Matrices & Determinants

Exercise – 1(A)

Q.1 (B)

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2^{\circ} A$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2^1 A$$

$$A^3 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = 2^2 A$$

Generalizing gives, $A^n = 2^{n-1} A$

Alternately

Characteristic equation of A is

$$\begin{vmatrix} 1-x & 1 \\ 1 & 1-x \end{vmatrix} = 0 \text{ or } x^2 = 2x$$

Hence by Cayley Hamilton theorem,

$A^2 = 2A$.

Now $A^3 = 2A^2 = 4A,\ A^4 = 2A^3 = 8A, \ldots,\ A^n = 2^{n-1} A$.

Q.2 (C)

$$E(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$E(\alpha)\, E(\beta) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\,\cos\beta - \sin\alpha\sin\beta & \cos\alpha\sin\beta + \sin\alpha\cos\beta \\ -(\sin\alpha\,\cos\beta + \cos\alpha\sin\beta) & -\sin\alpha\sin\beta\cos\alpha\cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha+\beta) & \sin\alpha(\alpha+\beta) \\ -\sin\alpha(\alpha+\beta) & \cos\alpha(\alpha+\beta) \end{bmatrix} = E(\alpha+\beta)$$

Q.3 (D)

$$A=\underbrace{\begin{bmatrix} k & 0 & 0 & . & . & 0 \\ 0 & k & 0 & . & . & . \\ 0 & 0 & k & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & k & 0 \\ 0 & . & . & . & 0 & k \end{bmatrix}}_{n\ colunm} \Bigg\} n\ rows$$

Hence by the property of diagonal matrices |A| will be product of the elements on principal diagonal i.e. k^n.

Q.4 (B)

Characteristic equation of A is

$$\begin{vmatrix} 3-x & -4 \\ 1 & -1-x \end{vmatrix} = 0 \ i.e.\ x^2 - 2x + 1 = 0.$$

Hence by Cayley Hamilton theorem,

$$A^2 = 2A - I.$$

Now $A^3 = 2A^2 - A = 3A - 2I,\ A^4 = 3A^2 - 2A = 4A - 3I, \ldots,\ A^n = nA - (n-1)I$

Hence $A^n = n\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - (n-1)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 2n+1 & -4n \\ n & 1-2n \end{bmatrix}$.

Q.5 (C)

Standard property of diagonal matrices.

Q.6 (B)

Given A $= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now $A^{-1} = \dfrac{(\text{adj } A)}{|A|}$

Here $|A| = \cos^2\theta + \sin^2\theta = 1$

$\therefore A^{-1} = (\text{adj } A)$

Q.7 Let the number be a , b , c , d (say)

1×1	matrix	$\rightarrow$			4
1×2	matrix	$\rightarrow$	4×3	$\rightarrow$	12
1×3	matrix	$\rightarrow$	$4 \times 3 \times 2$	$\rightarrow$	24
1×4	matrix	$\rightarrow$	$4 \times 3 \times 2 \times 1$	$\rightarrow$	24
2×1	matrix	$\rightarrow$	4×3	$\rightarrow$	12
2×2	matrix	$\rightarrow$	$4 \times 3 \times 2 \times 1$	$\rightarrow$	24
3×1	matrix	$\rightarrow$	$4 \times 3 \times 2$	$\rightarrow$	24
4×1	matrix	$\rightarrow$	$4 \times 3 \times 2 \times 1$	$\rightarrow$	24
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Q.8 (D)

Types of matrices which can be formed using 6 distinct numbers are

$1\times6,\ 2\times3,\ 3\times2\ \&\ 6\times1$.

In each of these elements can be rearranged in 6! ways.

Hence total number of matrices = 4(6!).

Q.9 (D)

from solution of Q.2 we know that $A_\alpha A_\beta = A_{\alpha+\beta}$

Now we have to check for $(A_\alpha)^n$

$$A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A_\alpha A_\beta = A_{\alpha+\beta} \Rightarrow A_\alpha^2 = A_\alpha \cdot A_\alpha = A_{2\alpha}$$

$\Rightarrow A_\alpha^3 = A_{2\alpha} \cdot A_\alpha = A_{3\alpha}$

Hence $(A_\alpha)^n = A_{n\alpha}$.

Q.10 (D)

A is orthogonal $\Rightarrow A^T A = AA^T = I_n$

$\Rightarrow |A^T A| = |AA^T| = 1$

$\Rightarrow |A||A| = 1$

$\Rightarrow |A|^2 = 1$

$\Rightarrow |A| = \pm 1$

Q.11 (A)

A is skew symmetric $\Rightarrow A^T = -A$

$\Rightarrow |A^T| = |-A|$

$\Rightarrow |A^T| = (-1)^n |A|$

$\Rightarrow |A^T| = -|A|$ [$\because$ odd order]

$\Rightarrow |A| = -|A|$

$\Rightarrow 2|A| = 0$

$\Rightarrow |A| = 0$

Q.12 (C)

$\Rightarrow AB = B$

$\Rightarrow (BA)B = B$ ($\because A = BA$)

$\Rightarrow B(AB) = B$

$\Rightarrow B^2 = B$ ……….(1)

Similarly $A^2 = A$ ……….(2)

$\therefore\ A^2 + B^2 = A + B$

Q.13 (C)

Standard property of diagonal matrices.

Q.14 (D)

$$(7A)^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & -7 \end{bmatrix}$$

$$\Rightarrow 7A = \frac{\begin{bmatrix} -7 & -2 \\ -4 & -1 \end{bmatrix}}{7-8}$$

$$\Rightarrow 7A = \begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & \frac{2}{7} \\ \frac{4}{7} & \frac{1}{7} \end{bmatrix}$$

Q.15 (B)

Standard fact : $\det(A^{-1}) = (\det(A))^{-1}$.

Q.16 (C)

Standard fact : AB = 0 implies at least one of A & B must be singular i.e.

$|A| = 0$ or $|B| = 0$.

Q.17 (B)

$$(aI_2 + bA)^2 = A$$

$$\left(\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} + \begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix}\right)^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a^2-b^2 & 2ab \\ -2ab & a^2-b^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$\Rightarrow a^2-b^2=0$ ....(1)

$\Rightarrow -2ab=-1$ ....(2)

$\Rightarrow 2ab=1$ ....(3)

$\Rightarrow a^2-b^2=0$ ....(4)

$$\Rightarrow a=b=\frac{1}{\sqrt{2}}$$

Q.18 (D)

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ -b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & b^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times a^2+c\times ab-b\times ac & 0\times ab+c\times b^2-b\times bc & 0\times ac+c\times bc-b\times c^2 \\ -c\times a^2+0\times ab+a\times ac & -c\times ab+0\times b^2+a\times bc & -c\times ac+0\times bc+a\times c^2 \\ -b\times a^2-a\times ab+0\times ac & -b\times ab-a\times b^2+0\times bc & -b\times ac-a\times bc+0\times c^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Q.19 (B)

$$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\frac{\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}}{\sec^2\theta} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix} = \begin{bmatrix} a\sec^2\theta & -b\sec^2\theta \\ b\sec^2\theta & a\sec^2\theta \end{bmatrix}$$

$\Rightarrow 1-\tan^2\theta = a\sec^2\theta \quad ....(1)$ $\qquad 2\tan\theta = b\sec^2\theta$

$\Rightarrow 1-\tan^2\theta = a(1+\tan^2\theta)$ $\qquad \Rightarrow b = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$\Rightarrow a = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$ $\qquad \boxed{b = \sin 2\theta}$

$\Rightarrow \boxed{a = \cos 2\theta}$

Q.20 (D)

$M^2 - \lambda M - I_2 = 0$

$$\Rightarrow \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} \lambda & 2\lambda \\ 2\lambda & 3\lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$5 - \lambda - 1 = 0$

$\Rightarrow \lambda = 4$.

Alternately

Find characteristic equation of M.

Q.21 (A)

$tr(A) = 60$

a + b + c + d = 60 (1)

$\boxed{b + c = 30}$ $\quad [\because a = b, c = d]$

b can take values from 1 – 14 [$\because b < c \ \& \in N$]

Q.22 (A)

$$\text{Det} = \begin{vmatrix} 5 & 4 & 3 \\ 1 & 1 & 1 \\ 100a+50+1 & 100b+40+1 & 100c+30+1 \end{vmatrix}$$

Applying $R_3 \to R_3 - 10R_1 - R_2$ gives

$$\text{Det} = \begin{vmatrix} 5 & 4 & 3 \\ 1 & 1 & 1 \\ 100a & 100b & 100c \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$ gives

$$\text{Det} = 100 \times \begin{vmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ a+c & b & c \end{vmatrix}$$

$$\text{Det} = 200 \times \begin{vmatrix} 4 & 4 & 3 \\ 1 & 1 & 1 \\ b & b & c \end{vmatrix} \qquad [\because\ a + c = 2b]$$

Hence determinant = 0.

Given matrix is singular.

Q.23 (A)

$$|A| = abc \begin{vmatrix} 1+\frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & 1+\frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ gives

$$|A| = abc \begin{vmatrix} 1+\frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Now applying $C_1 \to C_1 + C_2 + C_3$ gives

$$|A| = abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$|A| = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc \neq 0.$$

$\therefore$ A is non-singular.

Q.24 (A)

$A^2 - B^2 = (A - B)(A + B)$

$A^2 - B^2 = A^2 + AB - BA - B^2$

$\Rightarrow AB - BA = 0$

or AB = BA

Q.25 (C)

$$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

By $R_1 \to R_1 - (R_2 + R_3)$

$$= \frac{1}{abc}\begin{vmatrix} 0 & -2b^2 & -2a^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

Now by $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$= -\frac{2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2 & 0 \\ b^2 & 0 & c^2 \end{vmatrix}$$

$= -\frac{2}{abc}(-2a^2b^2c^2)$

= 4abc

Q.26 (A)

$$\begin{vmatrix} 8 & 27 & 19 \\ 27 & 64 & 37 \\ 64 & 125 & 61 \end{vmatrix} = \begin{vmatrix} 27 & 27 & 19 \\ 64 & 64 & 37 \\ 125 & 125 & 61 \end{vmatrix} \{\text{By } C_1 \to C_1 + C_3\}$$

= 0

Q.27 (B)

By $C_1 \to C_1 + C_2 + C_3$

$$2\begin{vmatrix} a+b+c & b+c & 19 \\ a+b+c & c+a & a+b \\ a+b+c & c+a & b+c \end{vmatrix}$$

By $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$$= 2\begin{vmatrix} a+b+c & -a & -b \\ a+b+c & -b & -c \\ a+b+c & -c & -a \end{vmatrix}$$

$$= 2(-1)^2\begin{vmatrix} a+b+c & a & b \\ a+b+c & b & c \\ a+b+c & c & a \end{vmatrix}$$

Now by $C_1 \to C_1 - (C_2 + C_3)$

$$= 2\begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$\therefore \boxed{\lambda = 2}$

Q.28 (B)

$$\Delta_1 = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$$

$$\Rightarrow \Delta_1 = (-1)\begin{vmatrix} b & a & c \\ y & x & z \\ p & o & r \end{vmatrix}$$

$$\Rightarrow \Delta_1 = \begin{vmatrix} y & x & z \\ b & a & c \\ q & p & r \end{vmatrix}$$

interchanging rows into columns gives

$$\Delta_1 = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix} = \Delta_2$$

Q.29 (B)

Apply $R_1 \to R_1 + R_2 + R_3$ to get

$$\Delta_1 = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix} = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix}$$

Now by $C_1 \to C_1 - C_2$ & $C_3 \to C_3 - C_2$

$$\Delta_1 = (x+y+z)\begin{vmatrix} 0 & 1 & 0 \\ x+y+z & y-z-x & x+y+z \\ 0 & 2z & -(x+y+z) \end{vmatrix} = (x+y+z)^3\begin{vmatrix} 0 & 1 & 0 \\ 1 & y-z-x & 1 \\ 0 & 2z & -1 \end{vmatrix}$$

Hence $\Delta_1 = (x+y+z)^3$.

Further

$$\Delta_2 = 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix} \quad \{By\ C_1 \to C_1 + C_2 + C_3\}$$

$$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 0 & y+z+x & 0 \\ 0 & 0 & z+x+y \end{vmatrix} \quad \{By\ R_2 \to R_2 - R_1\ \&\ R_3 \to R_3 - R_1\}$$

$= 2\ (x+y+z)^3$

$\therefore \Delta_2 = 2\Delta_1$

Q.30 (B)

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{vmatrix}$$

By $R_1 \to R_1 + R_2 + R_3$

$$\Delta_1 = \begin{vmatrix} 3 & 0 & 0 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{vmatrix}$$

$= 3(w^2 - w^4)$

$=3(w^2 - w)$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & w \\ 1 & w & w^2 \\ w^2 & w & 1 \end{vmatrix}$$

By $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2+w & 1 & w \\ 0 & w & w^2 \\ 0 & w & 1 \end{vmatrix}$$

$= (2+w)(w - w^3)$

$$\frac{\Delta_1}{\Delta_2} = \frac{3w(w-1)}{(2+w)(w-1)}$$

$$= \frac{3w}{2+w}$$

$$= i\sqrt{3}$$

Q.31 (A)

Property of determinants : Determinant formed by cofactors of the elements of an $n \times n$ determinant has value equal to $(n – 1)^{th}$ power of value of the determinant.

Q.32 (B)

$$\begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix} \times \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = \begin{vmatrix} ax+by+cz & cx+ay+bz & bx+cy+az \\ za+xb+cy & zc+xa+yb & zb+xc+ya \\ ya+zb+xc & yc+za+xb & yb+zc+xa \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

Q.33 (B)

$$\begin{vmatrix} ax & by & cz \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{xyz}\begin{vmatrix} ax & by & cz \\ x^2 & y^2 & z^2 \\ xyz & xyz & xyz \end{vmatrix}$$

$$= \frac{xyz}{xyz}\begin{vmatrix} a & b & c \\ x & y & z \\ xyz & xyz & xyz \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ x & y & z \\ yz & xz & xy \end{vmatrix}$$

Q.34 (D)

Split the given determinant about third column to get

$$\begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix} = 0$$

Take x, y, z common from I, II & III column respectively in first determinant.

$$xyz\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix} = 0$$

Apply $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$xyz(y-x)(z-x)\begin{vmatrix} 1 & x^2 & x^3 \\ 0 & x+y & x^2+xy+y^2 \\ 0 & z+x & z^2+zx+x^2 \end{vmatrix} - (y-x)(z-x)\begin{vmatrix} x & x^3 & 1 \\ 1 & x^2+xy+y^2 & 0 \\ 1 & z^2+zx+x^2 & 0 \end{vmatrix} = 0$$

Apply $R_3 \to R_3 - R_2$

$$(y-x)(z-x)(z-y)\left\{ xyz\begin{vmatrix} 1 & x^2 & x^3 \\ 0 & x+y & x^2+xy+y^2 \\ 0 & 1 & x+y+z \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ 1 & x^2+xy+y^2 & 0 \\ 0 & x+y+z & 0 \end{vmatrix} \right\} = 0$$

$$\Rightarrow (y-x)(z-x)(z-y)\left\{ xyz\left((x+y)(x+y+z)-\left(x^2+xy+y^2\right)\right)-(x+y+z)\right\} = 0$$

$$\Rightarrow xyz(xy+yz+zx)-(x+y+z)=0$$

$$\Rightarrow xy+yz+zx = \frac{x+y+z}{xyz}$$

Q.35 (A)

Representing the given determinant as product of two determinants gives

$$\Delta = \begin{vmatrix} 1 & 1 & 0 \\ a+b & c+d & 0 \\ ab & cd & 0 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 0 \\ c+d & a+b & 0 \\ cd & ab & 0 \end{vmatrix} = 0$$

Q.36 (D)

$$\sum_{p=1}^{5} D_p = \begin{vmatrix} \sum_{p=1}^{5} p & 15 & 8 \\ \sum_{p=1}^{5} p^2 & 35 & 9 \\ \sum_{p=1}^{5} p^3 & 25 & 10 \end{vmatrix}$$

$$\Rightarrow \sum_{p=1}^{5} D_p = \begin{vmatrix} 15 & 15 & 8 \\ 55 & 35 & 9 \\ 225 & 25 & 10 \end{vmatrix}$$

Now expand to get the answer.

Q.37 (D)

$$\begin{vmatrix} 1 & 3 & \pi \\ \ln e & 3 & \sqrt{5} \\ \log_{10} 10 & 3 & e \end{vmatrix} = 3\begin{vmatrix} 1 & 1 & \pi \\ 1 & 1 & \sqrt{5} \\ 1 & 1 & e \end{vmatrix} = 0$$

Q.38 (A)

Let 1st term, $a_1 = a$ & common ratio $= r$

Then the given determinant becomes

$$\begin{vmatrix} \log ar^{n-1} & \log ar^{n+1} & \log ar^{n+3} \\ \log ar^{n+5} & \log ar^{n+7} & \log ar^{n+9} \\ \log ar^{n+11} & \log ar^{n+13} & \log ar^{n+15} \end{vmatrix}$$

$$= \begin{vmatrix} \log a + (n-1)\log r & \log a + (n+1)\log r & \log a + (n+3)\log r \\ \log a + (n+5)\log r & \log a + (n+7)\log r & \log a + (n+9)\log r \\ \log a + (n+11)\log r & \log a + (n+13)\log r & \log a + (n+15)\log r \end{vmatrix}$$

$C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} \log a + (n-1)\log r & 2\log r & 4\log r \\ \log a + (n+5)\log r & 2\log r & 4\log r \\ \log a + (n+11)\log r & 2\log r & 4\log r \end{vmatrix} = 2\begin{vmatrix} \log a + (n-1)\log r & 2\log r & 2\log r \\ \log a + (n+5)\log r & 2\log r & 2\log r \\ \log a + (n+11)\log r & 2\log r & 2\log r \end{vmatrix} = 0$$

Q.39 (B)

$$\begin{bmatrix} b-c & c-a & a-b \\ b'-c' & c'-a' & a'-b' \\ b''-c'' & c''-a'' & a''-b'' \end{bmatrix} = 0 \quad (c_1 \to c_1 + c_2 + c_3)$$

$\therefore m = 0$

Q.40 (C)

$$\begin{vmatrix} \sin^2 A & \cos A \sin A & \cos^2 A \\ \sin^2 B & \sin B \cos B & \cos^2 B \\ \sin^2 C & \cos C \sin C & \cos^2 C \end{vmatrix} = \frac{1}{2}\begin{vmatrix} \sin^2 A & 2\cos A \sin A & \cos^2 A \\ \sin^2 B & 2\sin B \cos B & \cos^2 B \\ \sin^2 C & 2\cos C \sin C & \cos^2 C \end{vmatrix}$$

By $C_1 \to C_1 + C_3$

$$= \frac{1}{2}\begin{vmatrix} 1 & \sin 2A & \cos^2 A \\ 1 & \sin B & \cos^2 B \\ 1 & \sin 2C & \cos^2 C \end{vmatrix} = \frac{1}{4}\begin{vmatrix} 1 & \sin 2A & 2\cos^2 A \\ 1 & \sin B & 2\cos^2 B \\ 1 & \sin 2C & 2\cos^2 C \end{vmatrix}$$

By $C_3 \to C_3 - C_1$

$$= \frac{1}{4}\begin{bmatrix} 1 & \sin 2A & 1+\cos 2A \\ 1 & \sin 2B & 1+\cos 2B \\ 1 & \sin 3C & 1+\cos 2C \end{bmatrix}$$

By $C_3 \to C_3 - C_1$

$$= \frac{1}{4}\begin{bmatrix} 1 & \sin 2A & \cos 2A \\ 1 & \sin 2B & \cos 2B \\ 1 & \sin 3C & \cos 2C \end{bmatrix}$$

$$= \frac{1}{4}\left[(\sin 2B \cos 2C - \cos 2B \sin 2C) - (\sin 2A \cos 2C - \cos 2A \sin 2c) + (\sin 2A \cos 2B - \cos 2A \sin 2B)\right]$$

$$= \frac{1}{4}[\sin 2(B-C) + \sin 2(C-A) + \sin 2(A-B)]$$

$$= \frac{1}{4} \times 4 \sin(B-C) \sin(C-A) \sin(A-B)$$

$$= \sin(A-B)\, \sin(B-C)\, \sin(C-A)$$