Matrices & Determinants

Exercise – 1(B)

Q.1 (A)(B)(D)

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now $A^2 - 4A - 5I_3 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 4 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Further $A^2 - 4A - 5I_3 = 0 \Rightarrow A - 4I_3 - 5A^{-1} = 0$ or $A^{-1} = \frac{1}{5}(A - 4I_3)$.

$\left| A^2 \right| = |A|^2 = 25 \neq 0 \Rightarrow A^2$ is invertible.

Q.2 (A)(B)(C)(D)

Standard characteristics of a matrix

Q.3 (B)(D)

The given system of equations is $\begin{bmatrix} 1 & -1 & 3 \\ 2 & -1 & 1 \\ 1 & -2 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$

By $R_3 \to R_3 - R_1$ & $R_2 \to R_2 - 2R_1$ we get

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -5 \\ 0 & -1 & \alpha - 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

Further by $R_3 \to R_3 + R_2$ we get

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -5 \\ 0 & 0 & \alpha - 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

Hence the given system of equations has no solutions for $\alpha = 8$ and unique solution for any other value.

Q.4 (A)(B)(C)

$|\text{adj}(A)| = |A|^2$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}^2 = 1$$

Now, $\text{adj}(\text{adj}A) = |A|^2 \times A$

= A.

Further $|\text{adj}(\text{adj}(A))| = |A| = 1.$

Q.5 (A)(B)(C)

Given, a_1 , a_2, a_3............... in A.P.

Let $a_1 = a$, $a_2 = a + d$, $a_3 = a + 2d, \ldots\ldots\ldots\ldots\ldots$so on

$$\Rightarrow A = \begin{bmatrix} a & a+d & a+2d \\ a+3d & a+4d & a+5d \\ a+4d & a+5d & a+6d \end{bmatrix}$$

Hence, $|A| = \begin{vmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix} = 0$

As |A| = 0, hence option B is also correct.

Now, $|B| = \begin{vmatrix} a_1 & i\,a_2 \\ i\,a_2 & a_1 \end{vmatrix}$

$$= \begin{vmatrix} a & i(a+d) \\ i(a+d) & a \end{vmatrix}$$

$= a^2 + (a+d)^2 \neq 0$

Q.6 (B)(C)(D)

The given system of equations can be represented as

$$\begin{bmatrix} 1 & 3 & 2 \\ 1 & \lambda & 2 \\ 1 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ \mu \end{bmatrix}.$$

Now by linear row transformation $R_2 \to R_2 - R_1$ we get

$$\begin{bmatrix} 1 & 3 & 2 \\ 1 & \lambda & 2 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ \mu - 6 \end{bmatrix}$$

Clearly if $\mu = 6$, the system of equations has infinitely many solutions and otherwise no solution.

Note here that the conclusion drawn above is independent of the value of λ.

Q.7 (A)(C)(D)

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = - \begin{vmatrix} a^2 & (b-c)^2 & bc \\ b^2 & (c-a)^2 & ca \\ c^2 & (a-b)^2 & ab \end{vmatrix} \quad \{\text{By } C_2 \to C_2 - C_1\}$$

Now split the determinant about second column to get

$$= -\begin{vmatrix} a^2 & b^2+c^2 & bc \\ b^2 & c^2+a^2 & ca \\ c^2 & a^2+b^2 & ab \end{vmatrix} - \begin{vmatrix} a^2 & 2bc & bc \\ b^2 & 2ca & ca \\ c^2 & 2ab & ab \end{vmatrix}$$

$$= -\begin{vmatrix} a^2+b^2+c^2 & b^2+c^2 & bc \\ b^2+c^2+a^2 & c^2+a^2 & ca \\ c^2+a^2+b^2 & a^2+b^2 & ab \end{vmatrix} \quad \{\text{By } C_1 \to C_1 + C_2\}$$

$$= -\left(a^2+b^2+c^2\right)\begin{vmatrix} 1 & b^2+c^2 & bc \\ 1 & c^2+a^2 & ca \\ 1 & a^2+b^2 & ab \end{vmatrix}$$

$$= (a^2+b^2+c^2)\begin{vmatrix} 1 & b^2+c^2 & bc \\ 0 & b^2-a^2 & c(b-a) \\ 0 & a^2-c^2 & b(a-c) \end{vmatrix} \quad \{\text{By } R_2 \to R_2 - R_1 \ \& \ R_3 \to R_3 - R_1\}$$

$$= (a^2+b^2+c^2)\ (b-a)\ (a-c)\begin{vmatrix} 1 & b^2+c^2 & bc \\ 0 & b+a & c \\ 0 & a+c & b \end{vmatrix}$$

$$= (a^2+b^2+c^2)\ (a-b)\ (b-c)\ (c-a)\ (a+b+c)$$

Q.8 (A)(C)

Standard properties of determinants.

Q.9 (A)(B)(C)(D)

Standard properties of matrices.

Q.10 (B)(C)

Standard properties of determinants & matrices.

Q.11 (A)(B)

$f(x) = x^3 + 3x$

Now $f'(x) = 3x^2 + 3 > 0$ hence f(x) is an increasing function.

Minimum = f(0) = 0 & Maximum = f(1) = 4.

Q.12 (A)(B)(C)(D)

$$\begin{vmatrix} 1+\sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1+\cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & \cos^2 A & 2\sin 4\theta \\ 2 & 1+\cos^2 A & 2\sin 4\theta \\ 1 & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0 \quad \{\text{By } C_1 \to C_1 + C_2\}$$

Now apply $R_1 \to R_1 - R_2$ and then $R_2 \to R_2 - 2R_3$ to get

$$\begin{vmatrix} 0 & -1 & 0 \\ 0 & \sin^2 A & -2-2\sin 4\theta \\ 1 & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$$

$\Rightarrow 2+2\sin 4\theta = 0$

$\Rightarrow \sin 4\theta = -1$

Hence $\theta = -\frac{\pi}{8} \& \frac{3\pi}{8}$

Note here that solution is independent of A.

Q.13 (A)(B)(C)

Given that a_1, a_2, a_3 are in A.P & b_1, b_2, b_3 are in H.P.,

hence let $a_2 - a_1 = a_3 - a_2 = x$.

$$\therefore \Delta = \begin{vmatrix} a_1 - b_1 & a_1 - b_2 & a_1 - b_3 \\ a_2 - b_1 & a_2 - b_2 & a_3 - b_3 \\ a_3 - b_1 & a_3 - b_2 & a_3 - b_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 - b_1 & a_1 - b_2 & a_1 - b_3 \\ x & x & x \\ 2x & 2x & 2x \end{vmatrix} \quad \{\text{By } R_2 \to R_2 - R_1 ,\ R_3 \to R_3 - R_1\}$$

= 0.

Hence Δ is independent of all the variables.

Further as Δ = 0 hence option (B) & (C) are also correct.

Q.14 (A)(B)(C)(D)

AB = A ……….(1)

BA = B ………..(2)

Pre – multiply (1) by A

$A(AB) = A^2$

$A^2B = A^2$

Pre – multiply (2) by B

$B(BA) = B^2$

$B^2A = B^2$

Now, AB = A

A (BA) = A ……..$[\because B = BA]$

Similarly BA = B

B (AB) = B

Q.15 (A)(B)(C)

Given $\begin{vmatrix} x & a & b \\ a & x & a \\ b & b & x \end{vmatrix} = 0$.

$$\Rightarrow \begin{vmatrix} x & a-x & b-x \\ a & x-a & 0 \\ b & 0 & x-b \end{vmatrix} = 0 \ \{\text{By } C_2 \to C_2 - C_1 \ \& \ C_3 \to C_3 - C_1\}$$

$$\Rightarrow (x-a)(x-b)\begin{vmatrix} x & -1 & -1 \\ a & 1 & 0 \\ b & 0 & 1 \end{vmatrix} = 0$$

Now by $R_1 \to R_1 + R_2 + R_3$,

$$(x-a)(x-b)\begin{vmatrix} x+a+b & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-a)(x-b)(x+a+b) = 0.$$

Q.16 (A)(B)(C)

Standard properties of matrices.

Q.17 (A)(D)

Given $\begin{vmatrix} 1 & a & a^2 \\ 1 & x & x^2 \\ b^2 & ab & a^2 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 0 & x-a & x^2-a^2 \\ b^2 & ab & a^2 \end{vmatrix} = 0 \ \{\text{By } R_2 \to R_2 - R_1\}$$

$$\Rightarrow (x-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & x+a \\ b^2 & ab & a^2 \end{vmatrix} = 0$$

Now multiply first column by ‘a’ and take ‘a’ common from first & third row to get

$$(x-a)\begin{vmatrix} 1 & 1 & a \\ 0 & 1 & x+a \\ b^2 & b & a \end{vmatrix} = 0$$

$$\Rightarrow (x-a)(1-b)\begin{vmatrix} 1 & 1 & a \\ 0 & 1 & x+a \\ 0 & b & a(1+b) \end{vmatrix} = 0 \quad \{\text{By } R_3 \to R_3 - b^2R_1\}$$

$$\Rightarrow (x-a)(1-b)(a-bx) = 0$$

Alternately :

For x = a, first & second rows are identical hence (x – a) is a factor.

For $x = \frac{a}{b}$, second and third row are proportional, hence $\left(x - \frac{a}{b}\right)$ is a factor.

Q.18 (A)(C)

Characteristic equation of A is $\begin{vmatrix} a-x & b \\ c & d-x \end{vmatrix} = 0$ i.e. $x^2 - (a+d)x + ad - bc = 0$

Clearly a + d = 0 & k = ad – bc = |A|.

Q.19 (A)(C)

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 2 & 1+\cos^2\theta & 4\sin 4\theta \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \quad \{\text{By } C_1 \to C_1 + C_2\}$$

Now apply $R_1 \to R_1 - R_2$ and then $R_2 \to R_2 - 2R_3$ to get

$$\begin{vmatrix} 0 & -1 & 0 \\ 0 & \sin^2\theta & -2-4\sin 4\theta \\ 1 & \cos^2\theta & 1+2\sin 4\theta \end{vmatrix} = 0$$

$\Rightarrow 2+4\sin 4\theta = 0$

$\Rightarrow \sin 4\theta = -\dfrac{1}{2}$

Hence $\theta = \dfrac{7\pi}{24} \,\&\, \dfrac{11\pi}{24}$.

Q.20 (A)(C)(D)

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix} == \begin{vmatrix} 0 & 0 & 1 \\ \cos px & \sin px & 0 \\ \sin px & -\cos px & 0 \end{vmatrix} \times \begin{vmatrix} \cos dx & 1 & \cos dx \\ \sin dx & 0 & -\sin dx \\ 1 & a & a^2 \end{vmatrix}$$

Clearly the determinant of given matrix is independent of ‘p’ as first determinant in factorized representation is constant 1.

PASSAGE 1

Let $U_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ So that $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow x = 1 , y = -2 , z = 1$

$$\therefore U_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Similarly $U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}$ & $U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

Hence $U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}$

Q.21 (A)

Now $|U| = \begin{vmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{vmatrix}$

$$\Rightarrow |U| = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & 0 \\ 1 & -4 & 1 \end{vmatrix} \{\text{By } C_3 \to C_3 - C_2\}$$

$$\Rightarrow |U| = 3.$$

Q.22 (B)

$$\text{adj}(U) = \begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$$

$$\therefore U^{-1} = \frac{\text{adj}\,U}{3} = \begin{bmatrix} -1/3 & -2/3 & 0 \\ -7/3 & -5/3 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

sum of elements of $U^{-1} = 0$

Q.23 (A)

$$\begin{bmatrix}3 & 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2 & 2\\ -2 & -1 & -1\\ 1 & -4 & -3\end{bmatrix}\begin{bmatrix}3\\2\\0\end{bmatrix} = [-1 \quad 4 \quad 4]\begin{bmatrix}3\\2\\0\end{bmatrix}$$

$$=[5]$$

PASSAGE 2

Given $P = A^{-1}XA,\ A^TA = I\ \&\ X^TX = X$

Now $A^TA = I \Rightarrow A^{-1} = A^T\ \&\ X^TX = X \Rightarrow X^T = I$.

Also $X^TX = X \Rightarrow |X^TX| = |X|$ or $|X|^2 = |X| \Rightarrow |X| = 1\ \ (|x| \neq 0)$

Q.24 (B)

$P^T = \left(A^{-1}XA\right)^T \Rightarrow P^T = A^TX^T\left(A^{-1}\right)^T$ as $(ABC)^T = C^TB^TA^T$

$\Rightarrow P^T = A^{-1}X^TA,$ given $A^T = A^{-1}$

$\Rightarrow P^TP = \left(A^{-1}X^TA\right)\left(A^{-1}XA\right)$

$\Rightarrow P^TP = A^{-1}X^T\left(AA^{-1}\right)^{-1}XA$

$\Rightarrow P^TP = A^{-1}X^TXA$

$\Rightarrow P^TP = A^{-1}XA = P,$ given $X^TX = x$

Q.25 (C)

$|P| = |A^{-1}XA| \Rightarrow |P| = |A|^{-1}|X||A| = |X| = 1$

Q.26 (A)

$\Rightarrow A^{-1}X^{-2}A = \left(A^{-1}X^{-1}A\right)\left(A^{-1}X^{-1}A\right)$ $\quad\{$Multiply I between x^{-1} & x^{-1} & write it as $AA^{-1}\}$

$\Rightarrow A^{-1}X^{-2}A = \left(A^{-1}XA\right)^{-2} = P^{-2}$ $\quad\{$As $A^{-1}B^{-1}C^{-1} = (CBA)^{-1}\}$

Given $P = \begin{bmatrix}a & -a\\ a & a\end{bmatrix} \Rightarrow |P| = 2a^2$

As $|P| = 1$ hence $2a^2 = 1$.

Now $P^{-1} = \dfrac{\text{adj}(P)}{|P|} = \begin{bmatrix} a & a \\ -a & a \end{bmatrix}$

Hence $P^{-2} = \begin{bmatrix} a & a \\ -a & a \end{bmatrix} \times \begin{bmatrix} a & a \\ -a & a \end{bmatrix} = \begin{bmatrix} 0 & 2a^2 \\ -2a & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

PASSAGE 3

Let $a_{11} = a$, common ratio of G.P. be r and common difference of A.P. in 1st row be be d.

$$\text{Now } A = \begin{bmatrix} a & a+d & a+2d & . & . & a+(n-1)d \\ ar & ar+dr & ar+2dr & . & . & ar+(n-1)dr \\ ar^2 & ar^2+dr^2 & ar^2+2dr^2 & & & ar^2+(n-1)dr^2 \\ & & & & & \\ ar^{n-1} & ar^{n-1}+dr^{n-1} & ar^{n-1}+2dr^{n-1} & & & ar^{n-1}+(n-1)dr^{n-1} \end{bmatrix}.$$

Therefore $a_{ij} = (a+(j-1)d)r^{i-1}$ & $d_i = dr^{i-1}$.

Now $a_{24} = ar + 3dr = 1$, $a_{42} = ar^3 + dr^3 = \dfrac{1}{8}$ & $a_{43} = ar^3 + 2dr^3 = \dfrac{3}{16}$

From a_{42} & a_{43} we get $ar^3 = dr^3 = \dfrac{1}{16}$

From a_{24} we get $a = d = r = \dfrac{1}{2}$,

Q.27 (D)

$a_{4j} = (a+(j-1)d)r^3 \;\Rightarrow\; S_n = \sum_{j=1}^{n} a_{4j} = nar^3 + \dfrac{n(n-1)}{2}dr^3$

$\Rightarrow \lim_{n\to\infty} \dfrac{S_n}{n^2} = \dfrac{dr^3}{2} = \dfrac{1}{32}$.

Q.28 (C)

Now $\sum_{i=1}^{n} d_i = d\sum_{i=1}^{n} r^{i-1}$ or $\sum_{i=1}^{n} d_i = d\dfrac{1-r^n}{1-r}$

Hence $\sum_{i=1}^{n} d_i = 1 - \frac{1}{2^n}$

Q.29 (C)

$a_{ii} = \left(\frac{1}{2}\right)^i i \Rightarrow \sum_{i=1}^{\infty} a_{ii} = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots\infty \text{ terms} = 1.$

PASSAGE 4

Let $A = \begin{bmatrix} a_a & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ & $\vec{x} = x\hat{i} + y\hat{j} + zk$

Represent $\vec{x}$ as $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} \hat{i} & \hat{j} & k \end{bmatrix}$

Now $A\vec{x} = \begin{bmatrix} a_a & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} \hat{i} & \hat{j} & k \end{bmatrix}$

$\Rightarrow A\vec{x} = (a_{11}x + a_{12}y + a_{13}z)\hat{i} + (a_{21}x + a_{22}y + a_{23}z)\hat{j} + (a_{31}x + a_{32}y + a_{33}z)k$

Now $(A\vec{x}).\vec{x} = 0 \Rightarrow (a_{11}x + a_{12}y + a_{13}z)x + (a_{21}x + a_{22}y + a_{23}z)y + (a_{31}x + a_{32}y + a_{33}z)z = 0$

or $(a_{11}x^2 + a_{22}y^2 + a_{33}z^2) + (a_{12} + a_{21})xy + (a_{23} + a_{32})yz + (a_{13} + a_{31})zx = 0$

$\Rightarrow a_{11} = a_{22} = a_{33} = a_{12} + a_{21} = a_{23} + a_{32} = a_{13} + a_{31} = 0$

Q.30 (A)(D)

Matrix A is skew symmetric and singular.

Q.31 (A)(D)

$a_{13} = -2 \Rightarrow a_{31} = 2,\ a_{32} = 5 \Rightarrow a_{23} = -5$

Q.32 (A)(C)(D)

Conceptual question.

ASSERTION REASON TYPE

Q.33 (D)

Standard property of matrices : If AB = 0, then |A| = 0 or |B| = 0 but none of [A] & [B] is necessarily zero.

Q.34 (A)

Standard property of matrices : Determinant of an odd order skew symmetric matrix is zero.

Q.35 (C)

Let A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^2 = \begin{bmatrix} a^2+bc & ab+bd \\ ca+cd & d^2+bc \end{bmatrix}$

Now $ab+bd=0 \Rightarrow a+d=0 \ \left(b \neq 0\right)$.

Hence statement 1 is true.

$A^2 = I \Rightarrow |A|^2 = 1$
$\Rightarrow |A| = \pm 1$

Hence statement two is false.

Q.36 (D)

Standard property of matrices : Inverse of a singular matrix doesn’t exist.

Also AB = BC $\Rightarrow |A| = 0$ or $|B| = |C|$

Q.37 (C)

Statement 1 is standard characteristic of a diagonal matrix.

Now det. $(AB)^{-1}$ = (det. A) $(\det . B^{-1})$

= (– 3) det. $\left(1, 1, \frac{1}{2}\right)$

$= (-3)\left(\frac{1}{2}\right)$

$= -\frac{3}{2}$ $\therefore$ false.

MATRIX MATCH TYPE

Q.38 $A \to P, B \to P, C \to P$

(A) $\begin{vmatrix} 1 & 2 & \sqrt{3} \\ 5 & 13 & -6 \\ \frac{1}{2} & 1 & \frac{\sqrt{3}}{2} \end{vmatrix} = 1\left(\frac{13\sqrt{3}}{2} + 6\right) - 5\left(\frac{\sqrt{6}}{2} - \sqrt{3}\right) + \frac{1}{2}\left(-6\sqrt{2} - 13\sqrt{3}\right) = 0$

(B) $\begin{vmatrix} i & 1 & -i \\ -1 & i & 1 \\ i & -1 & i \end{vmatrix} = i(i^2+1) - 1(-i+i) - i(1+i^2) = 0$

(C) $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 4 \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix} = 0$ $\{\text{By } R_2 \to R_2 - R_3\}$

Q.39 $A \to Q, B \to S, C \to R$

(A) def. of skew – symmetric = 0

$\therefore k + 2 = 2$

(B) $\begin{vmatrix} x-1 & -1 & -2 \\ -1 & x-1 & -2 \\ -1 & -1 & x-2 \end{vmatrix} = \begin{vmatrix} x-1 & -1 & -2 \\ -x & x & 0 \\ -x & 0 & x \end{vmatrix}$ $\{\text{By } R_2 \to R_2 - R_1 \text{ \& } R_2 \to R_2 - R_1\}$

$= x^2 \begin{vmatrix} x-1 & -1 & -2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$

$$= x^2 \begin{vmatrix} x-2 & -1 & -2 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} \quad \{\text{By } C_1 \to C_1 + C_2\}$$

$$= x^2 (x-4)$$

(C) Given that B is skew symmetric & A is symmetric hence

$B^T = -B$ & $A^T = A$

Further $(A+B)(A-B) = (A-B)(A+B) \Rightarrow AB = BA$

Now $(AB)^T = B^T A^T \Rightarrow (AB)^T = -BA$

$(AB)^T = -AB$

Hence k must be an odd number.

Q.40 $A \to R, B \to S, C \to P, R$

(A) If A is nonsingular idempotent matrix, then $A = I_n$

$\therefore (A + I_n)^n = I + 127$

or $(2I_n)^n = 128 I_n$

or $2^n \; I_n^n = 128 I_n$

$\Rightarrow 2^n = 128 \Rightarrow n = 7$

(B) We have $(I - A)\,(I + A + A^2 + ... + A^7) = I$

$\Rightarrow I + A + A^2 + ... + A^7 - A - A^2 - A^3 .... - A^8 = I$

$I - A^8 = I$

Hence $A^8 = 0$.

(C) $a_{ij} = i^2 - j^2 \Rightarrow a_{ji} = j^2 - i^2$, hence $a_{ij} = -a_{ji}$.

A I a skew symmetric matrix.

For a skew symmetric matrix to be singular it must be of odd order.