vec{u}|\left|\vec{u}-2\hat{i}\right| = \left|\vec{u}-\hat{i}\right|^2$

$$2\,|x|\,\sqrt{(x-2)^2+3x^2} = [(x-1)^2+3x^2]$$

$$2\,|x|\,\sqrt{4x^2-4x+4} = 4x^2-2x+1$$

$$4\,|x|\,\sqrt{x^2-x+1} = 4x^2-2x+1$$

square $\quad 16x^2(x^2-x+1) = 16x^4+4x^2+1-16x^3-4x+8x^2$

$$16x^2 = 12x^2+1-4x$$

$$4x^2+4x-1 = 0$$

$$x = \frac{-4\pm\sqrt{16+16}}{8} = \frac{-4\pm4\sqrt{2}}{8} = \frac{-1\pm\sqrt{2}}{2} \text{ or } \frac{-(1+\sqrt{2})}{2}$$

$$2x = \sqrt{2}-1 \quad \text{or} \quad -(\sqrt{2}+1) \rightarrow \text{rejected}$$

hence $\quad |\vec{u}| = \sqrt{2}-1 = \sqrt{2}-\sqrt{1} \quad \Rightarrow \quad a = 2;\ b = 1$

$$(a+b)^3+(a-b)^3 = 27+1 = 28$$

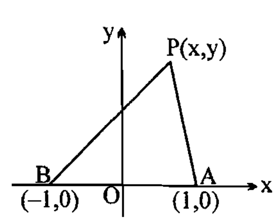
20 Given three points on the xy plane on O(0, 0), A(1, 0) and B(−1, 0). Point P is moving on the plane satisfying the condition $\left(\overrightarrow{PA}\cdot\overrightarrow{PB}\right)+3\left(\overrightarrow{OA}\cdot\overrightarrow{OB}\right) = 0$

If the maximum and minimum values of $\left|\overrightarrow{PA}\right|\left|\overrightarrow{PB}\right|$ are M and *m* respectively then find the value of $\dfrac{M^2+m^2}{17}$. [Ans. 2]

[Sol. Let P be (x, y)

$\overrightarrow{PA} = (1-x)\hat{i}-y\hat{j}$; $\quad \overrightarrow{PB} = (-1-x)\hat{i}-y\hat{j}$

$\therefore \quad \left(\overrightarrow{PA}\cdot\overrightarrow{PB}\right) = \left((x-1)\hat{i}+y\hat{j}\right)\cdot\left((x+1)\hat{i}+y\hat{j}\right) = (x^2-1)+y^2$



also $\quad 3\left(\overrightarrow{OA}\cdot\overrightarrow{OB}\right) = 3\hat{i}\cdot(-\hat{i}) = -3$

hence $\quad \left(\overrightarrow{PA}\cdot\overrightarrow{PB}\right)+3\left(\overrightarrow{OA}\cdot\overrightarrow{OB}\right) = 0$

$$x^2-1+y^2-3 = 0 = 0$$

$$x^2+y^2 = 4 \qquad \text{....(1)}$$

which gives the locus of P i.e. P move on a circle with centre (0, 0) and radius 2.

now $\quad \left|\overrightarrow{PA}\right|^2 = (x-1)^2+y^2$; $\left|\overrightarrow{PB}\right|^2 = (x+1)^2+y^2$

$\therefore \quad |\overrightarrow{PA}|^2 \, |\overrightarrow{PB}|^2 = (x^2 + y^2 - 2x + 1)(x^2 + y^2 + 2x + 1)$

$= (5 - 2x)(5 + 2x) \qquad$ [using $x^2 + y^2 = 4$]

$\therefore \quad |\overrightarrow{PA}|^2 \, |\overrightarrow{PB}|^2 = 25 - 4x^2 \qquad$ subject to $x^2 + y^2 = 4$

$\left. |\overrightarrow{PA}|^2 |\overrightarrow{PB}|^2 \right|_{min.} = 25 - 16 = 9$; (when $x = 2$ or -2)

and $\left. |\overrightarrow{PA}|^2 |\overrightarrow{PB}|^2 \right|_{max.} = 25 - 0 = 25$ (when $x = 0$)

$3 \le |\overrightarrow{PA}| \, |\overrightarrow{PB}| \le 5$

hence $M = 5$ and $m = 3 \quad \Rightarrow \quad M^2 + m^2 = 34$

(EX − 2(A))

33. and point satisfies the eqn of the plane

34. get plan equation of ABC by 3-point form

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

& get dist of D fro this plane

35. the vector eqns

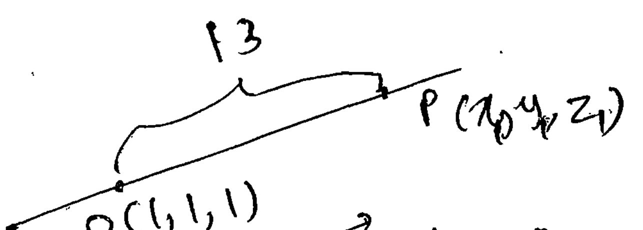
$$\vec{r} = \underbrace{2\hat{i} + 9\hat{j} + 13\hat{k}}_{\vec{a}_1} + \lambda \underbrace{(\hat{i} + 2\hat{j} + 3\hat{k})}_{\vec{b}_1}$$

$$\vec{r} = \underbrace{a\hat{i} + 7\hat{j} - 2\hat{k}}_{\vec{a}_2} + \mu \underbrace{(-\hat{i} + 2\hat{j} - 3\hat{k})}_{\vec{b}_2}$$

∴ two line intersect $\dfrac{(\vec{a}_1 - \vec{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} = 0$

36. angle betwee two plane is the ∠le between their normals

37.



OP = 13.

$\vec{}$ $3\hat{i} + 4\hat{j} + 12\hat{k}$

$$\frac{x-1}{3} = \frac{y-1}{4} = \frac{z-1}{12} = \lambda$$

∴ $x_1 = 3\lambda + 1$

$(x_1-1)^2+(y_1-1)^2+(z_1-1)^2 = 13^2$

$\therefore\ 9\lambda^2+16\lambda^2+144\lambda^2 = 13^2$

$\therefore\ \lambda = \pm 1$

$\Rightarrow (x, y, z) = (4, 5, 13)$

38. Let $\vec{n} = n_1\hat{i} + n_2\hat{j} + n_3\hat{k}$

$n_1x + n_2y + n_3z = q$

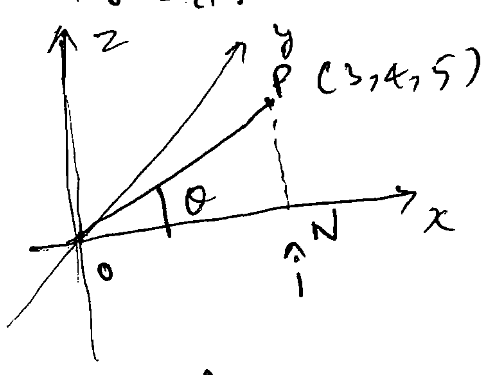
when it meets x-axis $y=0, z=0$.

$\therefore\ x = \dfrac{q}{n_1}$

$x = \dfrac{q}{\vec{n}\cdot\hat{i}}$

39 Equ of x-axis is

$\vec{r} = \lambda\hat{i}$



$\vec{OP} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$|OP\sin\theta| = PN$

$\therefore\ |\vec{OP}\times\hat{i}| = PN$

$|-4\hat{k} + 5\hat{j}| = PN$

40.

~~Observe that these are the ends of the~~

Solve for the lines

41. Observe that these are the ends of the body diagonal

42. Solve

43.

$\vec{r} \times \vec{p} = \vec{q} \times \vec{p}$

$\vec{r} \cdot \vec{s} = 0$

take dot product with $\vec{q}$

$\therefore \quad [\vec{r}\,\vec{p}\,\vec{q}] = 0$

$\therefore \quad \vec{r}, \vec{p}$ & $\vec{q}$ are coplanar.

$\Rightarrow \vec{r} = \alpha \vec{p} + \beta \vec{q}$

$\vec{r} \times \vec{p} = \vec{q} \times \vec{p}$

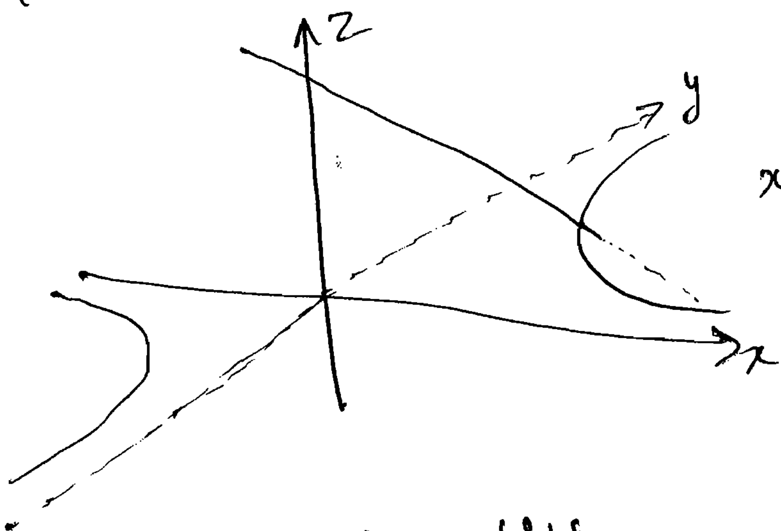
$\Rightarrow \beta = 1$

$\vec{r} \cdot \vec{s} = 0$

$\Rightarrow \quad \alpha\, \vec{p} \cdot \vec{s} + \vec{q} \cdot \vec{s} = 0$

$$\Rightarrow \alpha = \frac{-(\vec{q} \cdot \vec{s})}{(\vec{p} \cdot \vec{s})}$$

44.



$$\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1} = \lambda$$

$\therefore$ $x = 3\lambda + 2$

$y = 2\lambda - 1$

in xy plane $z = 0$

$\therefore \lambda = 1$

$\therefore$ $(3\lambda+2)(2\lambda-1) = c^2$

$\cancel{6\lambda^2 + \lambda - 2 - c^2 = 0}$

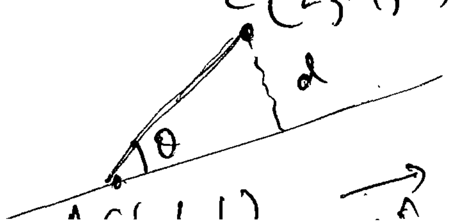
$\cancel{\lambda \in \mathbb{R}}$

$\therefore$ $\cancel{D \geq}$

$c^2 = 5$

$c = \pm\sqrt{5}$

46. Observe that A is a point on the given line



$$d = \frac{|\vec{AC} \times (6\hat{i} - 3\hat{j} + 2\hat{k})|}{|6\hat{i} - 3\hat{j} + 2\hat{k}|}$$

47.

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0.$$

$$a(0-c) - a(b-c) + c(c-0) = 0$$

$$-\cancel{ac} - ab + \cancel{ac} + c^2 = 0$$

$$c^2 = ab.$$

48. Direct formula.

$$\frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$$

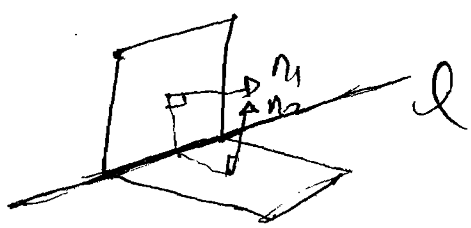
49. ~~[illegible]~~ straight forward

50.

(c)

$$\left| \frac{(\bar{a}_1 - \bar{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| \begin{cases} = 0 & \text{intersection} \\ \neq 0 & \text{non intersecting} \end{cases}$$

51. the line given with line of intersection of



$$3x - 2y + z = -3$$

$$4x - 3y + 4z = -1$$

$$\vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 4 & -3 & 4 \end{vmatrix}$$

$$= -5\hat{i} - 8\hat{j} - \hat{k}$$

$\vec{n_1} \times \vec{n_2}$ is the direction cosine of line of intersection

$\therefore (\vec{n_1} \times \vec{n_2}) \perp$ normal of the given plane.

$$\therefore (-5\hat{i} - 8\hat{j} - \hat{k}) \cdot (2\hat{i} - \hat{j} + m\hat{k}) = 0$$

$$-10 + 8 - m = 0$$

$$\therefore m = -2.$$

53. L is $(0, g, h)$

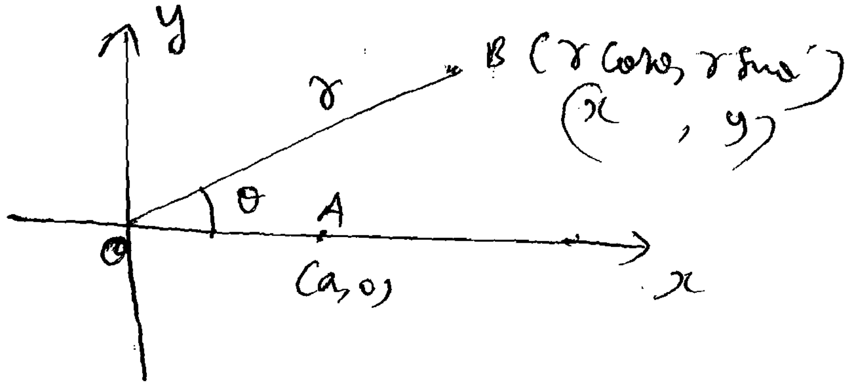
M is $(f, 0, h)$

O is $(0, 0, 0)$

$\therefore$ Plane is

$$\begin{vmatrix} x & y & z \\ 0 & g & h \\ \dots \end{vmatrix} = 0$$

54. let



$$\vec{OB} = r\cos\theta\,\hat{j} + r\sin\theta\,\hat{j}$$

$$\vec{OA} = a\hat{j}$$

$$\vec{OA} \times \vec{OB} = ar\sin\theta\,\hat{k} = \text{Constant} = c$$

$$\therefore \quad |r\sin\theta| = \frac{c}{a}$$

$$r^2\sin^2\theta = \frac{c^2}{a^2}$$

$$\therefore \quad y^2 = \frac{c^2}{a^2}$$

56. take

$$\cancel{\vec{x} = \alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}}$$

$$\vec{p} \times (\vec{x} \times \vec{p}) + \vec{q} \times (\vec{x} \times \vec{q}) + \vec{r} \times (\vec{x} \times \vec{r}).$$

$$= \vec{p} \times (\vec{p} \times$$

55. take $\vec{p} = \hat{i}$, $\vec{q} = \hat{j}$, $\vec{r} = \hat{k}$

& $\vec{x} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

56.

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} \qquad \& \ \vec{r_1} = \hat{i} - \hat{j}$$

$$= 5\hat{i} - 2\hat{j} - 3\hat{k}$$

$$\therefore \vec{r} \cdot (\vec{5} - 2\hat{j} - 3\hat{k}) = (\hat{i} - \hat{j}) \cdot (5\hat{i} - 2\hat{j} - 3\hat{k})$$

$$= 7.$$

57. Mid point satisfies the plane
& direction ~~cosines~~ Ratios of the line
joining the points is normal to the plane

58. straight forward.

~~59~~ 60. $|\vec{V}| = |\vec{r} \times \vec{\omega}|$

~~60~~ 61. Check all the Condition

62. let the plane make ~~xy~~ x, y & z intercepts
$(x_1, 0, 0)$ $(0, y_1, 0)$ & $(0, 0, z_1)$

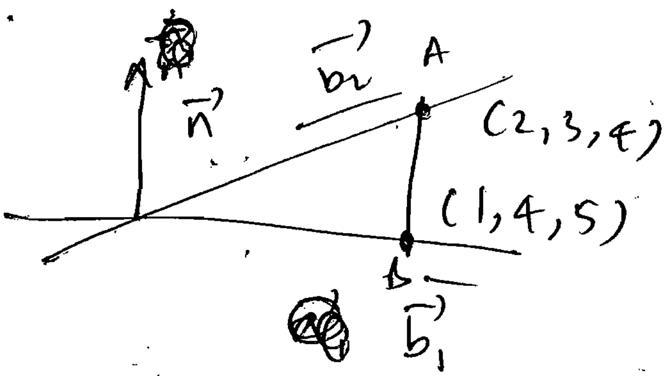
$$\therefore \frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1.$$

But Centroid $\left(\frac{x_1}{4}, \frac{y_1}{4}, \frac{z_1}{4}\right)$

$x \quad y \quad z$

$$\therefore \frac{\cancel{64}\, xyz}{6} = \cancel{64}k^3$$

63.



$$\vec{AB} = -\hat{i} + \hat{j} + \hat{k}$$

$$\vec{n} = \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix}$$

$$\vec{n} \cdot \vec{AB} = 0$$

$$\therefore \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$$-1(2k+1) - 1(1+k^2) + (2-k) = 0$$

64. get the Projection of $\vec{c}$ on the unit vector of the plane formed by $\vec{A}$ & $\vec{B}$

65. $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

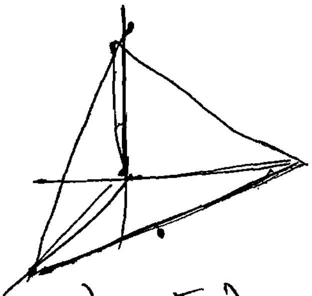
$(1+2\lambda)x + (-2+\lambda)y + (1-2\lambda)z$

$+ 2-3\lambda = 0$

$1+\cancel{2\lambda} + 4-2\lambda + \cancel{1-2\lambda} = 0$

$\Rightarrow 2\lambda = 6 \Rightarrow \lambda = 3$

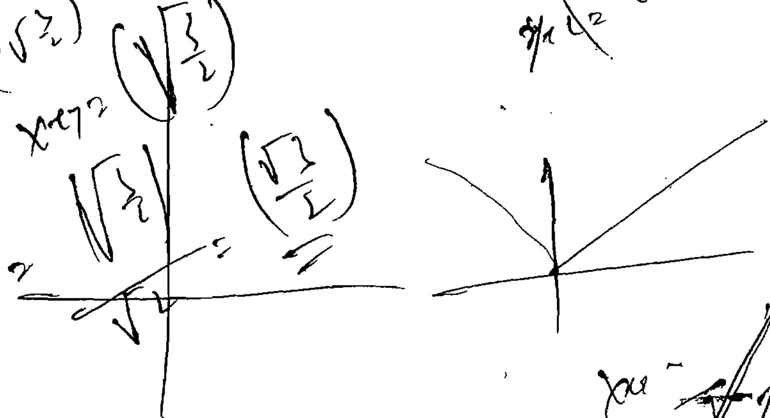
$7x + y - 5z = 7$

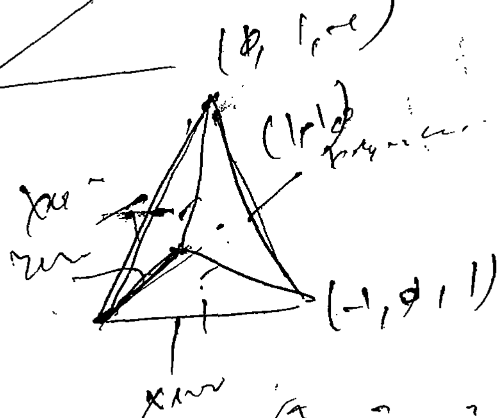


$x+y = x$

$\sqrt{\frac{2}{3}}, \quad x+y+z = \frac{\sqrt{6}}{2}$

$= \sqrt{\frac{3}{2}}$





a.

$x+y+\lambda_1(y+z)$

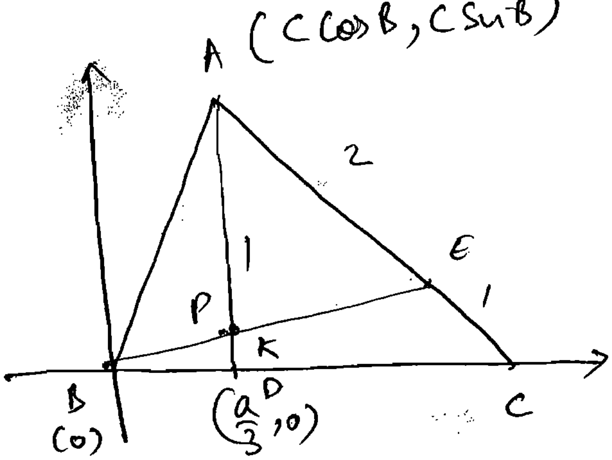
$x+z+\lambda_2(x+y+z\sqrt{6}) = 0$

$a(i-j) \quad (i+j-k)$

EX-2B.

1, 2. are standard results

3.



$\vec{BA} = c\cos B\,\hat{i} + c\sin B\,\hat{j}$

$\vec{BC} = a\hat{i}$

$\vec{BD} = \frac{a}{3}\hat{i}$

$$\vec{BE} = \frac{c\cos B\,\hat{i} + c\sin B\,\hat{j} + 2a\hat{i}}{3}$$

$$\vec{BP} = \frac{k[c\cos B\,\hat{i} + c\sin B\,\hat{j}] + \frac{a}{3}\hat{i}}{k+1}$$

$\vec{BP} \times \vec{BE} = 0.$

$$\therefore [(2a + c\cos B)\hat{i} + c\sin B\,\hat{j}] \times [(3kc\cos B + a)\hat{i} + 3kc\sin B\,\hat{j}] = 0.$$

$$3kc\sin B(2a + c\cos B) - c\sin B(3kc\cos B + a) = 0.$$

$$6\cancel{ack\sin B} - \cancel{ac\sin B} = 0.$$

$$k = \frac{1}{6}$$

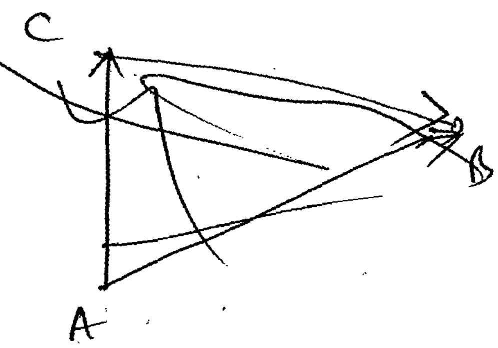
4. $\vec{a} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$

$\vec{b} = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$

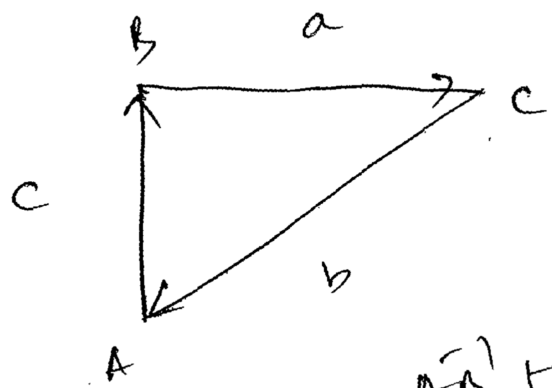
$\vec{a}\cdot\vec{b} = |a|\,|b|\cos(\alpha-\beta)$

~~$\Rightarrow \cos$~~

~~5.~~



5.



$\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$

$\therefore\ \vec{AB} + \vec{CA} = -\vec{BC}$

$(\vec{AB} + \vec{CA})^2 = (-\vec{BC})^2$

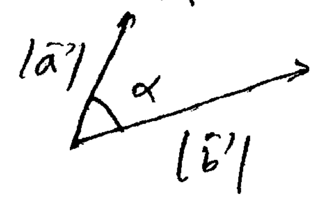
$\vec{AB}^2 + \vec{CA}^2 + 2\vec{AB}\cdot\vec{CA} = \vec{BC}^2$

$c^2 + b^2 + 2bc\cos(\pi - A) = a^2$

7.

$$\cancel{|\vec a|}|\vec b|\cos\alpha = \cancel{|\vec a|}|\vec c|\cos\beta \Rightarrow \frac{|\vec b|}{|\vec c|} = \frac{\cos\beta}{\cos\alpha}$$

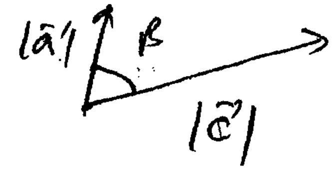
$$|\vec a||\vec b|\sin\alpha\,\hat n = |\vec a||\vec c|\sin\beta\,\hat n$$



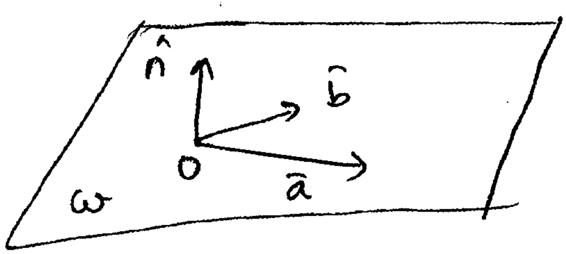
$$\cos\beta\sin\alpha\,\hat n = \sin\beta\cos\alpha\,\hat n$$

$$\Rightarrow \sin(\alpha-\beta)\,\hat n = 0.$$

$$\Rightarrow \alpha = \beta.$$



10.



$$\hat n = \frac{\vec a \times \vec b}{|\vec a \times \vec b|}$$

$$\vec n = \vec a \times \vec b = \begin{vmatrix} \hat i & \hat j & \hat k \\ 2 & 3 & -1 \\ 1 & 2 & 0 \end{vmatrix}$$

$$= +2\hat i + \hat j + \hat k$$

$$\vec n \times \vec a = \begin{vmatrix} \hat i & \hat j & \hat k \\ 2 & 1 & 1 \\ 2 & 3 & -1 \end{vmatrix}$$

11. $\theta_1, \theta_2, \theta_3$ are ∠le made with the co-ordinate plane.

$\frac{\pi}{2}-\theta_1, \frac{\pi}{2}-\theta_2, \frac{\pi}{2}-\theta_3$ " " axis

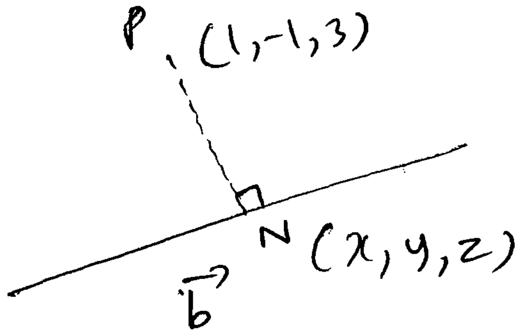
$$\therefore \cos^2\left(\frac{\pi}{2}-\theta_1\right) + \cos^2\left(\frac{\pi}{2}-\theta_2\right) + \cos^2\left(\frac{\pi}{2}-\theta_3\right) = 1$$

12. take $\vec{n_1} \times \vec{n_2}$

13. $$\frac{(a_1-a_2) \times (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}$$

14. take co-ordinates and solve

15.



$\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{PN} \cdot \vec{b} = 0$

$\therefore (x-1) + 2(y+1) + (z-3) = 0$

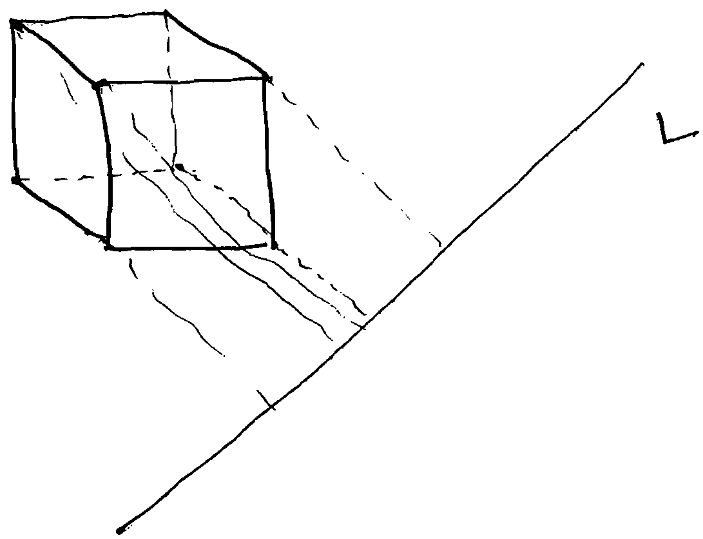
& also

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z+1}{1}$$

} solve

16. Refer to Ex 3(A) 15

17.



Prove it for a square & then proceed.

Ex - 3A

1. Observe that they are non-coplanar hence linearly independent

2. Diagonals of a $\parallel^{gm}$ bisect each other.

3. ~~Straight forward~~

$$\vec{OG} = \frac{\vec{a_1} + \vec{a_2} + \cdots + \vec{a_n}}{n}$$

where $\vec{OA_i} = \vec{a_i}$

$$\vec{GA_i} = \vec{OA_i} - \vec{OG}$$

$$= \vec{a_i} - \vec{OG}$$

$$\Sigma \vec{GA_i} = \Sigma \vec{a_i} - n\vec{OG}$$

$$= 0.$$

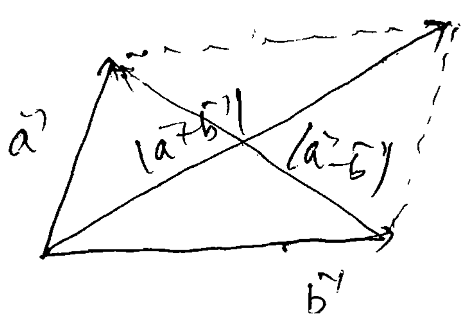
4. $$(\bar{e_1} + \bar{e_2} + \bar{e_3} + \cdots + \bar{e_n})^2 = 0.$$

$$\Rightarrow \quad n + 2\left(\sum_{1 \le i < j \le n} e_i e_j\right) = 0.$$

5. Put $\vec{c} = \frac{-\vec{a} - 2\vec{b}}{3}$

& proceed.

6.



hence above

or Squaring we get

$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$

7. $\vec{d} = (\vec{a}\cdot\vec{b})\vec{c} - (\vec{a}\cdot\vec{c})\vec{b}$

$\vec{a}\cdot\vec{d} = (\vec{a}\cdot\vec{b})(\vec{c}\cdot\vec{a}) - (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{a})$

$= 0.$

$\therefore \vec{a} \perp \vec{d}$

8. Straight forward

9. $\vec{e_3} = x\vec{e_1} + y\vec{e_2} + z(\vec{e_1}\times\vec{e_2})$

$\vec{e_3}\cdot\vec{e_1} = x = \cos\theta$

$\vec{e_3}\cdot\vec{e_2} = y = \cos\theta$

$|\vec{e_3}|^2 = 1 = x^2+y^2+z^2$

$\therefore z^2 = 1 - 2\cos^2\theta = 1 - 2yz$

10. $\dfrac{|\vec{b}\times\vec{c}|}{|\vec{b}||\vec{c}|} = \dfrac{\sqrt{15}}{4\cdot 1} = \sin\theta$

$\therefore \cos\theta = \dfrac{1}{4}$

$\gamma\vec{a} = \vec{b} - 2\vec{c}$

$\gamma^2\vec{a}^2 = \vec{b}^2 + 4\vec{c}^2 - 4\vec{b}\cdot\vec{c}$

$\gamma^2 = 16 + 4 - 4\times4\times1\cos\theta$

$\gamma^2 = 16$

12. $6x = 3y = 2z$

$$\Rightarrow x = \frac{y}{2} = \frac{z}{3} \qquad x-1 = \frac{y-2}{2} = \frac{z-3}{3}$$

$$\Rightarrow l_1 \parallel l_2$$

13. line of intersection is $\parallel$el to $\vec{n_1} \times \vec{n_2}$

$$\vec{n_1} = 3\hat{i} - \hat{j} + \hat{k}$$

$$\vec{n_2} = \hat{i} + 4\hat{j} - 2\hat{k}$$

14. $\Sigma \cos 2\theta_1 = 2\left(\Sigma \cos^2\theta_1\right) - 3 = -1.$

15. $\vec{r_1} \cdot \vec{r_2} = ab + bc + ca$

$$a^2+b^2+c^2 \geq ab+bc+ca$$

$$\Rightarrow \frac{\vec{r_1} \cdot \vec{r_2}}{|\vec{r_1}||\vec{r_2}|} = \frac{ab+bc+ca}{(a^2+b^2+c^2)} = \cos\theta.$$

$\cos\theta \leq 1$

But $a^2+b^2+c^2 \geq -2(ab+bc+ca)$

$$\Rightarrow \frac{ab+bc+ca}{a^2+b^2+c^2} \geq \frac{-1}{2}.$$

$$\therefore \theta \in \left[0, \frac{2\pi}{3}\right]$$

16.

$$\vec{n} = \vec{b} - \vec{a}$$

$$\vec{r_1} = \frac{\vec{a} + \vec{b}}{2}$$

$$\therefore \vec{r} \cdot \vec{n} = \vec{r_1} \cdot \vec{n}$$

$$\vec{r} \cdot (\vec{b} - \vec{a}) = \frac{|\vec{b}|^2 - |\vec{a}|^2}{2}.$$

18 eqn of the line

$$\underbrace{\frac{x-3}{k-2} = \frac{y-1}{-2-1}}_{\text{I}} = \frac{z-k}{1-k} \quad \text{II}$$

Say if $(-4, 2, 5)$ lies on the line

$$\text{I} \Rightarrow \quad \frac{-7}{k-2} = \frac{2-1}{-3}$$

$$21 = k - 2$$

$$k = 23$$

$$\frac{1}{-3} = \frac{5-k}{1-k}$$

$$\text{II} \Rightarrow \quad 1 - k = -15 + 3k$$

$$k = 4$$

19. $x = t+1$

$y = t-3$

$z = t\sqrt{2}+4.$

$\therefore$ line is

$$\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-4}{\sqrt{2}}$$

21

$\cancel{\cos^2 45^\circ + \cos^2 30^\circ + \cos^2 \gamma = 1}$

angle made by L with x-axis $= 45^\circ$

" " " " " y-axis $= 90^\circ - 30^\circ = 60^\circ$

" " " " " z-axis $= \theta$.

$\cos^2 45^\circ + \cos^2 60^\circ + \cos^2\theta = 1$ [sum of the squares of direction cosines]

$\frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = \frac{1}{4}$

$\therefore \cos\theta = \frac{1}{2}$ [θ is acute]

$\theta = 60^\circ$

$\therefore$ angle made with x-y plane $= 30^\circ$ [$90^\circ - 60^\circ$]

22. $\vec{b} = \hat{i}$ $\quad \vec{a} = a_1\hat{i} + \ldots \hat{j}$

$\vec{c} = \ldots \hat{j}$

23. $\hat{e} = \hat{i}$ or $\hat{j}$

~~24. normal to the plane $= \vec{AB} \times \vec{AC}$~~

~~$= (\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})$~~

~~$= \vec{b} \times \vec{c} - \vec{a} \times \vec{b}$~~

24. normal to the plane of ΔABC $= \vec{n_1} = (\vec{b} - \vec{a}) \times (\vec{a} - \vec{c})$

$= -\vec{n}$

$\therefore$ equation of the plane.

$$\vec{r} \cdot \vec{n_1} = g \cdot \vec{n_1}$$

25. let the centre be (r, r, r)

$\therefore$ Equation of the sphere

$$(x-r)^2 + (y-r)^2 + (z-r)^2 = r^2$$

$(1, 2, 0)$ satisfies it

$$(r-1)^2 + (2-r)^2 + \cancel{r^2} = \cancel{r^2}$$

$$2r^2 - 6r + 5 = 0$$

r_1 & r ...

25.

$$\Delta = \frac{1}{2}\left[a\vec{b} + b\vec{a}\right] \times \left[a\vec{b} - b\vec{a}\right]$$

$$= \frac{1}{2}\left[-ab\,\vec{b}\times\vec{a} + ab\,\vec{a}\times\vec{b}\right]$$

$$= ab\,\vec{a}\times\vec{b}$$

$$= a^2b^2\frac{\sqrt{3}}{2}.$$

9. EX-3(B)

let the P.Vs of A, B, C, D be $\vec{a}, \vec{b}, \vec{c}, \vec{d}$.

$$\therefore \quad \vec{AB}^2 + \vec{BC}^2 + \vec{CD}^2 + \vec{DA}^2 - \vec{AC}^2 - \vec{BD}^2$$

$$= (\vec{b}-\vec{a})^2 + (\vec{c}-\vec{b})^2 + (\vec{d}-\vec{c})^2 + (\vec{d}-\vec{a})^2 - (\vec{c}-\vec{a})^2 - (\vec{b}-\vec{d})^2$$

this simplifies to.

$$= \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + \vec{d}^2 + 2\vec{a}\cdot\vec{c} + 2\vec{b}\cdot\vec{d} - 2\vec{a}\cdot\vec{b} - 2\vec{b}\cdot\vec{c} - 2\vec{c}\cdot\vec{d} - 2\vec{a}\cdot\vec{d}$$

~~thus~~

$$= \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + \vec{d}^2 - 2(\vec{a}-\vec{d})(\vec{b}-\vec{c}) - \cancel{2}\,\vec{b}\cdot\vec{c} - 2\vec{a}\cdot\vec{d}$$

$$= \left[(\vec{b}-\vec{c}) - (\vec{a}-\vec{d})\right]^2 \geqslant 0. \quad -(1)$$

Equality iff

$$\vec{b}-\vec{c} = \vec{a}-\vec{d}$$

$$\Rightarrow BC \parallel AD \quad -(2)$$

(1) Can be further rearranged as

$$\left[(\vec{b}-\vec{a}) - (\vec{d}-\vec{c})\right]^2 \geqslant 0$$

Equality iff

$$\vec{b}-\vec{a} = \vec{d}-\vec{c}$$

$$\Rightarrow AB \parallel CD$$

19. $\because$ $\vec{a}, \vec{b}$ & $\vec{c}$ are coplanar.

$\vec{c}$ can be written as a linear combinati[on of] $\vec{a}$ & $\vec{b}$.

$$\therefore \quad \vec{c} = \alpha\vec{a} + \beta\vec{b} \qquad \alpha, \beta \text{ are scalars.}$$

$$\therefore \quad \Delta = \begin{vmatrix} \vec{a} & \vec{b} & \alpha\vec{a} + \beta\vec{b} \\ \vec{a}\cdot\vec{b} & \vec{b}^2 & \alpha\vec{a}\cdot\vec{b} + \beta\vec{b}^2 \\ \alpha\vec{a}^2 + \beta\vec{a}\cdot\vec{b} + \alpha\vec{a} & & \end{vmatrix}$$

$$\Delta = \begin{vmatrix} \vec{a} & \vec{b} & \alpha\vec{a} + \beta\vec{b} \\ \vec{a}\cdot b & \vec{b}^2 & \alpha\vec{a}\cdot\vec{b} + \beta\vec{b}^2 \\ \alpha\vec{a}^2 + \beta\vec{a}\cdot\vec{b} & \alpha\vec{a}\cdot\vec{b} + \beta\vec{b}^2 & (\alpha\vec{a} + \beta\vec{b})^2 \end{vmatrix}$$

$$C_3 \longrightarrow C_3 - \alpha C_1 - \beta C_2$$

$$\Delta = \begin{vmatrix} \cdots & 0 \\ \cdots & 0 \\ & 0 \end{vmatrix} = 0.$$

20 let

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

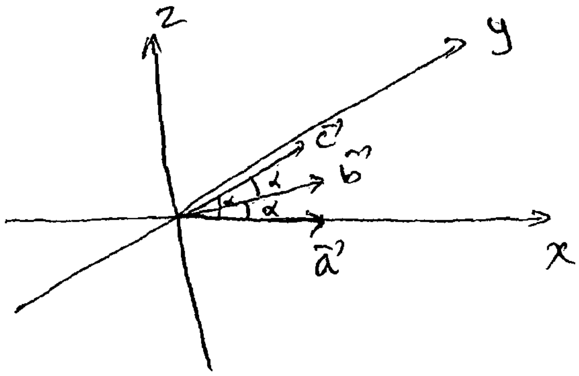
$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$[\vec{a}\,\vec{b}\,\vec{c}]^2 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \times \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} (a_1^2+a_2^2+a_3^2) & (a_1b_1+a_2b_2+a_3b_3) & (a_1c_1+a_2c_2+a_3c_3) \\ (b_1a_1+b_2a_2+b_3a_3) & (b_1^2+b_2^2+b_3^2) & (b_1c_1+b_2c_2+b_3c_3) \\ (c_1a_1+c_2a_2+c_3a_3) & (c_1b_1+c_2b_2+c_3b_3) & (c_1^2+c_2^2+c_3^2) \end{vmatrix}$$

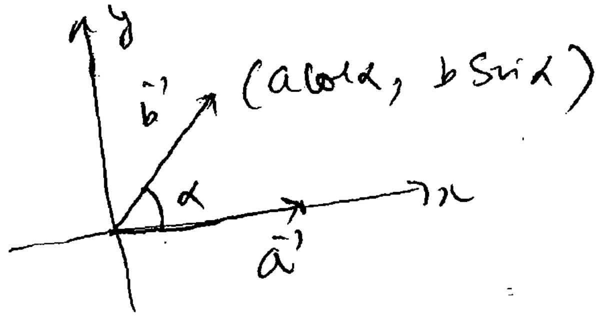
$= R.H.S.$

1.



take $\vec{a} = \hat{i}$

$\vec{b} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$



take

$\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{c}\cdot\vec{a} = \cos\alpha$

$\therefore \quad x = \cos\alpha$

$\vec{c}\cdot\vec{b} = x\cos\alpha + y\sin\alpha = \cos\alpha$

$\therefore \quad y = \dfrac{\cos\alpha - \cos^2\alpha}{\sin\alpha} \qquad y = \cos\alpha\,\dfrac{\cancel{2}\sin^2\frac{\alpha}{2}}{\cancel{2}\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$

$|\vec{c}| = 1 \qquad \therefore y = \cos\alpha\cot\frac{\alpha}{2}$

$\Rightarrow \quad x^2 + y^2 + z^2 = 1$

$\therefore \quad z^2 = 1 - \cos^2\alpha - \cos^2\alpha\cot^2\frac{\alpha}{2}$

$= 1 - \cos^2\alpha\,\operatorname{cosec}^2\frac{\alpha}{2}$

Volume of parallelopiped $= [\vec{a}\,\vec{b}\,\vec{c}]$

$= \begin{vmatrix} 1 & 0 & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix}$

25. $\vec{a}\times(\vec{a}\times\vec{c})+\vec{b}=\vec{0}$ $\quad |\vec{a}|=|\vec{b}|=1$

$(\vec{a}\cdot\vec{c})\vec{a}-(\vec{a}\cdot\vec{a})\vec{c}+\vec{b}=\vec{0}$ $\quad |\vec{c}|=2.$

let θ be the $\angle$le between $\vec{a}$ & $\vec{c}$

$2\cos\theta\,\vec{a}-\vec{c}+\vec{b}=0$

$(\vec{b})^2=1.$

$\Rightarrow \vec{c}^{\,2}+4\cos^2\theta\,\vec{a}^{\,2}-4\cos\theta.\,\vec{a}\cdot\vec{c}=(-\vec{b})^2$

$4+4\cos^2\theta-8\cos^2\theta=1$

$\Rightarrow \cos^2\theta=\frac{3}{4}.$

26. take $\vec{p}=a\hat{i}$, $\vec{q}=a\hat{j}$, $\vec{r}=a\hat{k}$

& proceed.

27. $\lambda\vec{x}+\vec{x}\times\vec{a}=\vec{b}$ $\quad -①$

take dot product with $\vec{a}$

$\lambda\vec{x}\cdot\vec{a}=\vec{a}\cdot\vec{b}$ $\quad -②$

Postmultiplying ① by $\vec{a}$

$\lambda(\vec{x}\times\vec{a})+(\vec{x}\times\vec{a})\times\vec{a}=\vec{b}\times\vec{a}$ $\quad -③$

$\lambda(\vec{x}\times\vec{a})+(\vec{x}\cdot\vec{a})\vec{a}-\vec{x}.|\vec{a}|^2=\vec{b}\times\vec{a}$

$= 2 \sin\alpha = \frac{1}{\sqrt{2}}$

$2^2 \sin^2\alpha = \frac{1}{2}$

$\sin^2\alpha - \sin^2\alpha \cot^2\alpha \operatorname{cosec}^2\frac{\alpha}{2} = \frac{1}{2}$

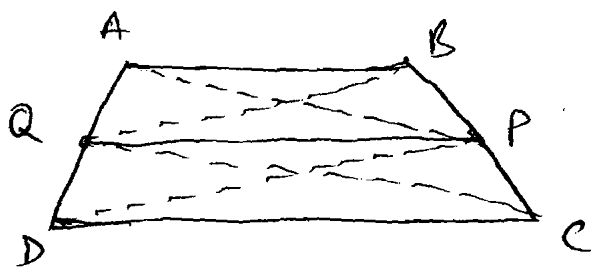
Solve for α.

23. observe that $\rightarrow$ $(\vec{b}' \times \vec{c}') \times \vec{a}' + (\vec{c}' \times \vec{a}') \times \vec{b}' + (\vec{a}' \times \vec{b}') \times \vec{c}' = 0$

[Jacobi].

$\Rightarrow$ the 3 vectors are coplanar as there is linear combination

24.



using similarity we can prove that

$PQ \parallel AB$ [B.P.T]

$\therefore$ area of ΔPDC = area of ΔQCD.

[Δ^{le} with common base & equal hieght]

area of ΔAPB = area of ΔAQB

$\therefore \Delta ABQ + \Delta QCD = \Delta ABP + \Delta PDC$

from ① $\vec{x} \times \vec{a} = \vec{b} - \lambda \vec{x}$

Substitute this in ③.

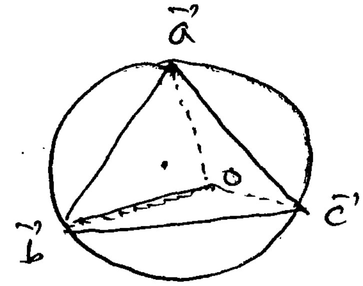
$$\lambda \vec{b} - \lambda^2 \vec{x} + \frac{(\vec{a}\cdot\vec{b})}{\lambda}\vec{a} - \vec{x}|\vec{a}|^2 = \vec{b} \times \vec{a}$$

$$\therefore \lambda \vec{b} + \frac{(\vec{a}\cdot\vec{b})}{\lambda}\vec{a} - \vec{b}\times\vec{a} = \vec{x}(\lambda^2 + |a|^2)$$

28. take

$$\vec{d} = x(\vec{b}\times\vec{c}) + y(\vec{c}\times\vec{a}) + z(\vec{a}\times\vec{b})$$

[$\because$ $\vec{a}$, $\vec{b}$, $\vec{c}$ are non coplanar we can take $\vec{d}$ as above]



here $\vec{d}$ is the P.V of the Centre of the Circumsphere

$$(\vec{d} - \vec{a})^2 = R^2$$

$$(\vec{d} - 0)^2 = R^2$$

$\therefore$ equating

$$|\vec{a}|^2 - 2\vec{a}\cdot\vec{d} = 0$$

$$|\vec{a}|^2 = 2\vec{a}\cdot\vec{d}$$

|||^{ly}

$$\vec{b}\cdot\vec{d} = \frac{|\vec{b}|^2}{2}$$

$$\vec{c}\cdot\vec{d} = \frac{|\vec{c}|^2}{2}$$

But $\vec{a}\cdot\vec{d} = x\,[\vec{a}\,\vec{b}\,\vec{c}]$

$$\therefore \quad x = \frac{|\vec{a}|^2}{2[\vec{a}\,\vec{b}\,\vec{c}]}$$

|||^{ly} $$y = \frac{|\vec{b}|^2}{2[\vec{a}\,\vec{b}\,\vec{c}]}$$

$$z = \frac{|\vec{c}|^2}{2[\vec{a}\,\vec{b}\,\vec{c}]}$$

~~hence to~~ put values of x, y, z and get $\vec{d}$.

$$\vec{d} = \frac{|\vec{a}|^2(\vec{b}\times\vec{c}) + |\vec{b}|^2(\vec{c}\times\vec{a}) + |\vec{c}|^2(\vec{a}\times\vec{b})}{2[\vec{a}\,\vec{b}\,\vec{c}]}.$$

~~29. Straight forward~~

~~32~~ 29. take

$\vec{a} = a\hat{i}$

$\vec{b} = b\cos\theta_1\hat{i} + b\sin\theta_1\hat{j}$

$\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$$y = \frac{c\cos\theta_2 - c\cos\theta_1\cos\theta_3}{\sin\theta_1}$$

$$x^2+y^2+z^2 = c^2$$

$$z^2 = c^2-x^2-y^2$$

volume of the ~~te~~tetrahedron

$$V = \frac{1}{6} \times [\vec{a}\,\vec{b}\,\vec{c}]$$

$$[\bar{a}\,\bar{b}\,\bar{c}] = abz\sin\theta_1$$

$$V^2 = \frac{1}{36} \times a^2b^2\sin^2\theta_1\,[c^2-x^2-y^2]$$

$$= \frac{1}{36} \times a^2b^2\left[c^2 - c^2\cos^2\theta_3 - \frac{(c\cos\theta_2 - c\cos\theta_1\cos\theta_3)^2}{\sin^2\theta_1}\right] \times \sin^2\theta_1$$

$$= \frac{1}{36}a^2b^2c^2\left[\sin^2\theta_1\cdot\sin^2\theta_3 - (\cos\theta_2-\cos\theta_1\cos\theta_3)^2\right]$$

$= R.H.S$ [~~Confirm it by~~

[one can observe by opening the determinant]

30. $(\vec{c}\cdot\vec{c})\vec{a} = \vec{c}$

Taking dot product with $\vec{c}$

$\cancel{(\vec{c}\cdot\vec{c})}\, \vec{a}\cdot\vec{c} = \cancel{(\vec{c}\cdot\vec{c})}$

$\vec{a}\cdot\vec{c} = 1.$

$\vec{a}\times(\vec{b}\times\vec{c}) + (\vec{a}\cdot\vec{b})\vec{b}$

$= (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} + (\vec{a}\cdot\vec{b})\vec{b}$

$= [1+(\vec{a}\cdot\vec{b})]\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$= (4-2\alpha-\sin\beta)\vec{b} + (\alpha^2-1)\vec{c}$

$\because$ $\vec{b}$ & $\vec{c}$ are non-collinear

we can equate

$4-2\alpha-\sin\beta = 1+\vec{a}\cdot\vec{b}$

$\alpha^2-1 = -\vec{a}\cdot\vec{b}$

adding

$\alpha^2-2\alpha+2-\sin\beta = 0$

$\Rightarrow \sin\beta = 1+(\alpha-1)^2$

31.

$\vec{a} \times \vec{c} = \vec{b} \times \vec{d} \quad -①$

$\vec{a} \times \vec{b} = \vec{c} \times \vec{d} \quad -②$

① - ②

$\vec{a} \times (\vec{c} - \vec{b}) = (\vec{b} - \vec{c}) \times \vec{d}$

$\vec{a} \times (\vec{c} - \vec{b}) = \vec{d} \times (\vec{c} - \vec{b})$

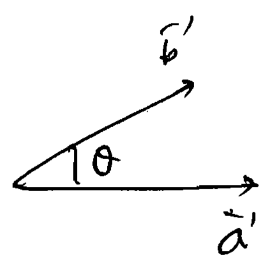
$(\vec{a} - \vec{d}) \times (\vec{c} - \vec{b}) = 0.$

$\Rightarrow (\vec{a} - \vec{d}) \parallel (\vec{c} - \vec{b})$

$\Rightarrow (\vec{a} - \vec{d}) \cdot (\vec{c} - \vec{b}) \neq 0.$

32.

$\vec{a} + \vec{b} = \sqrt{3}\,\vec{c}$



Squari

$\vec{a}^2 + \vec{b}^2 + 2\vec{a} \cdot \vec{b} = 3\vec{c}^2$

$1 + 1 + 2\cos\theta = 3$

$\cos\theta = \frac{1}{2}$

34.

$\vec{a} = cx\hat{i} - 6\hat{j} + 3\hat{k}$

$\vec{b} = x\hat{i} + 2\hat{j} + 2cx\hat{k}$

$\vec{a} \cdot \vec{b} < 0$ $\because$ $\vec{a}$ & $\vec{b}$ make obtuse angle $\forall x \in R$.

$cx^2 + 6cx - 12 < 0 \quad \forall x \in R.$

$c < 0$

& $36c^2 + 48c < 0$.

$3c^2 + 4c < 0$

$c \in \left(-\frac{4}{3}, 0\right)$.

35.

$\vec{c} = \hat{i}$

$\vec{b} = \cos\frac{\pi}{3}\hat{i} + \sin\frac{\pi}{3}\hat{j}$

$\vec{b} = \frac{\hat{i}}{2} + \frac{\sqrt{3}}{2}\hat{j}$

$\vec{a} = \hat{k}$

36. a, b, c are p^{th}, q^{th} & r^{th} term of a G.P.

then log a, log b, log c are p^{th}, q^{th} & r^{th} term of an A.P.

take dot product and we get its zero.

we get

$\therefore$ the vectors are $\perp^r$.

37. $\vec{a} \times \vec{x} = \vec{b}$

$\vec{a} \times (\vec{a} \times \vec{x}) = \vec{a} \times \vec{b}$

$(\vec{a} \cdot \vec{x})\vec{a} - |\vec{a}|^2 \vec{x} = \vec{a} \times \vec{b}$

$\vec{a} \cdot \vec{x} = c$.

$\therefore \quad c\vec{a} - |\vec{a}|^2 \vec{x} = \vec{a} \times \vec{b}$

$$\Rightarrow \vec{x} = \frac{c\vec{a} - \vec{a} \times \vec{b}}{|\vec{a}|^2}$$

38. Let $\hat{a} = \hat{i}$
$\hat{b} = \hat{j}$ for convenience.
$\vec{a} \times \vec{b} = \hat{k}$

Let the required unit vector be

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{r}|^2 = 1 \quad \Rightarrow x^2 + y^2 + z^2 = 1$

Let $\vec{r}$ make an angle 'θ' with the $\hat{i}, \hat{j}$ & $\hat{k}$

$\therefore \quad \vec{r} \cdot \hat{i} = \cos\theta = \vec{r} \cdot \hat{j} = \vec{r} \cdot \hat{k}$

$\therefore \cancel{\vec{r} = \cos\theta}$ $\quad \therefore x = \cos\theta$

$$x^2+y^2+z^2=1$$

$$\Rightarrow \cos^2\theta = \frac{1}{3}$$

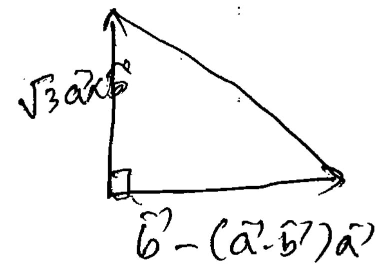
$$\therefore \cos\theta = \pm\frac{1}{\sqrt{3}}$$

$$\therefore \vec{r} = \pm\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$$

39. Straight forward.

40.
$$\vec{r}_1 = \sqrt{3}(\vec{a}\times\vec{b})$$

$$\vec{r}_2 = \vec{b} - (\vec{a}\cdot\vec{b})\vec{a}$$



$(\vec{a}\times\vec{b})$ is $\perp^r$ to both $\vec{a}$ & $\vec{b}$

$$\Rightarrow \vec{r}_1\cdot\vec{r}_2 = 0.$$

hence angle between $\vec{r}_1$ & $\vec{r}_2 = \pm\pi/2$

41. ~~Equation of cv_1 is $z=0$.~~

Apply Pythagoras & proceed.

~~vector normal to cv_2 is~~

$$= (\hat{i}-\hat{j})\times(\hat{j}+\hat{k})$$

$$= \hat{k}-\hat{j}-\hat{i}$$

$$= -\hat{i}-\hat{j}+\hat{k}.$$

42. Equation of the plane contain BCD

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1-1 & -2-1 & 3-1 \\ -3-1 & 1-1 & 2-1 \end{vmatrix} = 0$$

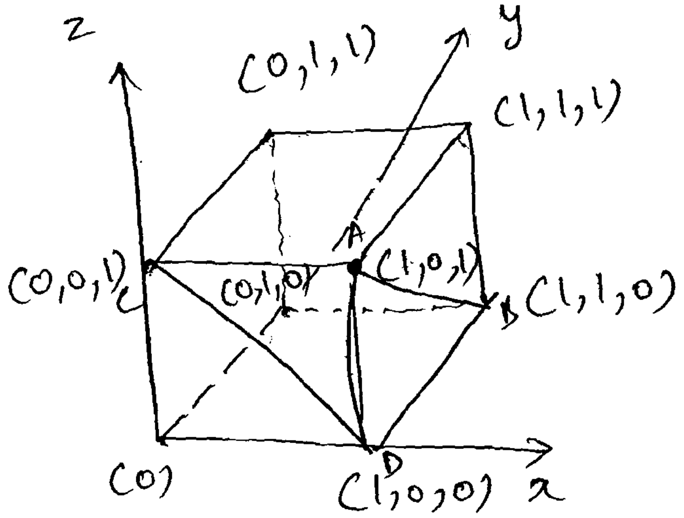
then apply distance of a point from a plane formula.

43. area of $\triangle^{le}$ with P.V of the vertices $\vec{a}, \vec{b}, \vec{c}$

is $= \frac{1}{2}|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}| = \frac{16\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$

$a^2 = 32 \qquad a = 4\sqrt{2}$

44.



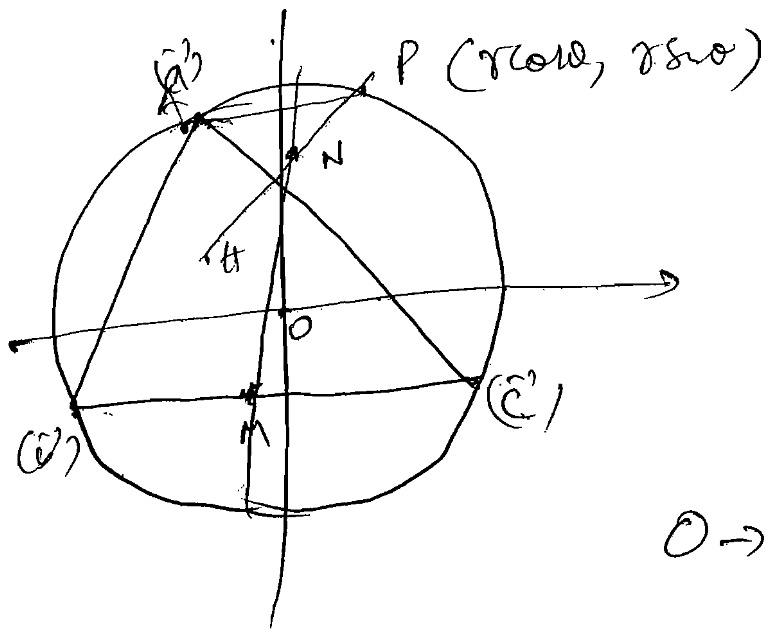
$\vec{AB} = (\hat{i}+\hat{j}) - (\hat{i}+\hat{k})$

$\vec{CD} = \hat{i} - \hat{k}$

angle between $\vec{AB} \times \vec{CD}$

$\cos\theta = \left| \frac{\vec{AB}\cdot\vec{CD}}{|\vec{AB}|\,|\vec{CD}|} \right|$

$= \frac{1}{}$



r = circumradius

O → circumcentre.

$\vec{OH} = \vec{a} + \vec{b} + \vec{c}$ [By Euler's line].

$\vec{OP} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$

$\vec{OM} = \dfrac{\vec{b} + \vec{c}}{2}$

$\vec{ON} = \dfrac{\vec{OH} + \vec{OP}}{2}$

$= \dfrac{\vec{a} + \vec{b} + \vec{c} + r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}}{2}$

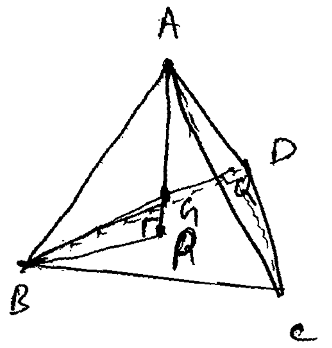
~~MN~~ $\vec{MN} = \dfrac{\vec{a} + r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}}{2}$

$\cancel{= \dfrac{(r\cos\theta + r\cos A)\hat{i} + (r\sin\theta + r\sin A)}{2}}$

$\vec{MN} = \dfrac{\vec{OA} + \vec{OP}}{2}$

$\vec{AP} = \vec{OP} - \vec{OA}$

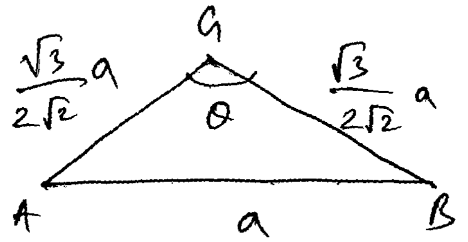
46.



$$BP = \frac{2}{3}\cdot\frac{\sqrt{3}}{2}a = \frac{a}{\sqrt{3}}$$

$$AP = \sqrt{a^2 - \frac{a^2}{3}} = \sqrt{\frac{2}{3}}\,a.$$

$$AG = \frac{3}{4}\cdot AP = \frac{\sqrt{3}}{2\sqrt{2}}\,a.$$



$$\cos\theta = \frac{\frac{3}{8}a^2 + \frac{3a^2}{8} - a^2}{2\times\frac{3a^2}{8}}$$

$$\cos\theta = -\frac{1}{3}$$

47.

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = |a|\,|b|\cdot\sin\frac{\pi}{6}\,\hat{n}$$

$$\hat{c} = \pm\hat{n} = c_1\hat{i} + c_2\hat{j} + c_2\hat{k}$$

$$(\vec{a}\times\vec{b})\cdot\hat{c} = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \pm\frac{1}{2}\sqrt{\Sigma a_i^2}\cdot\sqrt{\Sigma b_i^2}$$

48. CEVAS THEOREM.

49.

L.H.S $[\vec{b}\,\vec{c}\,\vec{d}]\,\vec{a} - [\vec{b}\,\vec{c}\,\vec{a}]\,\vec{d} + [\vec{c}\,\vec{a}\,\vec{d}]\,\vec{b} - [\vec{c}\,\vec{a}\,\vec{b}]\,\vec{d}$

$+ [\vec{a}\,\vec{b}\,\vec{d}]\,\vec{c} - [\vec{a}\,\vec{b}\,\vec{c}]\,\vec{d}$

But $[\bar{a}\,\bar{b}\,\bar{c}] = [\bar{b}\,\bar{c}\,\bar{a}] = [\bar{c}\,\bar{a}\,\bar{b}]$

$\therefore$ L.H.S $= [\vec{b}\,\vec{c}\,\vec{d}]\,\vec{a} + [\vec{c}\,\vec{a}\,\vec{d}]\,\vec{b} + [\bar{a}\,\bar{b}\,\bar{d}]\,\vec{c}$

$- 3\,[\bar{a}\,\bar{b}\,\bar{c}]\,\bar{d}$

take

$$\vec{d} = x\vec{a} + y\vec{b} + z\vec{c}$$

$$[\vec{b}\,\vec{c}\,\bar{d}] = x\,[\bar{b}\,\bar{c}\,\bar{a}]$$

$$\cancel{\therefore x = \frac{[\bar{b}\,\bar{c}\,\bar{d}]}{[\bar{a}\,\bar{b}\,\bar{c}]}}$$

||ly

$$[\bar{c}\,\bar{a}\,\bar{d}] = y\,[\bar{c}\,\bar{a}\,\bar{b}]$$

$$[\bar{a}\,\bar{b}\,\bar{d}] = z\,[\bar{a}\,\bar{b}\,\bar{c}]$$

$\therefore$ L.H.S $= [\bar{a}\,\bar{b}\,\bar{c}]\,\bar{d} - 3\,[\bar{a}\,\bar{b}\,\bar{c}]\,\bar{d}$

50. $\vec{a}\cdot\vec{b} \le |\vec{a}||\vec{b}|$

$[\because \vec{a}\cdot\vec{b} = |a||b|\cos\theta]$.

51. $A(x_1, y_1, z_1)$ $B(x_2, y_2, z_2)$ $C(x_3, y_3, z_3)$

$P(x, y, z)$

$$PA^2 + PB^2 + PC^2 = \Sigma(x-x_1)^2 + \Sigma(y-y_1)^2 + \Sigma(z-z_1)^2$$

$$= \cancel{(\Sigma x_1^2)x^2 - 2(\Sigma x_1)x +}$$

$$= 3x^2 - 2(\Sigma x_1)x + (\Sigma x_1^2)$$
$$+ 3y^2 - 2(\Sigma y_1)y + (\Sigma y_1^2)$$
$$+ 3z^2 - 2(\Sigma z_1)z + (\Sigma z_1^2)$$

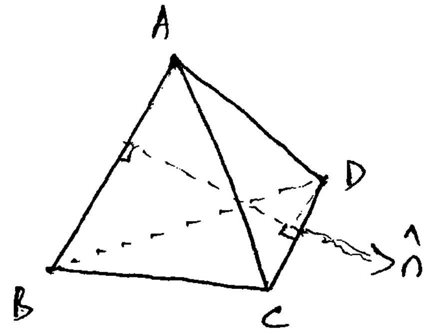
Each of the quadratic in x, y, z respectively are min when $x = -\frac{(-2\Sigma x_1)}{2\times 3}$

$$y = -\frac{(-2\Sigma y_1)}{2\times 3}$$

$$z = -\frac{(-2\Sigma z_1)}{2\times 3}$$

$x = \frac{\Sigma x_1}{3}$, $y = \frac{\Sigma y_1}{3}$, $z = \Sigma z$

52. Volume of the tetrahedron ABCD. $= \frac{1}{3}$ (area $\Delta BCD \times$ height)



$$= \left| \frac{1}{6} (\vec{BC} \times \vec{BD}) \cdot \vec{AB} \right|$$

$$= \frac{1}{6} \left| (\vec{BC} \times \vec{CD}) \cdot \vec{AB} \right|$$

$$= \frac{1}{6} \left| \vec{BC} \cdot (\vec{CD} \times \vec{AB}) \right|$$

$$= \frac{1}{6} \left| \vec{BC} \cdot \hat{n} \cdot ab \sin\theta \right|$$

$\vec{BC} \cdot \hat{n}$ is projection of BC on $\hat{n}$ $= d$..

$\therefore$ volume $= \frac{1}{6} abd \sin\theta$

53. use the diagram shown in Q.44 solution to get the vectors & proceed.

54. $l + m + n = 0 \quad l^2 + m^2 = n^2$

$\Rightarrow l^2 + m^2 = (l+m)^2$

$\Rightarrow ml = 0$

Case (i) $m = 0$

$\Rightarrow l = -n$

Case (ii) $l = 0$

$m = -n$

55.

$A(a,0,0)$, $B(0,b,0)$, $C(0,0,c)$

$\Delta ABC = \frac{1}{2}\left|(a\hat{i})\times(b\hat{j}) + (b\hat{j})\times(c\hat{k}) + (c\hat{k})\times(a\hat{i})\right|$

$= \frac{1}{2}\left|ab\hat{k} + bc\hat{i} + ac\hat{j}\right|$

56. $\vec{n_1} = \hat{i} + \hat{j} - 2\hat{k}$

$\vec{n_2} = 2\hat{i} - \cancel{2}\hat{j} + \hat{k}$

$\cos\theta = \dfrac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$

57.

(a) $x - 2y + z = 1$

$x + 2y - 2z = 5$

$2x - z = 6$

$z = 2x - 6$ or $x = \dfrac{z+6}{2}$

$\therefore$ $x - 2y + 2x - 6 = 1$

$\Rightarrow$ $2y = 3x - 7$

$y = \dfrac{3x-7}{2}$ $x = \dfrac{2y+7}{3}$

$\therefore$ the ln of intersection is

(b)

$$x = \frac{2y+7}{3} = \frac{z+6}{2} = \lambda \text{ [say]}$$

$\therefore$ $y = \frac{3\lambda-7}{2}$

$z = 2\lambda - 6$

$x = \lambda$

$\therefore$ put these values of x, y, z in the plane equation:

$$\lambda + \frac{3\lambda-7}{2} - 2(2\lambda-6) = 7$$

$$2\lambda + 3\lambda - 7 - 8\lambda + 24 = 14$$

$$-3\lambda = -3$$

$$\lambda = 1$$

$\therefore$ $(1, -2, -4)$ is the point of intersection.

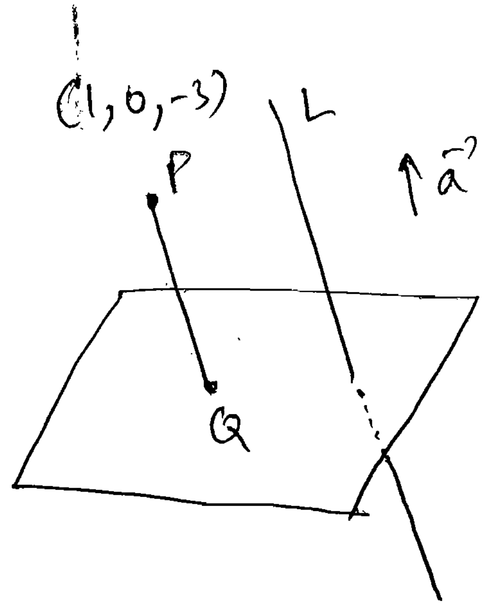
5 Q. If (a, b, c) is any point on the plane

$3x + 2y + z = 1$

$x^2+y^2+z^2$ is minimum is the shortest distance of the origin from the plane

$= \frac{1}{\quad}$

59.



L is || el to $\vec{a}$
$= 2\hat{i} + 3\hat{j} - 6\hat{k}$

~~Lost~~ eqn of PQ is
$\vec{r} = \hat{i} - 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$

Solve PQ with plane & get the co-ordinates of Q. & used distance formula to get PQ

60.
$x - cy - bz = 0 \quad - \omega_1$
$cx - y + az = 0. \quad - \omega_2$
$bx + ay - z = 0 \quad - \omega_3$

Put the condition that the direction ratio of the line of intersection of ω_1 & ω_2 is $\perp$ to the normal of ω_3 $(b\hat{i} + a\hat{j} - \hat{k})$

(to find line of intersection refer to Q. 57.)

61. The plane will be a tangent to the sphere. Centred at origin & radius $3p$.

$\therefore$ Eqn of sphere

$\therefore$ (x_1, y_1, z_1) be the point of contact

$\therefore$ $xx_1 + yy_1 + zz_1 = 9p^2$ is the

equation of the plane.

& $x_1^2 + y_1^2 + z_1^2 = 9p^2$

x intercept $= A\left[\frac{9p^2}{x_1}, 0, 0\right]$

y " $= B\left[0, \frac{9p^2}{y_1}, 0\right]$

z " $= C\left[0, 0, \frac{9p^2}{z_1}\right]$

Centroid $G\left(\frac{3p^2}{x_1}, \frac{3p^2}{y_1}, \frac{3p^2}{z_1}\right)$

x, y, z

~~take~~

$$\therefore \left(\frac{3p^2}{x}\right)^2 + \left(\frac{3p^2}{y}\right)^2 + \left(\frac{3p^2}{z}\right)^2 = x_1^2 + y_1^2 + z_1^2 = 9p^2$$

$$\therefore \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z} = \frac{1}{p^2}$$

62. ∵ use diagram of Q44 [Soln]

Get equation of Plane APQ by 3-point form & use distance of a point from plane formula.

63. Solve for vertices & proceed.

64. lines

$\vec{r} = \bar{a}_1 + \lambda \vec{b}_1$

$\vec{r} = \bar{a}_2 + \lambda \vec{b}_2$

are skew if

$$\frac{(\vec{a}_1 - \vec{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \neq 0.$$

65. Eqn of the line is

$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$

Solve it with the plane ~~(x-y+z)~~

66.

$x^2+y^2+z^2=25$

$2x+2y-z+12=0$

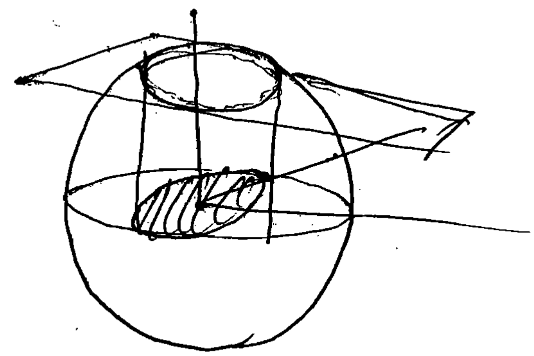
$\therefore \quad z = 2x+2y+12$

Solve sphere & plane

$\therefore \; x^2+y^2 + (2x+2y+12)^2 = 25$

$x^2+y^2 + 4x^2 + 4y^2 + 144 + 8xy, \ldots$

this a second degree curve is an ellipse which is nothing ~~to~~ but the projection of the $\odot^{le}$ of intersection on the xy plane



Center of ellipse is given by

$$\frac{\partial \phi}{\partial x} = 0 \quad \& \quad \frac{\partial \phi}{\partial y} = 0$$

$10x + 8y + 48 = 0$. $\qquad 10y + 8x + 48 = 0$

$5x + 4y + 24 = 0$. $\qquad 5y + 4x + 24 = 0$.

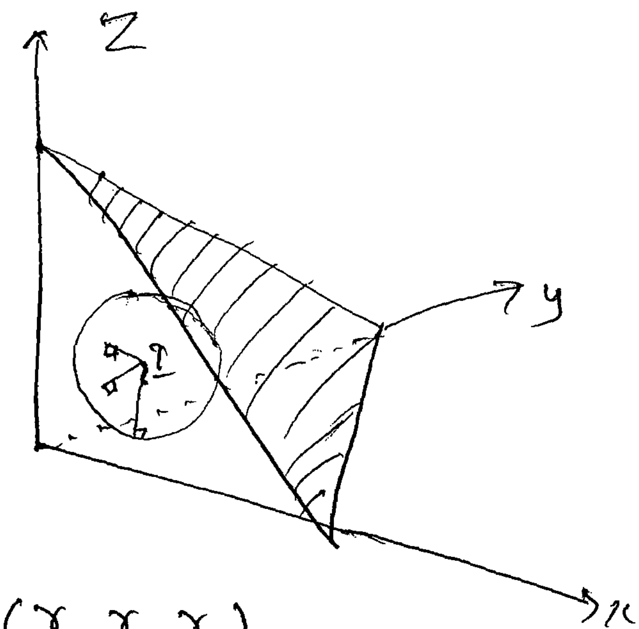
$$\left(-\frac{8}{3}, \frac{-8}{3}\right)$$

But $z = 2x + 2y + 12$

$$= -\frac{16}{3} - \frac{16}{3} + 12$$

$$= \frac{4}{3}$$

6Q.



$I(r, r, r)$

distance of I from plane $x+y+z=1$

$= r$.

$\therefore \dfrac{|3r-1|}{\sqrt{3}} = r$

$(3r-1)^2 = 3r^2$

$\Rightarrow 9r^2 - 6r + 1 = 3r^2$

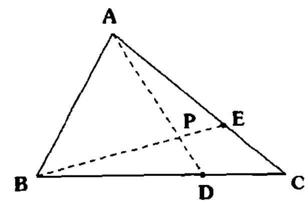
$6r^2 - 6r + 1 = 0$

$\frac{1}{\sqrt{3}} > r > 0$.

$\therefore r = \dfrac{3-\sqrt{3}}{6}$.

EXERCISE 3

1. ABC is a triangle . AD , BE are cevians such that $\frac{|BD|}{|DC|}=\frac{2}{1}$ and $\frac{|CE|}{|EA|}=\frac{1}{3}$. AD , BE meet at P. Find $\frac{|BP|}{|PE|}$.



Sol $\bar{d}=\frac{\bar{b}+2\bar{c}}{3}$, $\bar{e}=\frac{\bar{a}+3\bar{c}}{4}$

Eliminate $\bar{c}$ we get

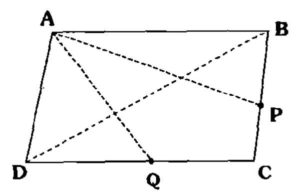
$$\frac{3\bar{d}-\bar{b}}{4}=\frac{4\bar{e}-\bar{a}}{3}$$

$$\therefore \quad \frac{2\bar{a}+9\bar{a}}{2+9}=\frac{3\bar{b}+8\bar{e}}{2+9} \qquad \text{(Dividing by 11)}$$

$$\therefore \quad \frac{2\bar{a}+9\bar{a}}{11}=\frac{3\bar{b}+8\bar{e}}{11}=\bar{p}$$

$$\therefore \quad \frac{BP}{PE}=\frac{8}{3}$$

2. ABCD is a parallelogram . P , Q are the mid-points of BC , CD respectively . Show that AP , AQ trisect BD .



Sol Let M be a point which divides BD in the ratio 1 : 2
Now, we wil have to show that : M lies on AP
We how

$$\bar{m}=\frac{2\bar{b}+\bar{d}}{3}$$

$$=\frac{\bar{b}+\left(\bar{b}+\bar{a}\right)}{3}$$

$\because$ diagonals risect each other in a 11th gm

$$\frac{\bar{b}+\bar{d}}{2}=\frac{\bar{a}+\bar{c}}{2} \qquad \therefore \qquad \bar{a}+\bar{c}=\bar{b}+\bar{d}$$

$$=\frac{\bar{b}+\bar{a}+\bar{c}}{3}$$

$$=\frac{\bar{a}+\left(\bar{b}+\bar{c}\right)}{3}$$

$$=\frac{\bar{a}+2\bar{p}}{3} \qquad \frac{\left(\bar{b}+\bar{c}\right)}{2}=\bar{p}$$

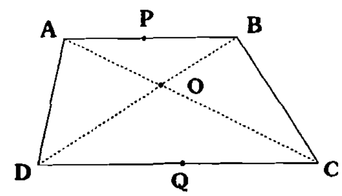
This shows that M lies on AP and divides AP in the ratio 2 : 1
Let N be a point which divides BD in the 2 : 1
Now, we will how to show that N lies on AQ
we how

$$\bar{A}=\frac{2\bar{b}+2\bar{d}}{3}=\frac{\bar{b}+\bar{d}+\bar{d}}{3}$$

$= \frac{\bar{a}+\bar{c}+\bar{d}}{3}$

$= \frac{\bar{a}+2\bar{q}}{3}$ $\qquad \frac{(\bar{c}+\bar{d})}{2}=\bar{q}$

This shows N lies on AQ.



3. In a □ ABCD , AB | | DC. P and Q are the mid-points of AB and DC respectively .AC , BD meet at O. Show that P , O , Q are collinear.

Sol Let $\frac{AP}{PC}=\frac{k}{1}$

$\because \quad \Delta APB \sim \Delta DPC$

$\frac{AP}{PC}=\frac{BP}{PD}=\frac{k}{1}$

We have

$\bar{p}=\frac{\bar{a}+k\bar{c}}{1+k}=\frac{\bar{b}+kd}{1+k}$

By the principle of equal ratoos

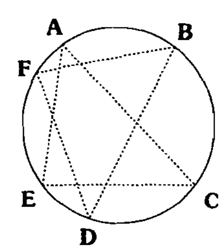
$\bar{p}=\frac{\bar{a}+k\bar{c}+\bar{b}+k\bar{d}}{2(1+k)}$

$=\frac{\bar{a}+\bar{b}+k(\bar{c}+\bar{d})}{2(1+k)}$

$=\frac{2\bar{m}+2\bar{n}}{2(1+k)}$

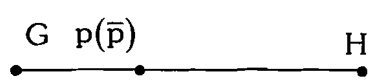
$\bar{p}=\frac{\bar{m}+k\bar{n}}{1+k}$

Hence p lies on MN and divides MN in the ratio k : 1



4. A , B , . . . , F are points on a circle. Consider pairs of triangles which do not have a common vertex , e.g. Δ ACE and Δ BDF. (There are ten pairs). Take the centroid of one triangle, and the orthocentre of the other , in every pair. Join them to get twenty lines in all. Show that these are concurrent .

Sol



$\bar{g}=\frac{\bar{b}+\bar{e}+\bar{f}}{3}$

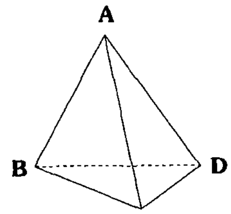
Now $\because$ $o(\bar{o})$ origin is the concurrent

$\bar{h}=\bar{a}+\bar{c}+\bar{d}$

$\bar{p}=\frac{3\bar{g}+1\bar{h}}{4}=\frac{\bar{a}+\bar{b}+\bar{c}+\bar{d}+\bar{e}+\bar{f}}{4}$

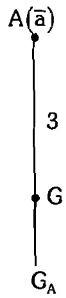
$\because$ $(\bar{p})$ expression is symmetric for $\bar{a},..,..,\bar{f}$

$\therefore$ We can say all to 20 lines are concurrent at p



5. ABCD is a tetrahedron. Prove that the principal medians (four in all) and the lateral medians (three in all) are concurrent .

Sol $\dfrac{(\bar{b}+\bar{c}+\bar{d})}{3}$



$\bar{g}=\dfrac{1\bar{a}+3\bar{g}}{3}$

$=\dfrac{\bar{a}+\bar{b}+\bar{c}+\bar{d}}{4}$

symmetric in $\bar{a},\bar{b},\bar{c},\bar{d}$

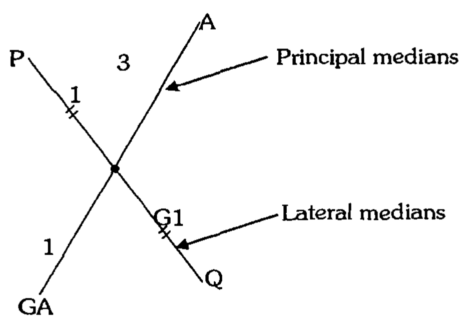
This shows that the four principle medians pass through $(\bar{g})$G.

$\bar{g}=\dfrac{(\bar{a}+\bar{b})+(\bar{c}+\bar{d})}{4}$

$=\dfrac{2\bar{p}+2\bar{q}}{4}$

$=\dfrac{\bar{p}+\bar{q}}{2}$

This shows that $(\bar{g})$G lies on PQ



6. $0<\alpha,\beta,\gamma<2\pi$ and $\alpha+\beta+\gamma=\pi$. Prove that $\Sigma\cos\alpha\ge-\dfrac{3}{2}$.

Deduce that , in any Δ ABC , $\Sigma\cos A\le\dfrac{3}{2}$.

Sol $0<\alpha,\beta,\gamma<\pi$

$\alpha+\beta+\gamma=2\pi$

$\cos\alpha+\cos\beta+\cos\gamma\ge-\dfrac{3}{2}$

We have $(\hat{e}_1+\hat{e}_2+\hat{e}_3)$

taking its self scalar product

$$\left(\hat{e}_1+\hat{e}_2+\hat{e}_3\right)\cdot\left(\hat{e}_1+\hat{e}_2+\hat{e}_3\right)\geq 0$$

$$3+2\left(\hat{e}_2\cdot\hat{e}_3+\hat{e}_3\cdot\hat{e}_1+\hat{e}_1\cdot\hat{e}_2\right)\geq 0$$

$$3+2(\cos\alpha+\cos\beta+\cos\gamma)\geq 0$$

$$\therefore \cos\alpha+\cos\beta+\cos\gamma\geq -\frac{3}{2}$$

Hence proved

Equality only when $\alpha=\beta=\gamma=\dfrac{2\pi}{3}$

S.T $\sum\cos A\leq\dfrac{3}{2}$

Proof : put $\alpha=(\pi-A)$

$\beta=(\pi-B)$

$\gamma=(\pi-C)$

$\therefore \quad \alpha+\beta+\gamma=2\pi$ and $0<\alpha,\beta,\gamma<\pi$

Hence,

$$\cos(\pi-A)+\cos(\pi-\beta)+\cos(\pi-C)\geq -3$$

$$\therefore \quad (\cos A+\cos B+\cos C)\geq -\frac{3}{2}$$

$$\therefore \quad \cos A+\cos B+\cos C\leq\frac{3}{2}$$

chelrycheff's inequality

Let $a_1, a_2, a_3 \ldots\ldots a_n$

$b_1, b_2, b_3 \ldots\ldots b_n$

be sequence of real nos having the same monotonocity i.e. if one of them is increasing so is then other same for decreasing.

then $\sum\dfrac{a_i b_i}{n}\geq\sum\dfrac{a_i}{n}\sum\dfrac{b_i}{n}$

C = only when all the a_i are equal b_i

or

If the montonicity are in opposite sense thendirection of inequality is rarrsed.

If $A+B+C=\pi$, then

$$\sum\cos A=4\pi\sin\frac{A}{2}+1$$

Let ABC be a triangle (A/R type)

A : $a\cos A+b\cos B+c\cos C\leq S$

B : $\pi\sin\dfrac{A}{2}\leq\dfrac{1}{8}$

we have, $\sum\cos A\leq\dfrac{3}{2}$

$\therefore \quad 1+4\pi\sin\frac{A}{2}\le\frac{3}{2}$

$4\pi\sin\frac{A}{2}\le\frac{1}{2}$

$\therefore \quad \pi\sin\frac{A}{2}\le\frac{1}{8}$

Now,

$\therefore$ Without loss of generality are an take $a\ge b\ge c$

$\because$ it is cyclic expression

$\therefore$ $A\ge B\ge C$

$\therefore$ $\cos A\le\cos B\le\cos C$

By cheby cheff's inequalety we have

$$\frac{a\cos A+b\cos B+c\cos C}{3}\le\frac{a+b}{3}\left(\frac{\cos A+\cos B+\cos C}{3}\right)\le\frac{\frac{3}{2}}{3}\le\frac{2}{3}\times\frac{1}{2}$$

$\therefore$ Even A is true and $\because \frac{\cos A+\cos B+\cos C}{3}\le\frac{3}{2}$ is used for its proof and it is equivalent to the statment R.

A & R true and R is correct explan of A

7. In a Δ ABC, the median CM $\perp$ the angle bisector AL (of $\angle$ BAC). If $|CM| = |AL|$, find cos A.

Sol Let A be the origin

$\Delta I\cong II$ $(ACP\cong APM)$

Rt angle at P gives similarity and one side is common so Ralio of sides is 1

$\therefore \quad |AC|=|AM|=\frac{1}{2}|AB|$

i.e $\quad |\bar{c}|=\frac{1}{2}|\bar{b}|$

$\Rightarrow \quad \frac{BL}{LC}=\frac{AB}{AC}$ (Apollonius priciple)

$=2$

$\therefore \quad \bar{\ell}=\frac{\bar{b}+2\bar{c}}{3}$

[Note : $|\overline{AB}|^2=|\bar{a}|^2+|\bar{b}|^2-2\bar{a}.\bar{b}$]

$|AL|=|CM|$

$\therefore \quad |AL|^2=|CM|^2$

$\therefore \quad \bar{\ell}\cdot\bar{\ell}=(\bar{m}-\bar{c})\cdot(\bar{m}-c)$

$\therefore \quad \frac{1}{9}(\bar{b}+2\bar{c})\cdot(\bar{b}+2\bar{c})=\frac{1}{4}(\bar{b}-2\bar{c})(\bar{b}-2\bar{c})$

$\therefore \quad 4\left(\left|\bar{b}\right|^2+4\left|\bar{c}\right|^2+4\bar{b}\cdot\bar{c}\right)=9\left(\left|\bar{b}\right|^2+4\left|\bar{c}\right|^2-4\bar{b}\cdot\bar{c}\right)$

$\therefore \quad 52\bar{b}\cdot\bar{c}=5|b|^2+20|c|^2$

$\qquad =5\times4|b|^2+20|c|^2$

$\therefore \quad 52\bar{b}\cdot\bar{c}=40|c|^2$

$\therefore \quad 52\left|\bar{b}\right|\cdot\left|\bar{c}\right|\cdot\cos A=40|c|^2$

$\therefore \quad 52\times2\left|\bar{c}\right|\cdot\left|\bar{c}\right|\cos A=40|c|^2$

$\therefore \quad \cos A=\dfrac{40}{2\times52}=\dfrac{20}{52}$

8. ABC and PQR are triangles. The perpendiculars from A , B , C to QR , RP , PQ ,are concurrent . Prove that the perpendicular from P , Q , R to BC , CA , AB are also concurrent .

Sol Let the $\perp$ from Q to CA and from to AB meet at N.

(taking mas origin) cyclic expression

$\left(\bar{b}-\bar{c}\right)\cdot\left(\bar{p}-\bar{h}\right)+\left(\bar{c}-\bar{a}\right)\cdot\left(\bar{q}-\bar{h}\right)+\left(\bar{a}-\bar{b}\right)\cdot\left(\bar{r}-\bar{h}\right)$

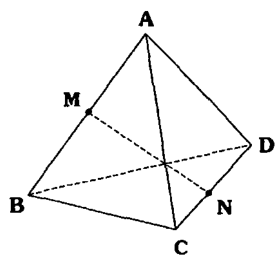
$=\left[\left(\bar{b}-\bar{c}\right)\cdot\bar{p}+\left(\bar{c}-\bar{a}\right)\cdot\bar{q}+\left(\bar{a}-\bar{b}\right)\cdot\bar{r}\right]-\left[\left(\bar{b}-\bar{c}\right)\cdot\bar{n}+\left(\bar{c}-\bar{a}\right)\cdot\bar{n}+\left(\bar{a}-\bar{b}\right)\cdot\bar{n}\right]$ this is 0

$=\left[\left(\bar{b}-\bar{c}\right)\cdot\bar{p}+\left(\bar{c}-\bar{a}\right)\cdot\bar{q}+\left(\bar{a}-\bar{b}\right)\cdot\bar{r}\right]$

$=\bar{b}\left(\bar{p}-\bar{r}\right)+\bar{c}\left(\bar{q}-\bar{p}\right)+\bar{a}\left(\bar{r}-\bar{p}\right)$ rach $= 0$

Hence proved $\left(\bar{b}-\bar{c}\right)\cdot\left(\bar{p}-\bar{h}\right)=0$

10. ABCD is a regular tetrahedron . MN is the lateral median , as shown. Show that AB , MN and DC are mutually orthogonal .If an edge of the solid has length 1, find the shortest distance between AB and DC.



Sol $\overline{AB}=3\hat{i}+2\hat{j}+0\hat{k}$

$\overline{PQ}=\bar{Q}-\bar{P}$

$\qquad =0\hat{i}+0\hat{j}+2\hat{k}$

$\overline{CD}=\bar{D}-\bar{C}$

$\qquad =-2\hat{i}+2\hat{j}+0\hat{k}$

$\therefore \quad \overline{AB}\cdot\overline{PQ}=0$

$\qquad \overline{PQ}\cdot\overline{CD}=0$

$\qquad \overline{AB}\cdot\overline{CD}=-4+4=0$

Hence proved

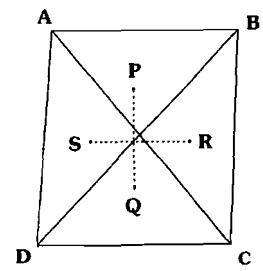
Note : The shortest distance between AB and CD is length PQ

$\because$ it is $\perp$ both

$|PQ|=2$ (edge is $|AB|=2\sqrt{2}$)

$\therefore$ In a regular tetrahedron the smallest distence between the two skew edges $=\dfrac{\text{side}}{\sqrt{2}}$

12. ABCD is any quadrilateral. AC , BD meet at O. P , Q are the orthocentres Δ OAB and Δ OCD, respectively. R,S are the centroids of Δ OBC and Δ ODA, respectively . Show that $PQ \perp RS$.



Sol Let o be origin

$$\overline{PQ}\cdot\overline{RS} = (\bar{q}-\bar{p})\cdot(\bar{s}-\bar{r})$$

$$= \left(\frac{\bar{a}+\bar{b}+\bar{o}}{3} - \frac{\bar{c}+\bar{d}+\bar{o}}{3}\right)\cdot(\bar{s}-\bar{r}) = \left(\frac{\bar{a}+\bar{b}-\bar{c}-\bar{d}}{3}\right)\cdot(\bar{s}-\bar{r})$$

$$\frac{9}{3}\left(\bar{c}+\bar{d}-\bar{a}-\bar{b}\right)\cdot(\bar{s}-\bar{r})$$

$$= \frac{1}{3}\left((\bar{c}-\bar{a})+(\bar{a}-\bar{b})\right)(\bar{s}-\bar{r})$$

$$= \frac{1}{3}\left[(\bar{c}-\bar{a})\cdot\bar{s}+(\bar{d}-\bar{b})\cdot\bar{s}-(\bar{c}-\bar{a})\cdot\bar{r}-(\bar{d}-\bar{b})\cdot\bar{r}\right]$$

$\because$ S is the orthocentre BS is $\perp$ AC

$\therefore$ $(\bar{c}-\bar{a})(\bar{s}-\bar{b}) = 0$

$\therefore$ $(\bar{c}-\bar{a})\cdot\bar{s} = (\bar{c}-\bar{a})\cdot\bar{b}$

CS is $\perp$ BD

$\therefore$ $(\bar{b}-\bar{d})\cdot(\bar{c}-\bar{s}) = 0$

$\therefore$ $-(\bar{b}-\bar{d})\cdot\bar{s} = (\bar{b}-\bar{d})\cdot\bar{c}$

III^{y} all and the replacing in 1

$$= \frac{1}{3}\left((\bar{c}-\bar{a})\cdot\bar{s}+(\bar{d}-\bar{b})\cdot\bar{s}-(\bar{c}-\bar{a})\cdot\bar{r}-(\bar{d}-\bar{b})\cdot\bar{r}\right)$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$(\bar{c}-\bar{a})\cdot\bar{b}+(\bar{d}-\bar{b})\cdot\bar{c}-(\bar{c}-\bar{a})\cdot\bar{d}-(\bar{d}-\bar{b})\cdot\bar{a}$$

13. A_i , $i = 1, 2, \ldots, n$, are arbitrary points on the surface of a unit sphere . Prove that

$$\sum_{1 \le i < j \le n} \sum |A_i A_j|^2 \le n^2 .$$

Sol $|A_iA_j|^2 = |\bar{a}_i|^2 + |\bar{a}_j|^2 - 2\bar{a}_i\cdot\bar{a}_j$

$$= 2 - 2\,\bar{a}_j\cdot\bar{a}_j \quad (\because |a_i| = 1)$$

The required expression

$$= 2\left(n^2-n\right)C_2 - 2\sum_j\sum_i\left(\bar{a}_i\cdot\bar{a}_j\right)$$

$$= n^2 - \left(n + 2\sum\sum\bar{a}_i\cdot\bar{a}_j\right)$$

$$= n^2 - \left(\bar{a}_1+\bar{a}_1+\bar{a}_2+\bar{a}_2+\ldots\bar{a}_n+\bar{a}_n+2\sum\sum\bar{a}_2+\bar{a}_j\right)$$

$$= n^2 - \left(\bar{a}_1+\ldots.+\bar{a}_n\right)\cdot\left(\bar{a}_1+\ldots+\bar{a}_n\right)\bar{a}_1\cdot\bar{a}_1 = 1$$

$\therefore \quad L.H.S \le n^2$

Equality only when $\bar{a}_1 + ... \bar{a}_n = 0$

This can happen for insland if the n points are ratices of a regular points.

14. $A_1B_1C_1$ in an equilateral triangle inscribed in a circle ω_1 , and P_1 is any point of ω_1 ; $A_2B_2C_2$ is an equilateral triangle inscribed in a circle ω_2 concentric with ω_1 and P_2 is any point of ω_2. Show that

$|P_1A_2|^2 + |P_1B_2|^2 + |P_1C_2|^2 = |P_2A_1|^2 + |P_2B_1|^2 + |P_2C_1|^2$

Sol The L.H.S.

$$= |\bar{p}_1|^2 + |\bar{a}_2|^2 - 2\bar{p}_1 \cdot \bar{a}_2 + |\bar{p}_1|^2 + |\bar{b}_2|^2 - 2\bar{p}_1 \cdot \bar{b}_2 + |\bar{p}_1|^2 + |\bar{c}_2|^2 - 2\bar{p}_1 \cdot \bar{c}_2$$

$$= 3R_1^2 + 3R_2^2 - 2\bar{p}_1 \cdot (\bar{a}_2 + \bar{b}_2 + \bar{c}_2)$$

$$= 3R_1^2 + 3R_2^2$$

$|||^y$ $R.H.S = 3R_1^2 + 3R_2^2$

15. For any two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, prove that

$(1 - \boldsymbol{a} \cdot \boldsymbol{b})^2 + |\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{b}|^2 = (1 + a^2)(1 + b^2)$

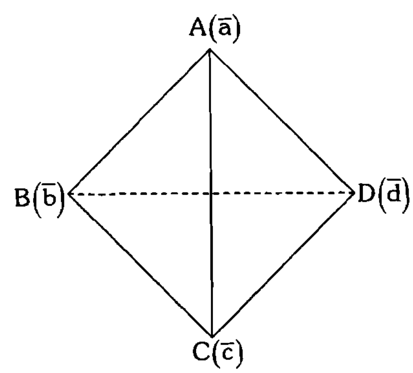
Sol $$L.H.S = 1 - 2\bar{a} \cdot \bar{b} + (\bar{a} \cdot \bar{b})^2 + |\bar{a}|^2 + |\bar{b}|^2 + |\bar{a} \times \bar{b}|^2 + 2\bar{a} \cdot \bar{b} + 2\bar{a} \cdot (\bar{a} \times \bar{b}) + 2\bar{b}(\bar{a} \times \bar{b})$$

$$= 1 + |\bar{a}|^2 + |\bar{b}|^2 + |\bar{a}|^2 + |\bar{b}|^2$$

$$= (1 + |\bar{a}|^2)(1 + |\bar{b}|^2) = R.H.S$$

16. In any tetrahedron , show that the vector sum of the areas of the faces , taken along the outward normals, is zero.

Sol Vector area of



1. face ABC is $\frac{1}{2}(\bar{a} \times b + \bar{b} \times \bar{c} + \bar{c} \times \bar{a})$

2. face ACD is $\frac{1}{2}(\bar{a} \times \bar{c} + \bar{c} \times \bar{d} + \bar{d} \times \bar{a})$

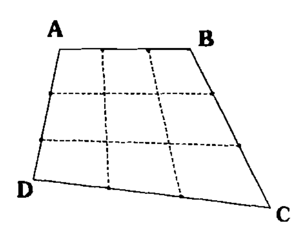
3. face BDC is $\frac{1}{2}(\bar{b} \times \bar{d} + \bar{d} \times \bar{c} + \bar{c} \times \bar{b})$

4. face ADB is $\frac{1}{2}(\bar{a} \times \bar{d} + \bar{d} \times \bar{b} + \bar{b} \times \bar{a})$

(Note : Each of the face to hav the area in the outword normal dicection we will be to it for all of them that they are taken in anticlockwise scnse)

It is dear that the vector sum is zero.

17. ABCD is a convex quadrilateral. Join the corresponding trisection points of pairs of opposite sides. Prove that the area of the central quadrilateral , so formed , is $\frac{1}{9}$ area (☐ ABCD)



Sol $M(\bar{m}) = \dfrac{2\bar{a} + \bar{b}}{3} \qquad N(\bar{n}) = \dfrac{2\bar{d} + \bar{c}}{3}$

The position vector of the point dividing MN in the ratio 1 : 2 is

$$\frac{2\overline{m}+\overline{n}}{3}=\frac{2\frac{\left(2\overline{a}+\overline{b}\right)}{3}+\frac{2\overline{d}+\overline{c}}{3}}{3}=\frac{2\left(\frac{2\overline{a}+\overline{d}}{3}\right)+\left(\frac{2\overline{b}+\overline{c}}{3}\right)}{3}$$

$$=\frac{2\overline{x}+\overline{y}}{3}$$

This shows that p divides MN in the ratio in 1 : 2

The position vector $P(\overline{p})$

$$\overline{p}=\frac{4\overline{a}+2\overline{b}+\overline{c}+2\overline{d}}{9}$$

lll^y

$$\overline{r}=\frac{\overline{a}+2\overline{b}+4\overline{c}+2\overline{d}}{9}$$

$$\overrightarrow{RP}=\overline{p}\cdot\overline{r}$$

$$=\frac{\overline{a}-\overline{c}}{3}=\frac{1}{3}\overrightarrow{CA}$$

lll^y

$$\overrightarrow{SQ}=\frac{1}{3}\overrightarrow{DB}$$

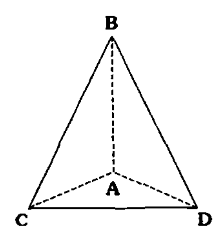
$\therefore \quad \overrightarrow{RP}\times\overrightarrow{SQ}=\frac{1}{9}\overrightarrow{CA}\times\overrightarrow{DB}$

$$\frac{1}{2}\left|\overrightarrow{RP}\times\overrightarrow{SQ}\right|=\frac{1}{9}\times\frac{1}{2}\left|\overrightarrow{CA}\times\overrightarrow{DB}\right|$$

$\therefore \quad$ Area of PQRS $=\frac{1}{9}$ Area of ABCD.

18. PYTHAGORAS' THEOREM IN 3-DIMENSIONS .

ABCD is a tetrahedron . All the face angles at A are 90^0.
S_B is the area of the face CAD. S_C , S_D and S_A are similarly defined. Prove that $S_A^2 = S_B^2 + S_C^2 + S_D^2$



Sol Area of face

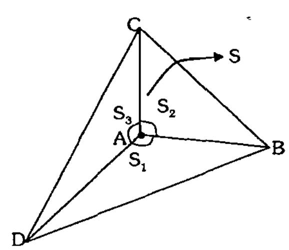
$ABD = S_1$

$ABC = S_2$

$ACD = S_3$

$BCD = S$

$T.P.T : S^2 = S_1^2 + S_2^2 + S_3^2$



Consider the vector area $\overline{S}_1, \overline{S}_2, \overline{S}_3, \overline{S}$ all along the outword normals

We have, $\overline{S}_1+\overline{S}_2+\overline{S}_3=-\overline{S}$

This implies

$$\left(\overline{S}_1+\overline{S}_2+\overline{S}_3\right)\cdot\left(\overline{S}_1+\overline{S}_2+\overline{S}_3\right)=\left(-\overline{S}\right)\cdot\left(-\overline{S}\right)$$

$$S_1^2+S_2^2+S_3^2+2\bar{S}_1\cdot\bar{S}_2+2\bar{S}_2\cdot\bar{S}_3+2\bar{S}_3\cdot\bar{S}_1=S^2$$

$$\qquad\qquad\downarrow\qquad\quad\downarrow\qquad\quad\downarrow$$

$$\qquad\qquad 0\qquad\quad 0\qquad\quad 0$$

$\therefore \quad S_1^2+S_2^2+S_3^2=S^2$

22. ***a*** , ***b*** are unit vectors . A vector ***c*** is such that ***c*** + (***c*** × ***a***) = ***b*** . Show that | ***a*** × ***b*** . ***c*** | ≤ $\frac{1}{2}$.

When does equality hold ?

Sol $\left[\vec{c}\times\vec{a}-\hat{b}\right]^2=(-\vec{c})^2$

$|\vec{c}\times\vec{a}|^2+\perp-2[cab]=c^2$

$\Rightarrow \quad c^2a^2\sin^2\theta+1-c^2=2[cab]-c^2(1-\sin^2\theta)+1=2[cab]$

$2[cab]=1-c^2\cos^2\theta\le\perp$

$[cab]\le\frac{1}{2}$

Equality holds when $\theta=900$

IN CHAPTER - 3.

1. take $\hat{b} = \hat{i}$ $\quad$ $\hat{b} \times \hat{c} = \hat{k}$

$\hat{c} = \hat{j}$

$$\vec{a} = \alpha \hat{i} + \beta \hat{j}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times \hat{k} = \alpha \hat{i} \times \hat{k} + \beta \hat{j} \times \hat{k}$$

$$= \beta \hat{i} - \alpha \hat{j}$$

$$|\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{\beta^2 + \alpha^2} = |\vec{a}|$$

2. Calculate $(\vec{a} \times \vec{b}) \times \vec{c}$ & equate $\hat{i}, \hat{j}, \hat{k}$ & solve for u, v, w obtain B.

3. Let $\vec{d} = \alpha \vec{a} + \beta \vec{b} + \gamma \vec{c}$

$$L.H.S = (\vec{b} \times \vec{c}) \times (\vec{a} \times \vec{d}) + (\vec{c} \times \vec{a}) \times (\vec{b} \times \vec{d}) + (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$$

$$= [\bar{b}\,\bar{c}\,\bar{d}]\vec{a} - [\bar{b}\,\bar{c}\,\bar{a}]\vec{d} + [\bar{c}\,\bar{a}\,\bar{d}]\bar{b} - [\bar{c}\,\bar{a}\,\bar{b}]\bar{d}$$

$$+ [\vec{a}\,\vec{b}\,\vec{d}]\vec{c} - [\bar{a}\,\bar{b}\,\bar{c}]\bar{d}$$

$$= [\bar{b}\,\bar{c}\,\bar{d}]\vec{a} + [\bar{c}\,\bar{a}\,\bar{d}]\bar{b} + [\bar{a}\,\bar{b}\,\bar{d}]\bar{c} - 3[\bar{a}\,\bar{b}\,\bar{c}]\bar{d}$$

$$[\bar{b}\,\bar{c}\,\bar{d}] = (\vec{b} \times \vec{c}) \cdot \vec{d} = [\bar{a}\,\bar{b}\,\bar{c}]\alpha$$

IIIly

$$[\bar{c}\,\bar{a}\,\bar{d}] = [\bar{a}\,\bar{b}\,\bar{c}]\beta$$

$$[\bar{a}\,\bar{b}\,\bar{d}] = [\bar{a}\,\bar{b}\,\bar{c}]\gamma$$

$$\therefore \quad L.H.S = [\bar{a}\,\bar{b}\,\bar{c}](\alpha\bar{a} + \beta\bar{b} + \gamma\bar{c}) - 3[\bar{a}\,\bar{b}\,\bar{c}]\bar{d}$$

$$= -2[\bar{a}\,\bar{b}\,\bar{c}]\bar{d}$$

4. Observe that $(\vec{b}\times\vec{c})\times\vec{a} + (\vec{c}\times\vec{a})\times\vec{b} + (\vec{a}\times\vec{b})\times\vec{c} = 0$

$\therefore$ $(\vec{b}\times\vec{c})\times\vec{a}$, $(\vec{c}\times\vec{a})\times\vec{b}$, $(\vec{a}\times\vec{b})\times\vec{c}$ are coplanar.

5. take $\bar{p} = a\hat{i}$, $\bar{q} = a\hat{j}$, $\bar{r} = a\hat{k}$

take $\vec{x} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

Solve for α, β, γ.

6. $|\vec{a}| = |\vec{b}| = 1$

$|\bar{c}| = 2$.

$\vec{a}\times(\vec{a}\times\vec{c}) + \vec{b} = 0$

$\vec{a}\times(\vec{a}\times\vec{c}) = -\vec{b}$

taking mod on both sides

$|\vec{a}\times(\vec{a}\times\vec{c})| = |\vec{b}|$

$|2\sin\theta| = 1$

$|\sin\theta| = \frac{1}{2}$

$\therefore\ \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

IN CHAPTER EXCERCISE-2.

1. ~~along~~

$$\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0} \quad -(1)$$

$\vec{a} \times$

$$2\vec{a}\times\vec{b} + 3\vec{a}\times\vec{c} = \vec{0}.$$

$$\Rightarrow \quad \vec{c}\times\vec{a} = \frac{2}{3}\vec{a}\times\vec{b}$$

$\vec{b}\times(1)$

$$\vec{b}\times\vec{a} + 3\,\vec{b}\times\vec{c} = 0$$

$$\Rightarrow \vec{b}\times\vec{c} = \frac{1}{3}\vec{a}\times\vec{b}$$

$$\therefore \vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = 2(\vec{a}\times\vec{b}).$$

2. Option (C)

3.

$$\hat{e} = x\hat{i} + y\hat{j}$$

$$|\hat{e}| = 1 \Rightarrow x^2+y^2=1$$

$$\cos 45^\circ = \frac{\hat{e}\cdot(\hat{i}+\hat{j})}{\sqrt{2}} = \frac{1}{\cancel{\sqrt{2}}}$$

$$\therefore x+y=1$$

$$y = 1-x$$

$$x^2 - 2x + \cancel{1} + x^2 = \cancel{1}$$

$$x = 0, 1$$

$$y = 1, 0$$

$$\therefore \hat{e} = \hat{i}, \hat{j}$$

$$\hat{e}\cdot(\hat{i}+\hat{j}+\hat{k}) = 1.$$

4. volume $= [\vec{a}\,\vec{b}\,\vec{c}]$

$$V = \begin{vmatrix} 1 & a & 1 \\ \dots \end{vmatrix} = 1 + a^3 \qquad a = \frac{1}{\sqrt{3}}$$

5. let $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a} \cdot \vec{b} = 1$

$\Rightarrow x + y + z = 1.$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$$

$= (y-z)\hat{i} - \hat{j}(x-z) + \hat{k}(x-y)$

$= \hat{j} - \hat{k}$

$\Rightarrow y = z$

$z - x = 1$

$x - y = -1$

$x + 2z = 1$

$3z = 2$

$z = \frac{2}{3}, \; x = -\frac{1}{3}$

$\therefore \; \vec{b} = -\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

Projecti of $\vec{b}$ on the vector $2\hat{i} - \hat{j} - 3\hat{k}$

$$= \left| \frac{\vec{b} \cdot (2\hat{i} - \hat{j} - 3\hat{k})}{|2\hat{i} - \hat{j} - 3\hat{k}|} \right|$$

$$= \left| \frac{-\frac{2}{3} + \frac{2}{3} - 2}{\sqrt{14}} \right| = \frac{2}{\sqrt{14}}.$$

6. $\vec{p} = |\vec{a}|\vec{b} + |\vec{b}|\vec{a}$

$\vec{q} = |\vec{a}|\vec{b} - |\vec{b}|\vec{a}$

$\vec{p}\cdot\vec{q} = 0$

$\therefore$ area of the $\Delta^{le} = \frac{1}{2}|p|\,|q|$

$$= \frac{1}{2}\sqrt{a^2b^2 + b^2a^2 + 2ab\,\vec{a}\cdot\vec{b}}$$

$$\times \sqrt{a^2b^2 + b^2a^2 \cancel{+} 2ab\,\vec{a}\cdot\vec{b}}$$

$$= \frac{1}{2}\sqrt{2a^2b^2 + a^2b^2} \times \sqrt{\cancel{2}a^2b^2 - a^2b^2}$$

$$= \frac{1}{2}a^2b^2\sqrt{3}.$$

7. option D.

8. $\vec{a}\cdot\vec{b} = 0$

$\Rightarrow \vec{a} \perp \vec{b}$

$(\vec{a}\times\vec{b})\times\vec{c} = 0$

$\therefore \vec{a}\times\vec{b} = k\vec{c}$

$(\vec{a}\times\vec{b})\cdot\vec{c} = 8$

$k|\vec{c}|^2 = 8$

$|(\vec{a}\times\vec{b})| = |a||b| \sin\frac{\pi}{2}$

$|k||c| = 4$

$\therefore |c| = 2$

$k = 2.$

$\vec{a}\times\vec{b} = 2\vec{c}$

IN CHAPTER EXCERCISE-1

1. $\vec{AB} = \vec{OB} - \vec{OA}$

$= 2\hat{i} - \hat{j} + \hat{k} - \hat{i} - \hat{j} + \hat{k}$

$= \hat{i} - 2\hat{j} + 2\hat{k}$

$|\vec{AB}| = 3.$

2. $\vec{PQ} = 6\hat{i} + 2\hat{j} + (a+2)\hat{k}$

$|\vec{PQ}| = 7$

$\therefore \quad 36 + 4 + (a+2)^2 = 49$

$(a+2)^2 = 9$

$a+2 = \pm 3$

$a = -5, 1.$

3. $\vec{a} \cdot \hat{k} = \cos\frac{\pi}{4} \qquad |\vec{a}| = 1$

$= \frac{1}{\sqrt{2}}$

$\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore \quad z = \frac{1}{\sqrt{2}}$

$\therefore \quad \vec{a} = x\hat{i} + y\hat{j} + \frac{\hat{k}}{\sqrt{2}}$

$\left|\vec{a} + \hat{i} + \hat{j}\right| = 1 = \left|(x+1)\hat{i} + (y+1)\hat{j} + \frac{\hat{k}}{\sqrt{2}}\right| = 1$

$(x+1)^2 + (y+1)^2 + \frac{1}{2} = 1$

$\therefore \quad (x+1)^2 + (y+1)^2 = \frac{1}{2}$

from the option $\quad x = -\frac{1}{2}, \; y = -\frac{1}{2}$

4. $|a+b|^2 = |a|^2 + |b|^2 + 2\vec{a}\cdot\vec{b}$

$25 = 9 + 16 + 2\,\vec{a}\cdot\vec{b}$

5. $\vec{a}+\vec{b}+\vec{c}=0$

$(\vec{a}+\vec{b}+\vec{c})^2=0$

$\vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}\cdot\vec{b})+2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a}=0.$

$1+1+1+2[\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}]=0$

$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-\frac{3}{2}$

6. ~~$\vec{a}$~~ If θ is the angle between $\vec{a}$ & $(\vec{a}+\vec{b}+\vec{c})$

the $\cos\theta = \dfrac{\vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})}{|\vec{a}|\,|\vec{a}+\vec{b}+\vec{c}|}$

$= \dfrac{\cancel{|\vec{a}|^2}}{\cancel{|\vec{a}|^2}\cdot\sqrt{3}}$

7. $|a+b+c|=\sqrt{3}$

8.

$|a|+|b|=|c|$

$\vec{a}+\vec{b}=\vec{c}$

$\cancel{|a|^2+|\vec{b}|^2}+\cancel{2}\vec{a}\cdot\vec{b}=|\vec{c}|^2=\cancel{|\vec{a}|^2}+\cancel{|\vec{b}|^2}+\cancel{2}|\vec{a}||\vec{b}|$

$\therefore$ angle between $\vec{a}$ & $\vec{b}=0$.

9. $\vec{OA}=7\hat{j}+10\hat{k}$, $\vec{OB}=-\hat{i}+6\hat{j}+6\hat{k}$, $\vec{OC}=-4\hat{i}+9\hat{j}+6\hat{k}$

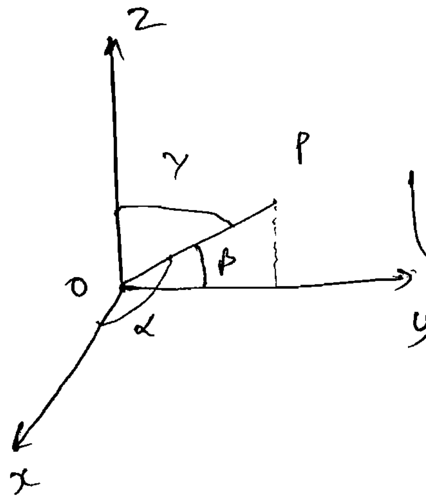
$\vec{AB}=-\hat{i}-\hat{j}-4\hat{k}$

$\vec{BC}=-3\hat{i}+3\hat{j}$

$\vec{CA}=4\hat{i}-2\hat{j}+4\hat{k}$

$\vec{AB}\cdot\vec{BC}=0$

10. $\vec{OP} = \hat{i} + 2\hat{j} + 3\hat{k}$



$\cos\beta = \frac{2}{\sqrt{14}}$, $\cos\gamma = \frac{3}{\sqrt{14}}$.

$$\cos\alpha = \frac{\vec{OP}\cdot\hat{i}}{|\vec{OP}|} = \frac{1}{\sqrt{14}}$$

$||OP| \sin\alpha| =$ distance from x-axis

$= \sqrt{14}\cdot\frac{\sqrt{13}}{\sqrt{14}} = \sqrt{13}$

$|OP \sin\beta| =$ distance from y-axis

$= \cancel{\sqrt{14}}\cdot\frac{\sqrt{10}}{\cancel{\sqrt{14}}} = \sqrt{10}$

$|OP \sin\gamma| = \cancel{\sqrt{14}}\cdot\frac{\sqrt{5}}{\cancel{\sqrt{14}}} = \sqrt{5}$

INCHAPTER EXCERCISE - 4

1. they are non - Coplanar

2. $\vec{r} = (\vec{a}\cdot\vec{b})\vec{c} - (\vec{a}\cdot\vec{c})\vec{b}$

$\vec{r}\cdot\vec{a} = 0$

$\therefore \vec{a} \perp \vec{r}$

3. they are ~~Mutually orthogonal~~ linearly independent

4. $(\vec{r} - \vec{g})$ ~~is coplanar~~ is $\perp^{r}$ to the normal to the plane $\vec{n} = \vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b}$

$\therefore (\vec{r} - \vec{g})\cdot\vec{n} = 0.$

5. Diagonals bisect each other at P.

$\vec{OP} = \dfrac{\vec{OA} + \vec{OC}}{2}$

$\vec{OP} = \dfrac{\vec{OB} + \vec{OD}}{2}$

$\therefore \vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OP}$

6. $\vec{GA_1} + \vec{GA_2} + \vec{GA_3} + \cdots + \vec{GA_n}$

$= (\vec{OA_1} + \vec{OA_2} + \cdots + \vec{OA_n}) - n\vec{OG}$

$= \cancel{(\vec{OA_1} + \vec{OA_2} + \cdots + \vec{OA_n})} - \cancel{n}\left(\dfrac{\vec{OA_1} + \vec{OA_2} + \cdots + \vec{OA_n}}{\cancel{n}}\right)$

$= \vec{0}.$

7.

$\vec{p} \perp \vec{q}$ $\quad$ $\vec{r} \perp \vec{s}$

$\therefore$ $\vec{p} \cdot \vec{q} = 0$ $\quad$ $\vec{r} \cdot \vec{s} = 0$

$\therefore$ $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + \vec{b}) = 0$

$6|\vec{a}|^2 + 3\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 5|\vec{b}|^2 = 0$

$6|\vec{a}|^2 - 7\vec{a} \cdot \vec{b} - 5|\vec{b}|^2 = 0$ — (1)

$(\vec{a} + 4\vec{b}) \cdot (-\vec{a} + \vec{b}) = 0$

$-|\vec{a}|^2 + \vec{a} \cdot \vec{b} - 4\vec{a} \cdot \vec{b} + 4|\vec{b}|^2 = 0$

$-|\vec{a}|^2 + 4|\vec{b}|^2 - 3\vec{a} \cdot \vec{b} = 0$ — (2)

(1) $\times 3 - 7 \times$ (2)

$18|\vec{a}|^2 - 15|\vec{b}|^2 + 7|\vec{a}|^2 - 28|\vec{b}|^2 = 0$

$\therefore 25|\vec{a}|^2 = 43|\vec{b}|^2$ tan $|\vec{a}| = 1$

$\therefore |\vec{b}| = \frac{5}{\sqrt{43}}$

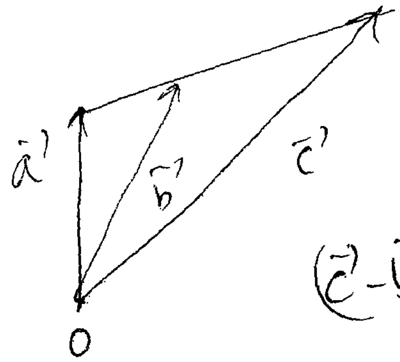
$\therefore$ $\cancel{\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{a}|}} =$

fm (1) $\quad 6 - 7\vec{a} \cdot \vec{b} - 5 \times \frac{25}{43} = 0$

$\frac{258 - 125}{43} = 7|a||b| \cos\theta$

$\frac{\cancel{133}^{19}}{43} = \cancel{7} \times 1 \times \frac{5}{\sqrt{43}} \cos\theta$

8.



$(\vec{c}-\vec{b})\times(\vec{b}-\vec{a})=\vec{0}$ & get $y=$

$\vec{c}-\vec{b}=\hat{i}+(y-1)\hat{j}$

$\vec{b}-\vec{a}=2\hat{i}-2\hat{j}$

$(\hat{i}+(y-1)\hat{j})\times(\hat{i}-\hat{j})=\vec{0}$

$-\hat{k}-(y-1)\hat{k}=\vec{0}$

$\therefore y=0.$