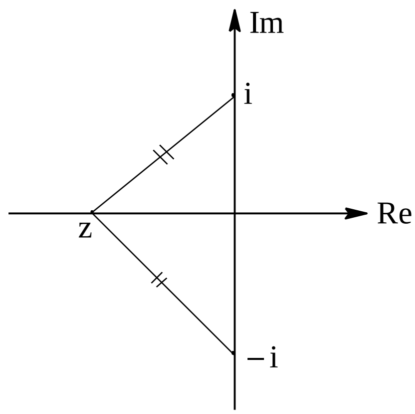
Complex Number

Exercise – 1(B)

Q.1 [B]

$$w = \frac{1 - iz}{z - i} = \frac{-i(z+i)}{(z-i)}$$



$$|w| = \left|\frac{z+i}{z-i}\right| = 1$$

$$|z+i| = |z-i|$$

z lies on real axis.

Q.2 [C]

$$|z| = |z_1 - z_2| = |-3-i| = 5$$

Q.3 [B]

$$z\bar{z} + a\bar{z} + \bar{a}z + b = 0;\ b \in R$$

radius of the circle $= |a|^2 - b > 0$

$$\therefore |a|^2 > b$$

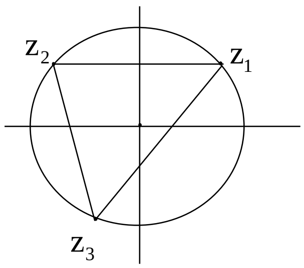
Q.4

$|z_1| = |z_2| = |z_3| = r$ (let's take)

Let $z_1 = re^{i\theta}$

$z_2 = re^{i\left(\theta + \frac{2\pi}{3}\right)}$

$z_3 = re^{i\left(\theta - \frac{2\pi}{3}\right)}$



$\therefore z_1 + z_2 + z_3 = r(0) = 0$

Q.5 [D]

$G\left(\frac{z_1 + z_2 + z_3}{3}\right) \qquad A(z_1)$

$\therefore$ mid point of AG $z = \dfrac{\frac{z_1 + z_2 + z_3}{3} + z_1}{2} = 0$

$\therefore 4z_1 + z_2 + z_3 = 0$

Q.6 [B]

$|z_1| = 12, \ |z_2 - 3 - 4i| = 5$

$|z_1 - z_2| = |z_1 + (-z_2 + 3 + 4i) + (-3 - 4i)|$

$||z_1| - |z_2 - 3 - 4i|| \le |z_1 + (-z_2 + 3 + 4i)| \le |z_1 - z_2| + |3 + 4i|$

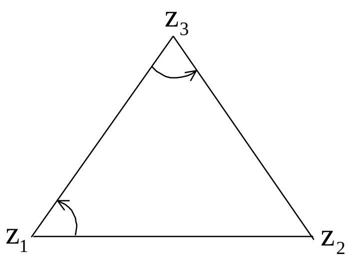
$7 \le |z_1 - z_2| + 5$

$\therefore |z_1 - z_2| \ge 2$

$|z_1 - z_2|_{min} = 2$

Q.7 [B]

For



$$\frac{z_3 - z_1}{z_2 - z_1} = e^{\frac{i\pi}{3}} = \frac{z_2 - z_3}{z_1 - z_3}$$

$$-(z_1 - z_3)^2 = (z_2 - z_3)(z_2 - z_1)$$

$$-\left(z_1^2 + z_3^2 - 2z_1z_3\right) = z_2^2 - z_1z_2 - z_2z_3 + z_1z_3$$

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$$

Q.8 [C]

Let $\arg(z) = \theta$

Then $\arg(-iz) = \arg(-i) + \arg(z) = \frac{-\pi}{2} + \theta$

$\therefore \arg(z) - \arg(-iz) = \frac{\pi}{2}$

Q.9 [C]

$$\text{Re}\left(\frac{z+4}{2z-i}\right) = \frac{1}{2}$$

$$\frac{z+4}{2z-i} + \frac{\bar{z}+4}{2\bar{z}+i} = 1$$

$$(z+4)(2\bar{z}+i) + (\bar{z}+4)(2z-i) = (2z-i)(2\bar{z}+i)$$

$$2|z|^2 + iz + 8\bar{z} + 4i + 2|z|^2 - i\bar{z} + 8z - 4i = 4|z|^2 + 2zi - 2i\bar{z} + 1$$

$$z i - i\bar{z} - 8\bar{z} - 8z + 4i + 1 = 0$$

$$z(i - 8) - \bar{z}(8 + i) + 4i + 1 = 0$$

$$z(8 - i) + \bar{z}(8 + i) - 4i - 1 = 0$$

This is equation of straight line

Q.10 [C]

$$\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$$

$$z_1^2 + z_2^2 = z_1 z_2$$

For equilateral triangle with vertices z_1, z_2, z_3

$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$$

if $z_3 = 0$

$$z_1^2 + z_2^2 = z_1 z_2$$

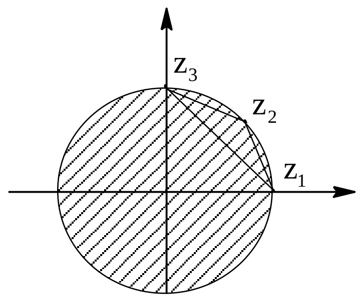
Q.11

$$z_1 = 1, z_2 = \frac{1+i}{\sqrt{2}}, z_3 = i$$

$$|z_1 - z_3| = \sqrt{2}$$

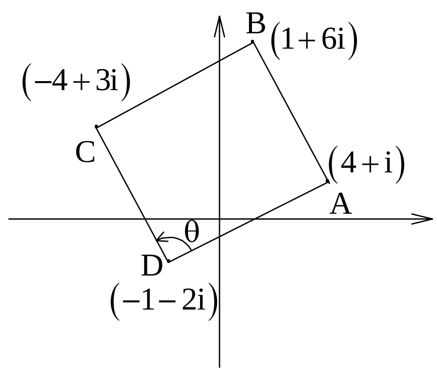
$$|z_2 - z_3| = \left|\frac{1+i}{\sqrt{2}} - i\right| = \left|\frac{1}{\sqrt{2}} + i\left(\frac{1}{\sqrt{2}} - 1\right)\right| = \sqrt{\frac{1}{2} + \frac{1}{2} + 1 - \sqrt{2}} = \sqrt{2 - \sqrt{2}}$$

$$|z_1 - z_2| = \sqrt{\left(1 - \frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}} = \sqrt{2 - \sqrt{2}}$$



Triangle is isoleces triangle

Q.12 [B]



$$|AB| = |BC| = |CD| = |DA|$$

$$\frac{Z_C - Z_D}{Z_A - Z_D} = \frac{-3+5i}{5+3i} = i = e^{\frac{i\pi}{2}}$$

$$\therefore \theta = \frac{\pi}{2}$$

$\therefore$ ABCD is square

Q.13 [B]

90^o

Q.14 [C]

By triangle inequality

$$||z_1| - |z_2|| \le |z_1 + z_2| \le |z_1| + |z_2|$$

Q.15

Q.16

$(z_1 - z_2) = \lambda(z_2 - z_3)$ for collinear

$$(3-2i) = \lambda\left(-2 + i\left(3 - \frac{a}{3}\right)\right)$$

$$3 = -2\lambda$$

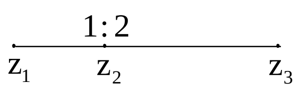
$$-2 = \lambda\left(3 - \frac{a}{3}\right)$$

$$-2 = \frac{-3}{2}\left(3 - \frac{a}{3}\right)$$

a = 5

Q.17 [B]

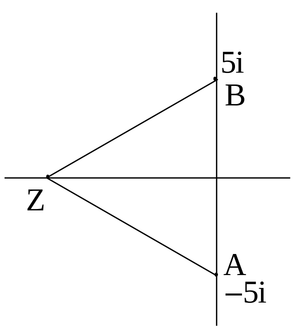
$$2z_1 - 3z_2 + z_3 = 0$$



$$z_2 = \frac{2z_1 + z_3}{2+1}$$

Collinear points.

Q.18 [A]



$$|Z - 5i| = |Z + 5i|$$

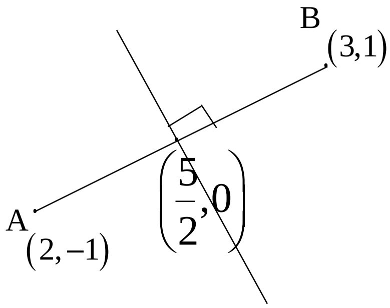
So locus of z will perpendicular bisector of AB or z lies on real axis.

$\therefore$ x = 0

Q.19 [A]

$$\left|Z-(2-i)\right|=\left|Z-(3+i)\right|$$

Locus of z will be perpendicular bisector of line segment AB. A(2, – 1), B (3, 1)



$$m_{AB}=\frac{1+1}{1}=2$$

$\therefore$ locus of z is

$$y-0=-\frac{1}{2}\left(x-\frac{5}{2}\right)$$

$$x+2y=\frac{5}{2}$$

Q.20 [D]

$$\left(Z-2-3i\right)=\frac{\pi}{4}$$

$$\text{amp}(x-2)+i(y-3)=\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{y-3}{x-2}\right)=\frac{\pi}{4}$$

y – 3 = x – 2

x – y + 1 = 0

Q.21

$$\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{6}$$

$$\arg(z-2)-\arg(z+2)=\frac{\pi}{6}$$

$$\tan^{-1}\left(\frac{y}{x-2}\right)-\tan^{-1}\left(\frac{y}{x+2}\right)=\frac{\pi}{6}$$

$$\tan^{-1}\left[\frac{\left(\frac{y}{x-2}-\frac{y}{x+2}\right)}{1+\left(\frac{y^2}{x^2-4}\right)}\right]=\frac{\pi}{6}$$

$$\frac{yx+2y-xy+2y}{x^2-4+y^2}=\sqrt{3}$$

$$x^2+y^2=4+\frac{4y}{\sqrt{3}}$$

$$x^2+y^2-\frac{4y}{\sqrt{3}}-4=0$$

$$z=\lambda+3+i\sqrt{5-\lambda^2}$$

It is a circle.

Q.22

$$x=\lambda+3 \ \& \ y=\sqrt{5-\lambda^2}$$

$$x=\lambda+3 \,\&\, y=\sqrt{5-\lambda^2}$$

$$y^2=5-(x-3)^2$$

$$(x-3)^2+y^2=5$$

Q.23 Repeated Question (21) of Ex. (2)

Q.24 [D]

$$|z-2|=2|z-3|$$

$$(x-2)^2+y^2=4\left((x-3)^2+\left(y^2\right)\right)$$

$$3x^2+3y^2-20x+32=0$$

$$x^2+y^2-\frac{20}{3}x+\frac{32}{3}=0$$

$$r=\sqrt{\left(\frac{10}{3}\right)^2-\frac{32}{3}}=\sqrt{\frac{4}{9}}=\frac{2}{3}$$

Q.25 [A]

$$i^{\frac{1}{3}}=\left(e^{i\frac{\pi}{2}}\right)^{\frac{1}{3}}=e^{i\frac{\pi}{6}}$$

$$\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$$

$$\frac{\sqrt{3}+i}{2}$$

Q.26 [D]

$$\left(-1+i\sqrt{3}\right)^{20}=\left[2\left(e^{i\frac{2\pi}{3}}\right)\right]^{20}$$

$$2^{20}e^{i\frac{40\pi}{3}}$$

$$1^{20}e^{i\frac{4\pi}{3}}=2^{20}\left(\frac{-1-i\sqrt{3}}{2}\right)$$

Q.27 [A]

$$z=\frac{\sqrt{3}+i}{2}=-i\left(\frac{-1+i\sqrt{3}}{2}\right)$$

$$z=-i\omega$$

$$z^{69}=\left(-i\right)^{69}\omega^{69}=-i$$

Q.28 [C]

$$(\sin\theta + i\cos\theta)^n = \left[\cos\left(\frac{\pi}{2} - \theta\right) + i\sin\left(\frac{\pi}{2} - \theta\right)\right]^n$$

$$\cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$$

Q.29

$$z = \left(\frac{\cos\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}} = (\cos\pi + i\sin\pi)^{\frac{1}{4}}$$

$$(-1)^{\frac{1}{4}}$$

$$z^4 = -1$$

$$z = \left(\cos\left(\frac{2k\pi + \pi}{4}\right) + i\sin\left(\frac{2k\pi + \pi}{4}\right)\right)$$

$$z_1 = e^{i\frac{\pi}{4}}$$

$$z_2 = e^{i\frac{3\pi}{4}}$$

$$z_3 = e^{i\frac{5\pi}{4}}$$

$$z_3 = e^{i\frac{7\pi}{4}}$$

Product of roots $= e^{i\left(\frac{\pi}{4} + \frac{3\pi}{4} + \frac{5\pi}{4} + \frac{7\pi}{4}\right)}$

$$e^{i(4\pi)} = 1$$

Q.30 [D]

$$\left(\frac{1 + \cos\theta + i\sin\theta}{i + \sin\theta + i\cos\theta}\right)^n = \cos n\theta + i\sin n\theta$$

$$\left[\frac{2\cos\frac{\theta}{2}\left(e^{i\frac{\theta}{2}}\right)}{2\cos\frac{\theta}{2}\left(\sin\frac{\theta}{2}+i\cos\frac{\theta}{2}\right)}\right]^n$$

$$\left[\frac{e^{i\frac{\theta}{2}}}{ie^{i\frac{\theta}{2}}}\right]^n=\left[-ie^{i\theta}\right]^n$$

$$(-i)^n(\cos n\theta+i\sin\theta)$$

So, n = 4

Q.31 [C]

$$\left[\frac{1-\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}}{1-\cos\frac{\pi}{10}-i\sin\frac{\pi}{10}}\right]^{10}=\left[\frac{2\sin\frac{\pi}{10}\left(\sin\frac{\pi}{20}+icoc\frac{\pi}{20}\right)}{\left(2\sin\frac{\pi}{20}\right)\left(\sin\frac{\pi}{20}-i\cos\frac{\pi}{20}\right)}\right]^{20}$$

$$\left[\frac{i\,e^{-i\frac{\pi}{20}}}{-ie^{i\frac{\pi}{20}}}\right]^{20}=\left[-e^{-i\frac{\pi}{10}}\right]^{20}=e^{-i2\pi}$$

1

Q.32 [D]

$$\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\left(\frac{2\pi k}{7}\right)\right)$$

$$\sum_{k=1}^{6}-ie^{i\frac{2\pi k}{7}} \qquad \text{Let } \alpha=e^{i\frac{2\pi}{7}}$$

$$-i\left[\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5+\alpha^6\right]$$

– i [–1] = i

Q.33 [B]

$$\cos\left(\theta+\frac{\theta}{2}+\frac{\theta}{2^2}+\frac{\theta}{2^3}+......\right)+\sin\left(\theta+\frac{\theta}{2}+\frac{\theta}{2^2}+\frac{\theta}{2^3}+.......\right)$$

$$\cos(2\theta)+i\sin 2\theta$$

Q.34 [C]

$$z^3=-1$$

$$z=-1,-\omega,-\omega^2$$

where ω is cube root of unity.

$$z_1z_2z_3=(-1)(-\omega)(-\omega^2)=-1$$

Q.35

$$\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)=\left[\frac{2\left(e^{i\frac{\pi}{3}}\right)}{2e^{-i\frac{\pi}{3}}}\right]^n=\left[e^{i\frac{2\pi}{3}}\right]^n$$

$$e^{i\frac{2\pi n}{3}}$$

it will be integer if n = 3.

Q.36 [C]

$$(1+\omega)^7=A+B\omega$$

$$(-\omega^2)^7=A+B\omega$$

$$(-\omega^2)=A+B\omega$$

$$1+\omega=A+B\omega$$

A = 1 = B

Q.37 [C]

$$(3t\omega+3\omega^2)^4=(-3\omega+\omega)^4=(-2\omega)^4=16\omega$$

Q.38 [A]

$$\left(3+\omega^2+\omega^4\right)^6=\left(3+\omega^2+\omega\right)^6=\left(2+0\right)^6=64$$

Q.39 [C]

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1+\omega+\omega^2=0$$

Q.40 [B]

Given question is wrong. Actual question is

$$\left(z+1\right)^3=8\left(z-1\right)^3$$

$$\left(\frac{z+1}{z-1}\right)=\left(8\right)^{\frac{1}{3}}$$

$$\frac{z+1}{z-1}=2,2\omega,2\omega^2$$

$$z=3,\frac{2\omega+1}{2\omega-1},\frac{2\omega^2+1}{2\omega^2-1}$$

$$\therefore z_1+z_2+z_3=\frac{27}{7}$$

$$\text{Re}\left(z_1+z_2+z_3\right)=\frac{27}{7}$$