COMPLEX NUMBER

EXERCISE – 1(A)

Q.1 [B]

$$\Rightarrow \sqrt{-2}\sqrt{-3} = \left(i\sqrt{2}\right)\left(i\sqrt{3}\right) = -\sqrt{6}$$

Q.2 [D]

$$\Rightarrow (1+i)^5 (1-i)^5 = \left(1-i^2\right)^5 = 2^5$$

Q.3 [B]

$$(1+i)^4 + (1-i)^4 = \left[(1+i)^2 + (1-i)^2\right]^2 - 2(1+i)^2(1-i)^2$$

$$\Rightarrow \left[(2i)+(-2i)\right]^2 - 2\left(1-i^2\right)^2$$

$$\Rightarrow -2.2^2 = -8$$

Q.4 [C]

$$(1+i)^8 + (1-i)^8 = \left[(1+i)^4 + (1-i)^4\right]^2 - 2(1+i)^4(1-i)^4$$

$$\Rightarrow [-8]^2 - 2\left(1-i^2\right)^4$$

$$\Rightarrow 64 - 2(2)^4 = 32$$

Q.5 [A]

$$(1+i)^6 + (1-i)^6 = \left[(1+i)^3 + (1-i)^3\right]^2 - 2(1+i)^3(1-i)^3$$

$$\Rightarrow \left[1+3i^2+3i+i^3+1-i^3+3i^2-3i\right]^2 - 2\left(1-i^2\right)^3$$

$$\Rightarrow [2-6]^2 - 2(2)^3$$

Q.6 [A]

$$\Rightarrow (1+i)^{10} = \left[(1+i)^2\right]^5 = (2i)^5 = 32i$$

Q.7 [A]

$\Rightarrow 1+i^2+i^3-i^6+i^8 = 1-1-i-i^2+1 = 2-i$

Q.8 [B]

$\because i^4 = 1$

$\Rightarrow \therefore i^{4n+\lambda} = i^{\lambda}$ where $n \in I$; $i^{4n+2} = i^2 = -1$

$\therefore$ given expression will become

$$\Rightarrow \frac{1-1+1-1+1}{-1+1-1+1-1} - 1 = -2$$

Q.9 [D]

For given equation to be true

$(1-i)^n = 2^n$

$\Rightarrow$ n = 4m; $m \in I$

$\Rightarrow$ min n = 4

Q.10 [A]

$$\left(\frac{-1+i}{1+i}\right)^n = \text{Real number}$$

$$\Rightarrow \left(\frac{-1+i}{1+i}\right)^n \left(\frac{1-i}{1-i}\right)^n = \frac{(1-i)^{2n}(-1)^n}{(1+1)^n} = \frac{(1+i^2-2i)^n(-1)^n}{2^n}$$

$$\Rightarrow \frac{2^n i^n}{2^n} = i^n$$

least n = 2

Q.11 [B]

$(a+ib)^5 = \alpha + i\beta$

$\Rightarrow i^5(-ai+b)^5 = \alpha + i\beta$

$$\Rightarrow (b-ai)^5 = \beta - \alpha i$$

Take complex conjugate then

$$\Rightarrow (b+ai)^5 = \beta + \alpha i$$

Q.12 [B]

$$\frac{1+2i}{1-i}$$

$$\Rightarrow \frac{1+2i}{1-i} \times \frac{1+i}{1+i}$$

$$\Rightarrow \frac{1+3i+2i^2}{2}$$

$$\Rightarrow \frac{-1+3i}{2} \qquad 2^{nd} \text{ quadrate}$$

Q.13 [A]

$$|z| = 1,\ w = \frac{z-1}{z+1} \qquad (z \neq -1)$$

Let $z = \cos\theta + i\sin\theta$

$$\Rightarrow \therefore w = \frac{(\cos\theta - 1) + i\sin\theta}{(\cos\theta + 1) + i\sin\theta} \neq 1$$

$$\Rightarrow \frac{-2\sin^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2} + \cos\frac{\theta}{2}}$$

$$\Rightarrow \frac{2i\sin\frac{\theta}{2}}{2\cos\frac{\theta}{2}} \left(\frac{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}} \right)$$

$$\Rightarrow w = i\tan\frac{\theta}{2}$$

$$\Rightarrow \therefore \text{Re}(w) = 0$$

Q.14 [C]

$$\frac{3+2i\sin\theta}{1-2i\sin\theta} = Ki; K \in R$$

$$\Rightarrow \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta} = Ki$$

$$\Rightarrow \therefore \frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0 \ \ (\text{Real part zero})$$

$$\Rightarrow \sin^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2 = \sin^2 60^o$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$$

Q.15 [B]

Given $x = \frac{1}{x}$

$$\Rightarrow x = \pm 1$$

Q.16 [C]

z = 1 + i

$$\Rightarrow z^2 = 1 + i^2 + 2i = 2i$$

Let z_1 is multiplication inverse

$$\Rightarrow \therefore z^2 z_1 = 1$$

$$\Rightarrow z_1 = \frac{1}{z^2} = \frac{1}{zi} = \frac{-i}{z}$$

Q.17 [B]

$$(x+iy)^{\frac{1}{3}} = a + ib$$

$$\Rightarrow x + iy = (a+ib)^3$$

$$\Rightarrow x = a^3 - 3ab^2$$

$$\Rightarrow y = -b^3 + 3a^2b$$

$$\Rightarrow \therefore \frac{x}{a} + \frac{y}{b} = \left(a^2 - 3b^2\right) + \left(-b^2 + 3a^2\right)$$

$$\Rightarrow 4\left(a^2 - b^2\right)$$

Q.18 [B]

$$\sqrt{3} + i = (a + ib)(c + id)$$

$$\Rightarrow \arg\left(\sqrt{3} + i\right) = \arg\left[(a + ib)(c + id)\right]$$

$$\Rightarrow \arg(a + ib) + \arg(c + id) = \tan^{-1}\frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan^{-1}\frac{b}{a} + \tan^{-1}\frac{d}{c} = \frac{\pi}{6}$$

Q.19 [C]

$$z_1 = 4 + i5$$

$$\Rightarrow z_2 = -3 + 2i$$

$$\Rightarrow \frac{z_1}{z_2} = \frac{4 + 5i}{-3 + 2i} = \frac{(4 + 5i)(-3 - 2i)}{9 + 4} = \frac{-12 + 10 - 15i - 8i}{13}$$

$$\Rightarrow \frac{-2}{13} - \frac{23}{13}i$$

Q.20 [C]

$$x + \frac{1}{x} = 2\cos\theta$$

$$\Rightarrow x^2 - 2x\cos\theta + 1 = 0$$

$$\Rightarrow x = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$$

$\Rightarrow x = \cos\theta \pm i\sin\theta$

Q.21 [A]

$3 - 2yi = 9^x - 7i$

$\Rightarrow 3 = 9^x; \qquad -2y = -7$

$\Rightarrow x = \frac{1}{2}$

$\Rightarrow y = \frac{7}{2}$

Q.22 [B]

$$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

$$\Rightarrow \left((1+i)x - 2i(3-i) + \left[(2-3i)y + i\right][3+i]\right) = i(3+1)$$

by comparing real & imaginary parts

$\Rightarrow x = 3,\ y = -1$

Q.23 [B]

$$x + iy = \frac{3}{2+\cos\theta + i\sin\theta} \times \left(\frac{2+\cos\theta - i\sin\theta}{2+\cos\theta - i\sin\theta}\right)$$

$$\Rightarrow \frac{6 + 2\cos\theta - 3i\sin\theta}{\left(4 + \cos^2\theta + 4\cos\theta + \sin^2\theta\right)}$$

$$\Rightarrow x = \frac{2(3+\cos\theta)}{5+4\cos\theta},\ y = \frac{-3\sin\theta}{5+4\cos\theta}$$

$$\Rightarrow x^2 + y^2 = \frac{4\left(9 + \cos^2\theta + 6\cos\theta\right) + 9\sin^2\theta}{(5+4\cos\theta)^2} = \frac{40 + 2y\cos\theta + 5\sin^2\theta}{(5+4\cos\theta)^2}$$

$$\Rightarrow \frac{8(3+\cos\theta)(5+4\cos\theta) - 3(5+4\cos\theta)^2}{(5+4\cos\theta)^2}$$

$\Rightarrow x^2 + y^2 = 4x - 3$

Q.24 [B]

$x = -5 + 2\sqrt{-4} = -5 + 4i$

$\Rightarrow x^2 - (-10)x + (41) = 0$

$\Rightarrow x^2 + 10x + 41 = 0$

$\Rightarrow x^2 = -10x - 41$

$\Rightarrow x^3 = -10x^2 - 41x = -10(-10x - 41) - 41x = 59x + 410$

$\Rightarrow \therefore x^4 + 9x^3 + 35x^2 - x + 4 = x^2 (x^2 + 35) + 9x^3 - x + 4$

$\Rightarrow x^2 (-10x - 6) + 9x^3 - x + 4$

$\Rightarrow -x^3 - 6x^2 - x + 4$

$\Rightarrow -59x + 410 + 60x + 41 \times 6 - x + 4$

$\Rightarrow -41 \times 4 + 4$

$\Rightarrow -160$

Q.25 [B]

(x + iy) (y – i3) = 4 + i

By comparing real & imaginary parts.

$\Rightarrow$ 2x + 3y = 4 ....(1)

$\Rightarrow$ 2y – 3x = 1 ....(2)

$\Rightarrow \therefore x = \frac{5}{13}, y = \frac{14}{13}$

Q.26 [D]

$z = \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha} = k;\ \ k \in R$

$$\Rightarrow z=\frac{(1-i\sin\alpha)(1-2i\sin\alpha)}{1+4\sin^2\alpha}$$

$$\Rightarrow \therefore \operatorname{Im}(z)=0$$

$$\Rightarrow -2\sin\alpha-\sin\alpha=0$$

$$\Rightarrow \alpha=n\pi;\quad n\in I$$

Q.27 [C]

$$z(2-i)=3+i$$

$$\Rightarrow z=\frac{3+i}{2-i}\times\frac{2+i}{2+i}=\frac{6-1+5i}{5}$$

$$\Rightarrow z=1+i=\sqrt{2}e^{\frac{i\pi}{4}}$$

$$\Rightarrow z^{20}=2^{10}e^{i5\pi}=2^{10}=1024$$

Q.28 [A]

$$\operatorname{Re}\left(\frac{z-8i}{z+6}\right)=0$$

$$\Rightarrow z=\frac{z+i(y-8)}{(x+6)+iy}=\frac{[x+i(y-8)][(x+6)-iy]}{(x+6)^2+y^2}$$

$$\Rightarrow \operatorname{Re}(z)=x(x+6)+y(y-8)=0$$

$$\Rightarrow x^2+y^2+6x-8y=0$$

Q.29 [B]

$$z=\frac{2+5i}{4-3i}\times\frac{4+3i}{4+3i}=\frac{8-15+26i}{25}$$

$$\Rightarrow z=\frac{-7}{25}+\frac{i26}{25}$$

$$\Rightarrow \bar{z}=\frac{-7}{25}-\frac{i26}{25}$$

Q.30 [B]

$z_1 + z_2 = \text{Real}$

$\Rightarrow z_1 z_2 = \text{Real}$

$\Rightarrow \therefore z_1 \,\&\, z_2$ are complex conjugate

$\Rightarrow z_1 = \overline{z_2}$

Q.31 [B]

z = x + iy (in 3rd quadrate)

$\Rightarrow x < 0, y < 0$

$\Rightarrow \bar{z} = x - iy = x + i(y)$ 2nd quadrate

Q.32 [A]

$(z+3)(\bar{z}+3)$

$\Rightarrow (z+3)(\overline{z+3})$

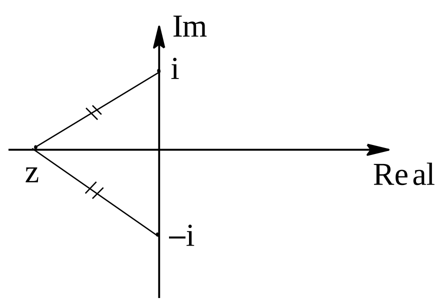
$\Rightarrow |z+3|^2$

Q.33 [A]

$\Rightarrow |z_1 z_2| = |z_1||z_2| = 1$

Q.34 [A]

$|z+1| = |z-i|$



Locus of z wier be Real axis

Q.35 [B]

$$\frac{z-1}{z+1}=ki;k\in R$$

$$\Rightarrow \text{Re}\left[\frac{(x-1)+iy}{(x+1)+iy}\times\frac{(x+1)-iy}{(x+1)-iy}\right]=0$$

$$\Rightarrow (x-1)(x+1)+y^2=0$$

$$\Rightarrow x^2+y^2=1$$

$$\Rightarrow |z|^2=1$$

$$\Rightarrow |z|=1$$

Q.36

$$\Rightarrow |2z-1|+|3z-2|$$

Q.37 [B]

$$|z_1+z_2|^2+|z_1-z_2|^2$$

$$\Rightarrow (z_1+z_2)(\overline{z_1}+\overline{z_2})+(z_1-z_2)(\overline{z_1}-\overline{z_2})$$

$$\Rightarrow 2\left(|z_1|^2+|z_2|^2\right)$$

Q.38

$$\frac{2z_1}{3z_2}=ki;k\in R$$

$$\Rightarrow \frac{2z_1\overline{z_2}}{3|z_2|^2}=ki$$

$$\Rightarrow \therefore \text{Re}\left(z_1\overline{z_2}\right)=0$$

$$\Rightarrow z_1\overline{z_2}+\overline{z_1}z_2=0$$

$$\Rightarrow \therefore \left|\frac{z_1 - z_2}{z_1 + z_2}\right|^2 = \left(\frac{z_1 - z_2}{z_1 + z_2}\right)\left(\frac{\overline{z_1} - \overline{z_2}}{\overline{z_1} + \overline{z_2}}\right)$$

$$\Rightarrow \frac{|z_1|^2 + |z_2|^2 - \left(z_1\overline{z_2} + z_2\overline{z_1}\right)}{|z_1|^2 + |z_2|^2 + \left(z_1\overline{z_2} + z_2\overline{z_1}\right)} = \frac{|z_1|^2 + |z_2|^2}{|z_1|^2 + |z_2|^2}$$

Q.39

$$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$$

$$\Rightarrow |z_1 + z_2 + z_3| = |\overline{z_1} + \overline{z_2} + \overline{z_3}| = \left|\frac{z_1\overline{z_1}}{z_1} + \frac{z_2\overline{z_2}}{z_2} + \frac{z_3\overline{z_3}}{z_3}\right|$$

$$\Rightarrow \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$$

Q.40

$$\left|z_1 + \sqrt{z_1^2 - z_2^2}\right| + \left|z_1 - \sqrt{z_1^2 - z_2^2}\right|$$

Let $z_3 = \sqrt{z_1^2 - z_2^2}$ $\quad z_3^2 = z_1^2 - z_2^2$

$$\Rightarrow \left[|z_1 + z_3| + |z_1 - z_3|\right]^2 = |z_1 + z_3|^2 + |z_1 - z_3|^2 + 2|z_1 + z_3||z_1 - z_3|$$

$$\Rightarrow 2\left(|z_1|^2 + |z_3|^2\right) + 2|z_1^2 - z_3^2|$$

$$\Rightarrow 2\left(|z_1|^2 + |z_3|^2\right) + 2|z_2|^2$$

$$\Rightarrow 2\left(|z_1|^2 + |z_2|^2\right) + 2|z_1^2 - z_2^2|$$

$$\Rightarrow \left(|z_1 - z_2|^2 + |z_1 + z_2|^2\right) + 2|z_1 - z_2||z_1 + z_2|$$

$$\Rightarrow \left[|z_1 - z_2| + |z_1 + z_2|\right]^2$$

Q.41 [C]

$$\left|\frac{z-4}{z-8}\right|=1$$

Locus of Z will be x = 6

$$\Rightarrow \therefore z = 6+iy$$

$$\Rightarrow \left|\frac{z-12}{z-8i}\right|=\frac{5}{3}$$

$$\Rightarrow 3\left|-6+iy\right|=5\left|6+i\left(y-8\right)\right|$$

$$\Rightarrow 9\left(36+y^2\right)=25\left(36+\left(y-8\right)^2\right)$$

$$\Rightarrow y^2-25y+136=0$$

$\Rightarrow$ y = 8, 17

$$\Rightarrow \therefore z = 6+8i, 6+i17$$

Q.42 [A]

$$|z-4|<|z-2|$$

$$\Rightarrow |z-4|^2<|z-2|^2$$

$$\Rightarrow |z|^2-4\left(\bar{z}+z\right)+16<|z|^2-2\left(z+\bar{z}\right)+4$$

$$\Rightarrow 2\left(z+\bar{z}\right)>12$$

$$\Rightarrow 2\left(2x\right)>12$$

$\Rightarrow$ x > 3

$\Rightarrow$ Re (z) > 3

Q.43 [B]

$$z=1+i\tan\alpha$$

$$\Rightarrow \pi<\alpha<\frac{3\pi}{2}$$

$\Rightarrow |z| = \sqrt{1+\tan^2\alpha} = |\sec\alpha|$

$\Rightarrow |z| = -\sec\alpha$ (3rd quadrant)

Q.44 [A, B]

$$\Rightarrow \left|\frac{|\bar{z}|^2}{z\bar{z}}\right| = \left|\frac{|z|^2}{|z|^2}\right| = 1 = \left|\frac{\bar{z}}{z}\right|$$

Q.45 [C]

$$|z_1 + z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right|$$

$$\Rightarrow |z_1 + z_2| = \left|\frac{z_1 + z_2}{z_1 z_2}\right|$$

$\Rightarrow |z_1 z_2| = 1$

Q.46 [D]

$z^2 + |z|^2 = 0$

$\Rightarrow z^2 = -|z|^2$

$\Rightarrow z = i|z|$

$\Rightarrow$ Real (z) = 0

$\Rightarrow \therefore z = iy$

So infinite solution.

Q.47

$\Rightarrow |z| = \max\{|z-2|, |z+2|\}$

Q.48 [C]

$z = -1 + i\sqrt{3}$ z lies in 2nd quadrant

$$\Rightarrow \therefore \arg(z) = \tan^{-1}\left(-\sqrt{3}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Q.49

$z = -1 - i\sqrt{3}$; z lies in 3rd quadrant

$$\Rightarrow \arg(z) = \pi + \tan^{-1}\left(\sqrt{3}\right) = \frac{4\pi}{3}$$

Q.50 [A]

$$z = \frac{1 + i\sqrt{3}}{\sqrt{3} + i} \times \frac{\sqrt{3} - i}{\sqrt{3} - i} = \frac{\sqrt{3} + \sqrt{3} + 3i - i}{4}$$

$$\Rightarrow z = \frac{\sqrt{3} + i}{2}$$

$$\Rightarrow \arg(z) = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

Q.51 Repeated Q.50

Q.52

$$z = \frac{13 - 5i}{4 - 9i}$$

$\Rightarrow$ arg (z) = arg (13 – 5i) – arg (4 – 9i)

$$\Rightarrow \tan^{-1}\left(\frac{-5}{13}\right) - \tan^{-1}\left(\frac{-9}{4}\right)$$

$$\Rightarrow \left[-\tan^{-1}\frac{5}{13}\right] - \left[-\tan^{-1}\frac{9}{4}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{9}{4} - \frac{5}{13}}{1 + \frac{9}{4} \times \frac{5}{13}}\right] = \tan^{-1}\left(\frac{97}{97}\right) = \frac{\pi}{4}$$

Q.53 [C]

$$z = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$$

$$\Rightarrow \arg(z) = \arg\left(1 - i\sqrt{3}\right) - \arg\left(1 + i\sqrt{3}\right)$$

$$\Rightarrow \left(-\frac{\pi}{3}\right) - \left(\frac{\pi}{3}\right) = \frac{-2\pi}{3} = \frac{4\pi}{3}$$

Q.54 [D]

$$z = 1 - \cos\alpha + i\sin\alpha$$

$$\Rightarrow \text{amp}(z) = \tan^{-1}\left(\frac{\sin\alpha}{1 - \cos\alpha}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\sin^2\frac{\alpha}{2}}\right)$$

$$\Rightarrow \tan^{-1}\left(\cot\frac{\alpha}{2}\right) = \frac{\pi}{2} - \frac{\alpha}{2}$$

Q.55 [B]

$$z = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = 1 \cdot e^{\frac{i\pi}{6}}$$

$$\Rightarrow |z| = 1, \arg(z) = \frac{\pi}{6}$$

Q.56 [B]

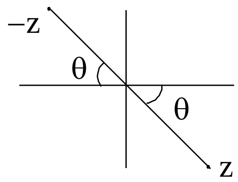
$$\arg(z) = \theta$$

$$\Rightarrow \arg(\bar{z}) = -\theta$$

Q.57

arg (z) < 0

Let arg (z) = $-\theta$; $\theta > 0$



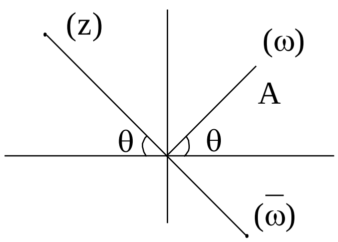
then arg (– z) – arg (z)

$(\pi-\theta)-(-\theta)=\pi$

Q.58 [D]

$|z|=|\omega|$

$\Rightarrow \arg(z)+\arg(\omega)=\pi$



$\Rightarrow \therefore z=-\bar{\omega}$

Q.59

Re(z) < 0

$\Rightarrow$ Im (z) = 0

$\Rightarrow \arg(z)\pi$

Q.60 [D]

if $\arg(z)=\theta$

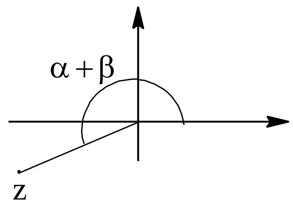
$\Rightarrow$ then $\arg(\bar{z})=-\theta$

Q.61 [C]

$\arg(z_1)=\alpha$

$\Rightarrow \arg(z_2)=\beta$

given $\alpha+\beta>\pi$



$\Rightarrow z_1z_2 = r_1r_2e^{i(\alpha+\beta)}$

Principal argument $\arg(z_1 \times z_2) = -(2\pi - \alpha + \beta)$

$\Rightarrow \alpha+\beta-2\pi$

Q.62 [B]

z = – 1

$\Rightarrow \arg\left(z^{\frac{2}{3}}\right) = \frac{2}{3}\arg(z) = \frac{2}{3}\arg(-1) = \frac{2}{3}\pi$

Q.63 [B]

z = x + iy

$\Rightarrow$arg (z – 1) = arg (z + 3i)

$\Rightarrow \tan^{-1}\frac{y}{x-1} = \tan^{-1}\frac{y+3}{x}$

$\Rightarrow \therefore \frac{y}{x-1} = \frac{y+3}{x}$

$\Rightarrow$xy = xy + 3x – y – 3

$\Rightarrow \frac{x-1}{y} = \frac{1}{3}$

Q.64 [C]

$(1+i)^n + (1-i)^n$

$$\Rightarrow\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^n+\left[\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)\right]^n$$

$$\Rightarrow 2^{\frac{n}{2}}\left[2\cos^n\frac{\pi}{4}\right]$$

$$\Rightarrow\left(\sqrt{2}\right)^{n+2}\cos\left(\frac{n\pi}{4}\right)$$

Q.65 [A]

$$y=\cos\theta+i\sin\theta$$

$$\Rightarrow\frac{1}{y}=\bar{y}=\cos\theta-i\sin\theta$$

$$\Rightarrow y+\frac{1}{y}=2\cos\theta$$