Solutions

COMPLEX NUMBERS

Ex. 2(B)

Q.1 (A), (C)

If vertices of an equilateral triangle are $Z_1, Z_2 \& Z_3$, then $Z_1^2 + Z_2^2 + Z_3^2 = Z_1Z_2 + Z_2Z_3 + Z_3Z_1$.
Now if $Z_1 = \omega \& Z_2 = \omega^2$, then $Z_3 = 1$ or -2.

Q.2 (A), (C)

Opposite sides of rhombus are parallel, hence $\frac{Z_1 - Z_4}{Z_2 - Z_3}$ is purely real.

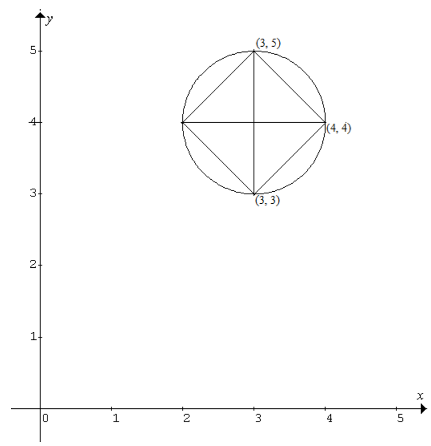
Diagonals of a rhombus are equal in length and mutually perpendicular, hence

$\frac{Z_1 - Z_3}{Z_2 - Z_4}$ is purely imaginary $\& |Z_1 - Z_3| = |Z_2 - Z_4|$.

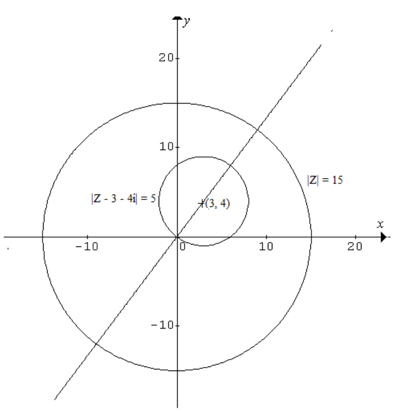
QS will bisect the angle between PS & RS, hence $\text{amp}\frac{Z_1 - Z_4}{Z_2 - Z_4} = \text{amp}\frac{Z_2 - Z_4}{Z_3 - Z_4}$.

Q.3 (B), (C)

As the rectangle is of maximum area hence it must be a square of side length $\sqrt{2}$.
Now the points on $|Z - 3 - 4i| = 1$ at a distance of $\sqrt{2}$ from (4, 4) are (3, 5) & (3, 3).



Q.4 (A), (D)



Minimum distance between any point on the bigger circle and any point on the smaller circle will be equal to radius of bigger circle – diameter of smaller circle i.e. 5. Hence

$|Z_1 - Z_2|_{min} = 5$

Maximum distance between any point on the bigger circle and any point on the smaller circle will be equal to radius of bigger circle + diameter of smaller circle i.e. 25. Hence

$|Z_1 - Z_2|_{max} = 25$

Q.5 (A), (D)

If vertices of an equilateral triangle are Z_1, Z_2 & Z_3, then $Z_1^2+Z_2^2+Z_3^2=Z_1Z_2+Z_2Z_3+Z_3Z_1$.

Hence $Z^2+Z^2+(1-Z)^2=-Z^2-Z(1-Z)+(1-Z)Z \Rightarrow 4Z^2-2Z+1=0$.

$\therefore \sum Z=\frac{1}{2}$ & $\prod Z=\frac{1}{4}$.

Q.6 (B), (C)

Let locus of point P with affix Z be C(Z) and points A & B have affixes Z_1& Z_2.

$C(Z): \frac{|Z-Z_1|}{|Z-Z_2|}=3 \Rightarrow C(Z): (Z-Z_1)(\bar{Z}-\bar{Z}_1)=9(Z-Z_2)(\bar{Z}-\bar{Z}_2)$.

or $C(Z): 8|Z|^2+(Z_1-9Z_2)\bar{Z}+(\bar{Z}_1-9\bar{Z}_2)Z+9|Z_2|^2-|Z_1|^2=0$.

Hence locus of P is a circle.

Now $C(Z_1)=9\left(|Z_1|^2-Z_2\bar{Z}_1-\bar{Z}_2Z_1+|Z_2|^2\right)=9(Z_1-Z_2)(\bar{Z}_1-\bar{Z}_2)$ or $C(Z_1)=9|Z_1-Z_2|^2>0$.

Hence A lies outside the locus of P.

Also $C(Z_2)=-|Z_2|^2-Z_1\bar{Z}_2-\bar{Z}_1Z_2-|Z_1|^2=-(Z_1-Z_2)(\bar{Z}_1-\bar{Z}_2)$ or $C(Z_2)=-|Z_1-Z_2|^2<0$.

Hence B lies inside the locus of P.

Q.7 (A), (D)

Let Z = x + yi, then as given $\frac{y}{x+a}=\frac{1}{\sqrt{3}}$ & $\frac{y}{x-a}=-\sqrt{3} \Rightarrow \frac{y}{x}=\sqrt{3}$.

Also $\frac{y}{x+a}=\frac{1}{\sqrt{3}}$ & $\frac{y}{x-a}=-\sqrt{3} \Rightarrow x=\frac{a}{2}$ & $y=\frac{\sqrt{3}a}{2}$, hence |Z| = a.

Q.8 (A), (C)

$-i=\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2} \Rightarrow (-1)^{1/3}=\cos\left(\frac{2k\pi}{3}+\frac{\pi}{2}\right)+i\sin\left(\frac{2k\pi}{3}+\frac{\pi}{2}\right)$, where $k=0,1,2$.

or $i, -\frac{\sqrt{3}}{2}-\frac{i}{2}$ & $\frac{\sqrt{3}}{2}-\frac{i}{2}$.

Q.9 (B), (C)

$\text{amp}(Z_1Z_2)=0$ & $|Z_1Z_2|=1 \Rightarrow Z_2Z_1=1$,

Also $|Z_1|=|Z_2|=1 \Rightarrow Z_1=\frac{1}{Z_2}=\bar{Z}_2$ or $Z_1=\bar{Z}_2$.

Q.10 (B), (C)

$\left|Z_1^2 - Z_2^2\right| = \left|\bar{Z}_1^{\,2} + \bar{Z}_2^{\,2} - 2\bar{Z}_1\bar{Z}_2\right| \Rightarrow \left|Z_1^2 - Z_2^2\right| = \left|\bar{Z}_1 - \bar{Z}_2\right|^2$

but $|Z| = |\bar{Z}|$, hence $|Z_1 - Z_2||Z_1 + Z_2| = |Z_1 - Z_2|^2$

$\therefore\ |Z_1 + Z_2| = |Z_1 - Z_2|$ as $Z_1 \neq Z_2$.

Now $|Z_1 \pm Z_2|^2 = |Z_1|^2 + |Z_2|^2 \pm 2\left(Z_1\bar{Z}_2 + Z_2\bar{Z}_1\right) \& |Z_1 + Z_2| = |Z_1 - Z_2| \Rightarrow Z_1\bar{Z}_2 + Z_2\bar{Z}_1 = 0$

Or $\dfrac{Z_1}{Z_2} + \left(\dfrac{\bar{Z}_1}{\bar{Z}_2}\right) = 0$ which implies $\dfrac{Z_1}{Z_2}$ is purely imaginary and $\arg\left(\dfrac{Z_1}{Z_2}\right) = \dfrac{\pi}{2}$.

Q.11 (A), (C), (D)

$Z' = Z \times e^{i\alpha}\ \&\ Z'' = Z \times e^{-i\alpha} \Rightarrow Z' \times Z'' = Z^2$.

Hence Z', Z & Z" are in G.P.

Clearly $Z' + Z'' = Z\left(e^{i\alpha} + e^{-i\alpha}\right)$ or $Z' + Z'' = 2Z\cos\alpha$.

Also $(Z')^2 + (Z'')^2 = Z^2\left(e^{2i\alpha} + e^{-2i\alpha}\right)$ or $(Z')^2 + (Z'')^2 = 2Z^2\cos 2\alpha$.

Q.12 (A), (B), (C)

$|Z_1 + iZ_2| \le |Z_1| + |Z_2| \Rightarrow |Z_1 + iZ_2| \le 17$.

$|Z_1 + (1+i)Z_2| \ge \left||Z_1| - |(1+i)Z_2|\right| \Rightarrow |Z_1 + (1+i)Z_2| \ge 13 - 4\sqrt{2}$.

$\left||Z_2| - \dfrac{4}{|Z_2|}\right| \le \left|Z_2 + \dfrac{4}{Z_2}\right| \le |Z_2| + \dfrac{4}{|Z_2|} \Rightarrow 3 \le \left|Z_2 + \dfrac{4}{Z_2}\right| \le 5$.

Now $\dfrac{|Z_1|}{\left|Z_2 + \dfrac{4}{Z_2}\right|} = \dfrac{13}{\left|Z_2 + \dfrac{4}{Z_2}\right|} \Rightarrow \dfrac{13}{5} \le \dfrac{|Z_1|}{\left|Z_2 + \dfrac{4}{Z_2}\right|} \le \dfrac{13}{3}$.

Q.13 (A), (B), (D)

$\left|\dfrac{Z - i/2}{Z + 1}\right| = \dfrac{m}{2}$ will be a circle for all values of m except m = 2.

For m = 2 this eq. will represent a straight line.

Q.14 (A), (D)

The equation $\arg(Z) = \dfrac{\pi}{6}$ represents the ray $\sqrt{3}y = x$.

The equation $\left|Z - 2\sqrt{3}i\right| = r$ represents the circle $x^2 + \left(y - 2\sqrt{3}\right)^2 = r^2$.

Solving the two equations simultaneously, we get $4y^2 - 4\sqrt{3}y + 12 - r^2 = 0$.

For this equation to have distinct real roots $48-16\left(12-r^2\right)>0$ or $r^2>9$.

Hence r > 3 & $[r]\neq 2$.

Q.15 (B), (C)

$$\left|\frac{1}{Z_2}+\frac{1}{Z_1}\right|=\left|\frac{1}{Z_2}-\frac{1}{Z_1}\right|\Rightarrow\left|Z_1+Z_2\right|=\left|Z_1-Z_2\right|\Rightarrow Z_1\bar{Z}_2+Z_2\bar{Z}_1=0.$$

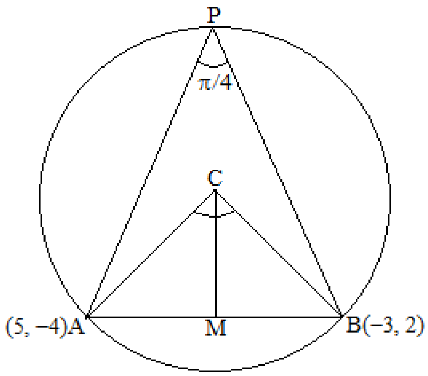
Or $\frac{Z_1}{Z_2}+\frac{\bar{Z}_1}{\bar{Z}_2}=0\Rightarrow\frac{Z_1}{Z_2}$ is purely imaginary .

Hence $\angle POQ=\frac{\pi}{2}$.

Triangle OPQ is right angled with PQ as hypotenuse.

Circum center will be midpoint of PQ i.e. $\frac{Z_1+Z_2}{2}$.

Q.16 (A), (C), (D)



$AM=10$ & $\angle ACM=\frac{\pi}{4}\Rightarrow CA=\frac{5}{\sin\frac{\pi}{4}}$

or radius $=5\sqrt{2}$.

Also coordinates of M are (1, -1).

Now slope of CM is 4/3 and CM = 5.

Coordinates of C will be $\left(1+5\times\frac{3}{5},-1+5\times\frac{4}{5}\right)$ i.e. $(4,3)$ or

$\left(1-5\times\frac{3}{5},-1-5\times\frac{4}{5}\right)$ i.e. $(-2,-5)$.

Length of arc APB = $5\sqrt{2}\times\frac{3\pi}{2}$ i.e. $\frac{15\pi}{\sqrt{2}}$.

Q.17 (A), (B), (C), (D)

Let $\sqrt{5-12i}=x+iy$, then $x^2-y^2=5$ & $2xy=-12$.

Now $x^2+y^2=\sqrt{\left(x^2-y^2\right)^2+4x^2y^2}=13\Rightarrow x=\pm3$ & $y=\pm2$, but $xy<0$, hence $\sqrt{5-12i}=3-2i$ & $-3+2i$.

Similarly let $\sqrt{-5-12i}=x+iy$, then $x^2-y^2=-5$ & $2xy=-12$.

Now $x^2+y^2=\sqrt{\left(x^2-y^2\right)^2+4x^2y^2}=13\Rightarrow x=\pm2$ & $y=\pm3$, but $xy<0$, hence $\sqrt{5-12i}=2-3i$ & $-2+3i$.

Now possible values of $Z=\sqrt{5-12i}+\sqrt{5-12i}=5(1-i)$, $1+i$, $-5(1+i)$ & $-1+i$.

Possible values of $\operatorname{Arg}(Z)=\frac{\pi}{4},-\frac{\pi}{4},\frac{3\pi}{4},-\frac{3\pi}{4}$.

COMPREHENSION TYPE

Paragraph I

$$|Z_1+Z_2|^2=|Z_1|^2+|Z_2|^2+Z_1\overline{Z}_2+Z_2\overline{Z}_1.$$

Q.1 (b)

$|Z_1+Z_2|^2=|Z_1|^2+|Z_2|^2 \Rightarrow Z_1\overline{Z}_2+Z_2\overline{Z}_1=0$, hence $Z_1\overline{Z}_2$ is purely imaginary.

Q.2 (b)

$Z_1\overline{Z}_2+Z_2\overline{Z}_1=0 \Rightarrow \dfrac{Z_1}{Z_2}+\dfrac{\overline{Z}_1}{\overline{Z}_2}=0$, hence $\dfrac{Z_1}{Z_2}$ is purely imaginary.

Q.3 (c)

As $\dfrac{Z_1}{Z_2}$ is purely imaginary therefore $i\dfrac{Z_1}{Z_2}$ is purely real.

Q.4 (c)

As $\dfrac{Z_1}{Z_2}$ is purely imaginary therefore $\arg\left(\dfrac{Z_1}{Z_2}\right)$ i.e. $\arg(Z_1)-\arg(Z_2)=\pm\dfrac{\pi}{2}$.

Paragraph II

$$Z=\frac{1-i\sin\theta}{1+i\sin\theta}\Rightarrow Z=\frac{\cos^2\theta}{1+\sin^2\theta}-\frac{2\sin\theta}{1+\sin^2\theta}i$$

Q.5 (c)

If Z is purely real, then $\dfrac{2\sin\theta}{1+\sin^2\theta}=0 \Rightarrow \theta=n\pi$.

Q.6 (d)

If Z is purely imaginary, then $\dfrac{\cos^2\theta}{1+\sin^2\theta}=0 \Rightarrow \theta=(2n+1)\dfrac{\pi}{2}$.

Q.7 (d)

$|Z|=1\Rightarrow \cos^4\theta+4\sin^2\theta=\left(1+\sin^2\theta\right)^2$ or $\cos^2\theta+\sin^2\theta=1$.

Hence Z is unimodular for all real values of θ.

Q.8 (d)

$\arg(Z)=\dfrac{\pi}{4} \Rightarrow -\dfrac{2\sin\theta}{\cos^2\theta}=\tan\dfrac{\pi}{4}$ or $\sin^2\theta-2\sin\theta-1=0$.

Now $\sin^2\theta - 2\sin\theta - 1 = 0$ gives $\theta = n\pi + (-1)^n \sin^{-1}\left(1-\sqrt{2}\right)$.

Paragraph III

$$\left||Z| - \frac{4}{|Z|}\right| \le \left|Z - \frac{4}{Z}\right| \le |Z| + \frac{4}{|Z|} \Rightarrow \left||Z| - \frac{4}{|Z|}\right| \le 2 \ \& \ |Z| + \frac{4}{|Z|} \ge 2.$$

Now $|Z| + \dfrac{4}{|Z|} \ge 2 \Rightarrow |Z|^2 - 2|Z| + 4 \ge 0$, which is true for all $|Z|$.

Similarly $-2 \le |Z| - \dfrac{4}{|Z|} \le 2 \Rightarrow |Z|^2 + 2|Z| - 4 \ge 0 \ \& \ |Z|^2 - 2|Z| - 4 \le 0.$

Q.9 (a)

$\sqrt{5} - 1 \le |Z| \le \sqrt{5} + 1.$

Hence the difference in least & the greatest values of |Z| is 2.

Q.10 (b)

$$\left|Z - \frac{4}{Z}\right| = 2 \Rightarrow |Z|^2 + \frac{16}{|Z|^2} - 4\frac{Z}{\overline{Z}} - 4\frac{\overline{Z}}{Z} = 4 \ \Rightarrow \left(|Z| - \frac{4}{|Z|}\right)^2 - 4\frac{Z^2}{|Z|^2} - 4\frac{\overline{Z}^2}{|Z|^2} + 4 = 0$$

But for greatest & least $|Z|$, $\left(|Z| - \dfrac{4}{|Z|}\right)^2 = \left|Z - \dfrac{4}{Z}\right|^2$. Hence $\dfrac{Z^2}{|Z|^2} + \dfrac{\overline{Z}^2}{|Z|^2} = 2$ i.e. $\dfrac{2\,\mathrm{Re}\left(Z^2\right)}{|Z|^2} = 2$.

or $\mathrm{Re}\left(Z^2\right) = |Z|^2 \Rightarrow \cos 2\theta = 1$ or $\theta = \pi \ \& \ 0$.

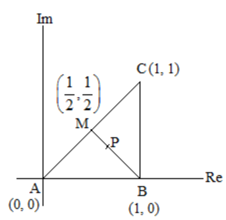
Now $\arg\left(\dfrac{Z_1}{Z_2}\right) = \arg(Z_1) - \arg(Z_2) = \pi.$

Q.11 (b)

$|Z - Z_1| = |Z - Z_2| \Rightarrow \left|Z - \left(\sqrt{5} + 1\right)\right| = \left|Z - \left(\sqrt{5} - 1\right)\right|.$

Hence locus of Z will be the line passing through $\left(\sqrt{5}, 0\right)$ and parallel to the Imaginary axis.

Paragraph IV



Let Z = x + iy so that P(Z) be (x, y), then $\dfrac{y}{x-1} = \dfrac{0 - \frac{1}{2}}{1 - \frac{1}{2}}$ or $x + y = 1$.

Now $\mu = Z^2 + 1 = x^2 - y^2 + 2ixy$.

Q.12 (a)

$\mu = h + ki$, then $Z^2 + 1 = h + ki \Rightarrow x^2 - y^2 = h - 1, 2xy = k \ \& \ x + y = 1 \Rightarrow x^2 + y^2 = 1 - k.$

Now $2x^2 = h - k \,\&\, 2y^2 = 2 - h - k$

$\therefore 4x^2y^2 = k^2 \Rightarrow (h-k)(2-h-k) = k^2$ **or** $(h-1)^2 = -2\left(k - \frac{1}{2}\right)$

Hence locus of (μ) is the parabola $(x-1)^2 = -2\left(y - \frac{1}{2}\right)$.

Q.13 (c)

Locus of m is $(x-1)^2 = -2\left(y - \frac{1}{2}\right)$, whose axis is x = 1 i.e. $Z + \overline{Z} = 2$.

Q.14 (b)

Locus of m is $(x-1)^2 = -2\left(y - \frac{1}{2}\right)$, whose directrics is y = 1 i.e. $Z - \overline{Z} = 2i$.

ASSERTION REASONING TYPE

Q.1 (A)

Greatest possible value of principal argument of any complex number is π, hence
$\arg(Z_1Z_2) = 2\pi \Rightarrow \text{Arg}(Z_1) = \text{Arg}(Z_2) = \pi.$
Hence both the statements are true & statement 2 is the correct explanation of statement I.

Q.2 (C)

Standard concept : $1 + \omega^n + \omega^{2n} = \begin{cases} 3, & \text{if } n = 3m \\ 0, & \text{otherwise} \end{cases}$, hence statement 2 is false.

Now $P(x) = x^3 + x^2 + x = x(1 + x + x^2) = x(x - \omega)(x - \omega^2)$, hence roots of $P(x)$ are $0, \omega \,\&\, \omega^2$.

Now let $Q(x) = (1+x)^n - 1 - x^n$, then

$Q(\omega) = (1+\omega)^n - 1 - \omega^n$ or $Q(\omega) = (-\omega^2)^n - 1 - \omega^n$.

As n is odd integer & not a multiple of 3, hence $Q(\omega) = -(1 + \omega^n + \omega^{2n}) = 0$.

Similarly $Q(\omega^2) = 0$ & $Q(0) = 0$.

Hence P(x) divides Q(x) and statement 1 is true.

Q.3 (D)

Now $|Z_1 \pm Z_2|^2 = |Z_1|^2 + |Z_2|^2 \pm 2(Z_1\overline{Z}_2 + Z_2\overline{Z}_1) \,\&\, |Z_1 + Z_2| = |Z_1 - Z_2| \Rightarrow Z_1\overline{Z}_2 + Z_2\overline{Z}_1 = 0$

Or $\frac{Z_1}{Z_2} + \left(\frac{\overline{Z}_1}{\overline{Z}_2}\right) = 0$ which implies $\frac{Z_1}{Z_2}$ is purely imaginary and $\arg\left(\frac{Z_1}{Z_2}\right) = \frac{\pi}{2}$.

Hence the triangle AOB is right angled at O.

Now the point $P\left(\frac{Z_1+Z_2}{2}\right)$ is midpoint of AB, hence its circumcenter.

Q.4 (D)

Statement 2 is a standard property of an ellipse.
Equation of an ellipse having $S(Z_1) \& S'(Z_2)$ as foci and major axis = 2a is
$|Z-Z_1|+|Z-Z_2|=2a$.
But for the given equation distance between (1, 0) & (8, 0) is more than 5, but in ellipse distance between foci is less than the major axis.
Statement 1 is false.

Q.5 (A)

If $1, \alpha, \alpha^2, \ldots, \alpha^{n-1}$ represent n roots of unity, then product of these is 1, hence statement 2 is true.

Similarly if $Z=\left(e^{i\alpha}\right)^{3/5}$, then $Z^5=e^{3i\alpha}$. Let Z_k denote roots of Z for $k=0,1,2,3,4$, then

$Z_k=e^{\frac{2k\pi+3\alpha}{5}i}$ or $Z=e^{\frac{2k\pi}{5}i}e^{\frac{3\alpha}{5}i}$, where $e^{\frac{2k\pi}{5}i}$ denotes 5 roots of unity.

Now $\prod_{k=0}^{4} Z_k=\left(\prod_{k=0}^{4} e^{\frac{2k\pi}{5}i}\right)\left(\prod_{k=0}^{4} e^{\frac{3\alpha}{5}i}\right) \Rightarrow \prod_{k=0}^{4} Z_k=e^{3i\alpha}$ i.e. $\cos 3\alpha+i\sin 3\alpha$..

Hence both the statements are true & statement 2 is the correct explanation of statement

MATRIX MATCH TYPE

Q.1 $\mathbf{(A)\to(s),(B)\to(r),(C)\to(p),(D)\to(q)}$

(A) $Z^4=1 \Rightarrow Z=\cos\frac{k\pi}{2}+i\sin\frac{k\pi}{2}$ for $k=0,1,2,3$.

(B) $Z^4=-1 \Rightarrow Z=\cos\frac{(2k+1)\pi}{4}+i\sin\frac{(2k+1)\pi}{4}$ for $k=0,1,2,3$.

(C) $Z^4=i \Rightarrow Z=\cos\frac{(4k+1)\pi}{8}+i\sin\frac{(4k+1)\pi}{8}$ for $k=0,1,2,3$.

(D) $Z^4=-i \Rightarrow Z=\cos\frac{(4k-1)\pi}{8}+i\sin\frac{(4k-1)\pi}{8}$ for $k=0,1,2,3$.

Q.2 $\mathbf{(A)\to(q),(B)\to(s),(C)\to(p),(D)\to(r)}$

(A) $P(Z)$ lies on perpendicular bisector of line segment joining $A(1,0) \& B(0,1)$.

(B) $\left|Z+\overline{Z}\right|+\left|Z-\overline{Z}\right|=2 \Rightarrow |x|+|y|=1,$ where $Z=x+iy.$

(C) $\left|Z+\overline{Z}\right|=\left|Z-\overline{Z}\right| \Rightarrow |x|=|y|,$ where $Z=x+iy.$

(D) Let $Z=x+iy$ & $\frac{2}{Z}=h+ik,$ then $h=\frac{2x}{x^2+y^2}$ & $k=-\frac{2k}{x^2+y^2}.$

But $|Z|=1 \Rightarrow x^2+y^2=1,$ hence $h^2+k^2=4.$

Q.3 $(A)\to(p),(r);(B)\to(p),(q),(r),(t);(C)\to(p),(r),(s);(D)\to(p),(q),(r),(s),(t)$

Let Z_1, Z_2, Z_3 & Z_4 represent the points A, B, C & D.

(p) $Z_1-Z_4=Z_2-Z_3 \Rightarrow \frac{Z_1+Z_3}{2}=\frac{Z_2+Z_4}{2}$, hence AC & BD i.e. diagonals bisect each other.

This property is true in case of a parallelogram, a rhombus, a rectangle or a square.

(q) $|Z_1-Z_3|=|Z_2-Z_4| \Rightarrow$ AC = BD i.e. Diagonals are of equal length.

This property is true in case of a rectangle or a square.

(r) $\frac{Z_1-Z_2}{Z_3-Z_4}$ is purely real hence $\arg\left(\frac{Z_1-Z_2}{Z_3-Z_4}\right)=0$ or $\pi.$ Hence AB || CD.

This property is true in case of a parallelogram, a rhombus, a rectangle or a square.

(s) $\frac{Z_1-Z_3}{Z_2-Z_4}$ is purely imaginary hence $\arg\left(\frac{Z_1-Z_3}{Z_2-Z_4}\right)=\pm\frac{\pi}{2}.$ Hence $AC \perp BD$.

This property is true in case ofa rhombus or a square.

(t) $\frac{Z_1-Z_2}{Z_3-Z_2}$ is purely imaginary hence $\arg\left(\frac{Z_1-Z_2}{Z_3-Z_2}\right)=\pm\frac{\pi}{2}.$ Hence $AB \perp BC$.

This property is true in case ofa rectangle or a square.

Q.4 $(A)\to(q),(r);(B)\to(p),(s);(C)\to(q),(s);(D)\to(p),(r)$

(a) $Z^2-Z+1=0$

$$\Rightarrow Z=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm\sqrt{3}i}{2}$$

$$\Rightarrow \text{Hence } \frac{\pi}{3} \text{ or } \frac{-\pi}{3}$$

(b) $Z^2+Z+1=0$

$$\Rightarrow Z=\frac{-1\pm i\sqrt{3}}{2}$$

$\Rightarrow$ Hence, $\frac{2\pi}{3}$ or $\frac{-2\pi}{3}$

(c) $Z^2 = \frac{-1-i\sqrt{3}}{2} = \omega^2$

$\Rightarrow Z = \omega$ or $Z = -\omega$

$\Rightarrow \frac{2\pi}{3}$ or $\frac{-\pi}{3}$

(d) $Z^2 = \frac{-1+i\sqrt{3}}{2} = \omega = \omega \times \omega^3 = \omega^4$

$\Rightarrow \therefore Z = \pm\omega^2$

$\Rightarrow$ Hence, principle values of arg (Z) are $\frac{-2\pi}{3}$ or $\frac{\pi}{3}$