APPLICATIONS OF DERIVATIVES
EXERCISE 2(A)

1. $y = x^{1/3}(x-1)$

$$\frac{dy}{dx} = \frac{4}{3}x^{1/3} - \frac{1}{3}\cdot\frac{1}{x^{2/3}} = \frac{1}{3x^{2/3}}[4x-1]$$

hence f is $\uparrow$ for $x > \frac{1}{4}$

and f $\downarrow$ for $x < \frac{1}{4}$ $\quad\Big]$ $x^{2/3}$ *is always positive and at* $x = 1/4$ *the curves has a local* minima

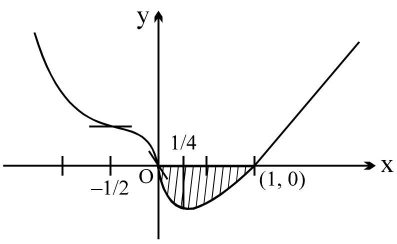
now $f'(x) = \frac{4}{3}x^{1/3} - \frac{1}{3}\cdot x^{-2/3}$ (non existent at x = 0, vertical tangent)

$$f''(x) = \frac{4}{9}\cdot\frac{1}{x^{2/3}} + \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{x^{5/3}}$$

$$= \frac{2}{9x^{2/3}}\left[2+\frac{1}{x}\right] = \frac{2}{9x^{2/3}}\left[\frac{2x+1}{x}\right]$$

$\therefore$ $f''(x) = 0$ at $x = -\frac{1}{2}$ (inflection point)

graph of f (x) is as



$$A = \int_0^1 \left(x^{4/3} - x^{1/3}\right)dx = \left.\frac{3}{7}x^{3/7} - \frac{3}{4}x^{4/3}\right]_0^1$$

$$= \left|\frac{3}{7} - \frac{3}{4}\right| = 3\left|\frac{4-7}{28}\right| = \frac{9}{28} \Rightarrow \textbf{(D)} \,]$$

2. $\frac{dy}{dx} =$ slope fo tangent

$-\frac{1}{t^2} = -\frac{b}{a}$ $\quad\therefore\ \frac{a}{b} = t^2 > 0$ $\quad\Rightarrow$ a and b are of same sign.

3. $f'(x) = \sqrt{1-x^4} > 0$ in $(-1, 1)$ $\Rightarrow$ f is $\uparrow$

Now $f(x) + f(-x) = \int_0^x \sqrt{1-t^4}\,dt + \int_0^{-x} \sqrt{1-t^4}\,dt \Rightarrow \int_0^x \sqrt{1-t^4}\,dt + \left(-\int_0^y \sqrt{1-y^4}\,dy\right)$ $(t = -y)$

$= 0 \Rightarrow$ f (x) is odd

again $f''(x) = \frac{-4x^3}{2\sqrt{1-x^4}}$ which vanished at x = 0 and changes sign $\Rightarrow$ (0, 0) is inflection since f is well defined in [–1, 1] $\Rightarrow$ A, B, C, D]

4. Since intercepts are equal in magnitude but opposite in sign $\Rightarrow \left.\frac{dy}{dx}\right|_P = 1$

now $\frac{dy}{dx} = x^2 - 5x + 7 = 1 \Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x = 2$ or 3]

5. $h(x) = \frac{\ell n\left(f(x)\,.\,g(x)\right)}{\ell n\,a} = \frac{\ell na^{\{a^{|x|}.\text{sgn}\,x\}+[a^{|x|}.\text{sgn}\,x]}}{\ell na}$

$= \left\{a^{|x|}\ \text{sgn}\,x\right\} + \left[a^{|x|}\ \text{sgn}\,x\right] = a^{|x|}\ \ \text{sgn}\,x \qquad (\because\ \{y\} + [y] = y)$

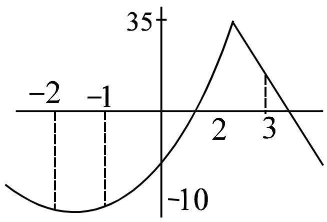
$= \begin{bmatrix} a^{x} & \text{for} & x > 0 \\ 0 & \text{for} & x = 0 \\ -a^{-x} & \text{for} & x < 0 \end{bmatrix} \Rightarrow$ h (x) is an odd function]

6. $f'(x) = 100\,x^{99} + \cos x$

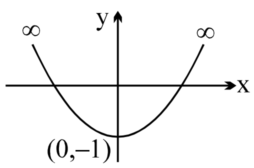
for $x \in (0,1)$ and $\left(0, \frac{\pi}{2}\right)$, cosx and x are both +ve $\Rightarrow \uparrow$

for $x \in \left(\frac{\pi}{2}, \pi\right)$, x > 1 hence $100\,x^{99}$ obviously > cosx $\Rightarrow \uparrow$]

7. Note that f (x) is continuous at x= 2 and f is decreasing for (2, 3) and increasing for [–1, 2] .
At x = 2 f has a maxima hence (A) is not correct.]



8. Graph of y = f (x) $\Rightarrow$ (A) and (C)



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9. If f and g are inverse then (fog)(x) = x
f ' [g(x)] g ' (x) = 1
if f is increasing $\Rightarrow$ f ' > 0 $\Rightarrow$ sign of g ' is also + ve $\Rightarrow$ (A) is correct
If f is decreasing $\Rightarrow$ f ' < 0 $\Rightarrow$ sign of g ' is – ve $\Rightarrow$ (B) is false
since f has an inverse $\Rightarrow$ f is bijective $\Rightarrow$ f is injective $\Rightarrow$ (C) is correct
inverse of a bijective mapping is bijective
$\Rightarrow$ g is also bijective $\Rightarrow$ g is onto $\Rightarrow$ (D) is correct]

10. $f(x) = \ln(1 - \ln x)$
domain (0, e)

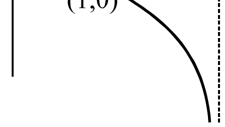
$f'(x) = -\frac{1}{(1-\ln x)}\cdot\frac{1}{x} < 0 \Rightarrow$ decreasing $\forall$ x in its domain $\Rightarrow$ (A) & (B) are incorrect

$f'(1) = -1 \Rightarrow$ (C) is also incorrect

also $f(1) = 0$; $\lim_{x\to e^{-1}} f(x) \to -\infty$; $\lim_{x\to 0^{+}} f(x) \to \infty$

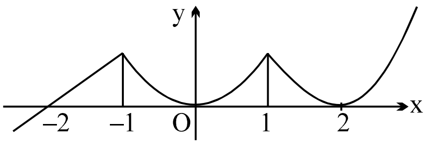
$f''(x) = \frac{-\ln x}{x^2(1-\ln x)^2}$

$f''(1) = 0$ which is a point of inflection
graph is as shown



y axis and x = e are two asynmptotes]

11. f is obvious continuous $\forall$ x $\in$ R and not derivable at – 1 and 1
f '(x) changes sign 4 times at – 1, 0, 1, 2
local maxima at 1 and – 1
local minima at x = 0 and 2]



12. Domain is x $\in$ R

Also $f(x) = \left[\cos\left(\tan^{-1}(\sin\theta)\right)\right]^2$ where $\cot\theta = x$

$$= \left[\cos\left(\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)\right]^2 = (\cos\phi)^2 \text{ where } \tan\phi = \frac{1}{\sqrt{1+x^2}}$$

$$= \left(\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right)^2$$

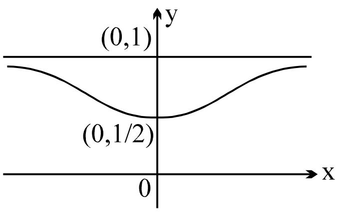
$$g(x) = \frac{1+x^2}{2+x^2} = 1 - \frac{1}{2+x^2}$$

range is $\left[\frac{1}{2}, 1\right)$; $f'(x) = \frac{2x}{(2+x^2)^2}$

hence $f'(0) = 0$

also $\lim_{x\to\infty} f(x) = 1$

hence (B), (C), (D)]



13. Let the tangent line be y = ax + b
The equation for its intersection with the upper parabola is

$x^2 + 1 = ax + b$

$x^2 - ax + (1 - b) = 0$

This has a double root when $a^2 - 4(1 - b) = 0$ or $a^2 + 4b = 4$

For the lower parabola

$ax + b = -x^2$

$x^2 + ax + b = 0$

This has a double root when $a^2 - 4b = 0$

subtract these two equations to get $8b = 4$ or $b = 1/4$

add them to get $2a^2 = 4$ or $a = \pm\sqrt{2}$

The tangent lines are $y = \sqrt{2}x + \frac{1}{2}$ and $y = -\sqrt{2}x + \frac{1}{2}$

14. $f(x) = \int_0^{\pi} \cos t \cos(x - t)\,dt$ ....(1)

$$= \int_0^{\pi} -\cos t \cdot \cos(x - \pi + t)\,dt$$

$f(x) = \int_0^{\pi} \cos t \cdot \cos(x + t)\,dt$ ....(2)

(1) + (2) gives

$$2f(x) = \int_0^{\pi} \cos t\,(2\cos x \cdot \cos t)\,dt$$

$\therefore \quad f(x) = \cos x \int_0^{\pi} \cos^2 t \, dt = 2 \cos x \int_0^{\pi/2} \cos^2 t \, dt$

$f(x) = \frac{\pi \cos x}{2}$ Now verify. Only (A) & (B) are correct.

15. (A) $f(x) = x - \tan^{-1}x$

$f'(x) = 1 - \frac{1}{1+x^2} = \frac{x^2}{1+x^2} > 0 \quad \Rightarrow \quad$ f is increasing in (0, 1)

$f(x) > f(0)$ but $f(0) = 0$

$f(x) > 0 \quad \Rightarrow \quad x > \tan^{-1}x$ in (0, 1)

(B) $f(x) = \cos x - 1 + \frac{x^2}{2}$

$f'(x) = -\sin x + x = x - \sin x > 0$ in (0, 1) $\Rightarrow$ **(B) is not correct**

(C) $f(x) = 1 + x \, ln\left(x + \sqrt{1+x^2}\right) - \sqrt{1+x^2}$

$$f'(x) = x\left(\frac{1+\frac{1}{2}\cdot\frac{2x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}\right) + ln\left(x+\sqrt{1+x^2}\right) - \frac{x}{\sqrt{1+x^2}}$$

$$= \frac{x}{\sqrt{1+x^2}} + ln\left(x+\sqrt{1+x^2}\right) - \frac{x}{\sqrt{1+x^2}} > 0 \ \forall \ x \in R$$

$\Rightarrow$ (C) is true

(D) $f(x) = x - \frac{x^2}{2} - ln(1+x)$

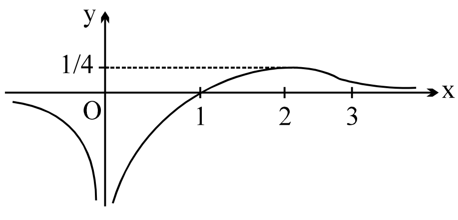
$f'(x) = (1-x) - \frac{1}{1+x} = \frac{(1-x^2)-1}{1+x} = -\frac{x^2}{1+x} < 0 \Rightarrow$ **(D) is correct**

hence f (x) is decreasing in (0, 1)

$\therefore \quad f(x) < f(0)$

$f(x) < 0 \quad \Rightarrow \quad x - \frac{x^2}{2} < ln(1+x)$]

16. $f'(x) = \frac{2-x}{x^3}$ and $f''(x) = \frac{x-3}{x^4}$. Now interpret



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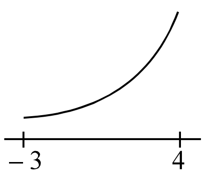
17. (A) f (x) has no relative minimum on (–3, 4)

(B) f (x) is continuous function on [–3, 4]

$\Rightarrow$ f (x) has min. and max. on [–3, 4] by IVT

(C) $f''(x) > 0 \quad \Rightarrow \quad$ f (x) is concave upwards on [–3, 4]



(D) $f(3) = f(4)$
By Rolle's theorem
$\exists\ c \in (3, 4)$, where $f'(c) = 0$
$\Rightarrow$ $\exists$ critical point on $[-3, 4]$

18. (A) False, e.g. $f(x) = \sin\sqrt{x}$
(B) True, from IVT
(C) True as $\lim\limits_{x\to\infty} \sin^{-1}\left(1+\frac{1}{x}\right) = \sin^{-1}$(a quantity greater than one) $\Rightarrow$ not defined
(D) True, as the line passes through the centre of the circle.

19.

(A) Let $\ell = \lim\limits_{x\to 0} \dfrac{x\int_0^x e^{t^2}dt}{-(e^x - x - 1)}\left(\frac{0}{0}\right) = \lim\limits_{x\to 0} \dfrac{x\int_0^x e^{t^2}dt}{-x^2\left(\frac{e^x - x - 1}{x^2}\right)} = -2\lim\limits_{x\to 0}\dfrac{\int_0^x e^{t^2}dt}{x}\left(\frac{0}{0}\right) = -2\lim\limits_{x\to 0}\dfrac{e^{x^2}}{1} = -2$

(B) $14x^2 - 7xy + y^2 = 2$

$\dfrac{dy}{dx} = \dfrac{28x - 7y}{7x - 2y}$ ....(1)

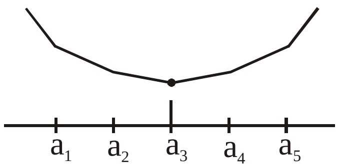
if $x = 1$ then $14 - 7y + y^2 = 2$ $\Rightarrow$ $y^2 - 7y + 12 = 0$ $\Rightarrow$ $y = 3$ or 4
hence L (1, 3) and M (1, 4)

slope of tangent at L = $\dfrac{28-21}{7-6} = 7$; slope of tangent at M = $\dfrac{28-28}{7-8} = 0$

equation of tangent at L and M are
$y - 3 = 7(x - 1)$ $\Rightarrow$ $y = 7x - 4$
and $y - 4 = 0(x - 1)$ $\Rightarrow$ $y = 4$

hence $N = \left(\frac{8}{7}, 4\right)$ $\Rightarrow$ **(C)**

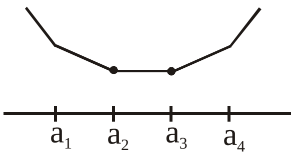
(C) If n is odd then graph of f (x) is



a_3 is the only point where

f (x) has its minimum value

If n is even then graph of f (x) is



From a_2 to a_3 at all values of x, f (x) is minimum.

(D) $2lc + m = (lb^2 + mb)\ \dfrac{-(la^2 + my)}{b-a} = l(b^2 - a^2) + m(b - a) = l(b + a) + m$; $c = \dfrac{a+b}{2}$

20. We have $f'(x) = 5\sin^4 x\cos x - 5\cos^4 x\sin x = 5\sin x\cos x(\sin x - \cos x)(1 + \sin x\cos x)$

$\therefore$ $f'(x) = 0$ at $x = \dfrac{\pi}{4}$. Also $f'(0) = f'\left(\dfrac{\pi}{2}\right) = 0$

Hence $\exists$ some $c \in \left(0, \frac{\pi}{2}\right)$ for which $f'(c) = 0$ (By Rolle's Theorem) $\Rightarrow$ **(C)** is correct.

Also in $\left(0, \frac{\pi}{4}\right)$ f is decreasing and in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ f is increasing $\Rightarrow$ minimum at $x = \frac{\pi}{4}$

As $f(0) = f\left(\frac{\pi}{2}\right) = 0$ $\Rightarrow$ 2 roots $\Rightarrow$ **(D)** is correct.]

21. $f(x) = \tan^{-1}(x)$ is defined on R and is strictly increasing but do not have its range R]

22. $f(0) = 1$; $f(2) = 2$
$f(1^-) = f(1^+) = f(1) = 2$]

23. $f(x) = \ln(2 + x) - \frac{2x+2}{x+3}$ is continuous in $(-2, \infty)$

$$f'(x) = \frac{1}{x+2} - \frac{4}{(x+3)^2} = \frac{(x+3)^2 - 4(x+2)}{(x+2)(x+3)^2}$$

$$= \frac{x^2+2x+1}{(x+2)(x+3)^2} = \frac{(x+1)^2}{(x+2)(x+3)^2} > 0 \qquad (f'(x) = 0 \text{ at } x = -1)$$

$\Rightarrow$ f is increasing in $(-2, \infty)$

also $\lim_{x \to -2^+} f(x) \to -\infty$ and $\lim_{x \to \infty} f(x) \to \infty$ $\Rightarrow$ unique root]

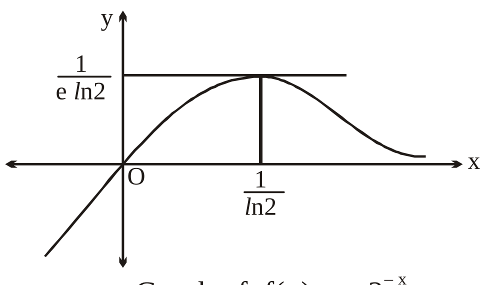
24. Let $f(x) = 0$ has two roots say $x = r_1$ and $x = r_2$ where $r_1, r_2 \in [a, b]$
$\Rightarrow$ $f(r_1) = f(r_2)$
hence $\exists$ there must exist some $c \in (r_1, r_2)$ where $f'(c) = 0$
but $f'(x) = x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$
for $|x| \geq 1$, $f'(x) = (x^6 - x^5) + (x^4 - x^3) + (x^2 - x) + 1 > 0$
for $|x| \leq 1$, $f'(x) = (1 - x) + (x^2 - x^3) + (x^4 - x^5) + x^6 > 0$
hence $f'(x) > 0$ for all x
$\therefore$ Rolles theorem fails $\Rightarrow$ $f(x) = 0$ can not have two or more roots.]

25. Consider the example of $f(x) = e^x$ and $f'(x) = e^x$ both increasing]

Paragraph for question nos. 26 to 27

(i) We have $f(x) = x\,2^{-x}$
So, $f'(x) = 2^{-x}(1 - x \ln 2)$
and $f''(x) = 2^{-x} \ln 2 (x \ln 2 - 2)$

Clearly, f(x) is increasing in $\left(-\infty, \frac{1}{\ln 2}\right)$ and decreasing in $\left(\frac{1}{\ln 2}, \infty\right)$.



Graph of $f(x) = x\,2^{-x}$

(ii) . . . . $\in \left(0, \frac{1}{e\, ln2}\right)$.

(iii) Given $f(x) = x\, 2^{-x}$ and $g(x) = \max.\{f(t) : x \le t \le x + 1\}$

As f(x) is increasing in $\left(-\infty, \frac{1}{ln2}\right)$, hence maximum value of g(x) occurs at $t = x + 1$

$\therefore \quad g(x) = f(x + 1) = (x + 1)\, 2^{-(x+1)}$

$$\text{Let } I = \int_0^{\frac{1}{ln2}-1} g(x)\,dx = \int \underbrace{(x+1)}_{\substack{I \\ (I.B.P.)}} \underbrace{2^{-(x+1)}}_{II} dx$$

$$= -\left.\frac{(x+1)\, 2^{-(x+1)}}{ln\, 2}\right]_0^{\frac{1}{ln\,2}-1} + \frac{1}{ln2} \int_0^{\frac{1}{ln\,2}-1} 2^{-(x+1)} dx$$

$$= -\left.\frac{(x+1)}{ln2}\, 2^{-(x+1)}\right]_0^{\frac{1}{ln2}-1} - \left.\frac{1}{ln^2 2}\, 2^{-(x+1)}\right]_0^{\frac{1}{ln2}-1}$$

$$= -\frac{1}{ln2}\left[\frac{1}{ln2}\frac{1}{e} - \frac{1}{2}\right] - \frac{1}{ln^2 2}\left[\frac{1}{e} - \frac{1}{2}\right] = -\frac{1}{e\, ln^2 2} + \frac{1}{2\, ln2} - \frac{1}{e\, ln^2 2} + \frac{1}{2\, ln^2 2}$$

$$= \frac{1}{2\, ln^2 2} + \frac{1}{2\, ln2} - \frac{2}{e\, ln^2 2}$$ **Ans.**]

Paragraph for question nos. 29 to 31

(1) $$\lim_{x\to 0^+} x\, ln\left(1+\frac{1}{x}\right) = \lim_{x\to 0^+} \frac{ln\left(\frac{x+1}{x}\right)}{\frac{1}{x}} \quad \left(\frac{\infty}{\infty}\right)$$

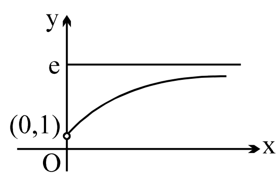
Using L'Hospital's Rule

$$l = \lim_{x\to 0} -\left(\frac{1}{x+1} - \frac{1}{x}\right)x^2 = \lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{x+1}\right)\cdot x^2 = \lim_{x\to 0} \frac{1}{x(x+1)}\cdot x^2 = \lim_{x\to 0} \frac{x}{(x+1)} = 0 \text{ Ans.}$$

(2) $\lim_{x\to 0} f(x) = 1$ (can be verified)

$\lim_{x\to\infty} f(x) = e$

Also f is increasing for all x > 0 $\Rightarrow$ (D) (can be verified)



(3) $$l = \left(\prod_{k=1}^{n}\left(1+\frac{n}{k}\right)^{k/n}\right)^{1/n}$$ {given $f(x) = (1 + 1/x)^x$ and $f(k/n) = \left(1+\frac{n}{k}\right)^{k/n}$ }

taking log,

$$ln\, l = \lim_{n\to\infty} \frac{1}{n}\cdot\sum_{k=1}^{n} ln\left(1+\frac{n}{k}\right)^{k/n} = \lim_{n\to\infty}\frac{1}{n}\cdot\sum_{k=1}^{n}\frac{k}{n}\, ln\left(1+\frac{1}{k/n}\right)dx$$

$$= \int_0^1 \underbrace{x}_{II}\, \underbrace{ln\left(1+\frac{1}{x}\right)}_{I} dx = \left. ln\left(1+\frac{1}{x}\right)\cdot\frac{x^2}{2}\right]_0^1 + \int_0^1 \left(\frac{1}{x} - \frac{1}{x+1}\right)\cdot\frac{x^2}{2}\, dx$$

$$= \left(\frac{1}{2}\ln 2 - 0\right) + \frac{1}{2}\int_0^1 \frac{x+1-1}{x+1}dx = \frac{1}{2}\ln 2 + \frac{1}{2}\left[x - \ln(x+1)\right]_0^1$$

$$= \frac{1}{2}\ln 2 + \frac{1}{2}\left[(1 - \ln 2) - 0\right] = \frac{1}{2}$$

$l = \sqrt{e}$

Paragraph for question nos. 32 to 34

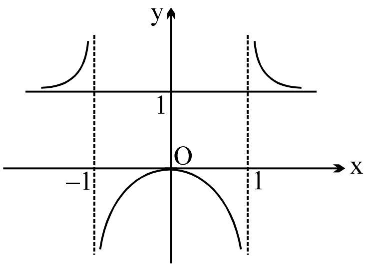
$y = \dfrac{x^2}{x^2-1}$; not defined at $x = \pm 1$

$= 1 + \dfrac{1}{x^2-1}$; $\qquad y' = -\dfrac{2x}{(x^2-1)^2}$

$\dfrac{dy}{dx} = 0 \quad \Rightarrow \quad x = 0$ (point of maxima)

as $\quad x \to 1^+,\ y \to \infty \quad ; \quad x \to 1^-,\ y \to -\infty$

|||ly $\quad x \to -1^+,\ y \to -\infty \quad ; \quad x \to -1^-,\ y \to \infty$



The graph of $y = \dfrac{x^2}{x^2-1}$ is as shown

verify all alternativels from the graph.

Paragraph for question nos. 35 to 37

(i) $a = 1$

$f(x) = 8x^3 + 4x^2 + 2bx + 1$

$f'(x) = 24x^2 + 8x + 2b = 2(12x^2 + 4x + b)$

for increasing function, $\quad f'(x) \ge 0 \quad \forall\ x \in R$

$\therefore \quad D \le 0 \quad \Rightarrow \quad 16 - 48b \le 0$

$\Rightarrow \quad b \ge \dfrac{1}{3} \quad \Rightarrow \quad$ **(C)**

(ii) if $b = 1$

$f(x) = 8x^3 + 4ax^2 + 2x + a$

$f'(x) = 24x^2 + 8ax + 2 \quad$ or $\quad 2(12x^2 + 4ax + 1)$

for non monotonic $f'(x) = 0$ must have distinct roots

hence $D > 0$ i.e. $\quad 16a^2 - 48 > 0 \quad \Rightarrow \quad a^2 > 3$;

$\therefore \quad a > \sqrt{3}$ or $a < -\sqrt{3}$

$\therefore \quad a \in 2, 3, 4, ...$

sum = 5050 – 1 = 5049 Ans.

(iii) If x_1, x_2 and x_3 are the roots then $\log_2 x_1 + \log_2 x_2 + \log_2 x_3 = 5$

$\log_2(x_1x_2x_3) = 5$

$x_1x_2x_3 = 32$

$-\dfrac{a}{8} = 32$

$\Rightarrow \quad a = -256$ **Ans.**]

38. (A) R; (B) R, S, T ; (C) Q; (D) Q

(A) $$I = \int_{\sqrt{2}-1}^{\sqrt{2}+1} \frac{(x^2+1)^2-(x^2-1)}{(x^2+1)^2}dx = \int_{\sqrt{2}-1}^{\sqrt{2}+1}\left(1-\frac{(x^2-1)}{(x^2+1)^2}\right)dx = 2 - \underbrace{\int_{\sqrt{2}-1}^{\sqrt{2}+1}\frac{(x^2-1)}{(x^2+1)^2}dx}_{I_1}$$

$$I_1 = \int_{1/a}^{a}\frac{(x^2-1)}{(x^2+1)^2}dx \text{ where } (a = \sqrt{2}+1); \quad \text{put } x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2}dt$$

$$= \int_a^{1/a}\frac{\frac{1}{t^2}-1}{\left(\frac{1}{t^2}+1\right)^2}\cdot\left(-\frac{1}{t^2}\right)dt = -\int_a^{1/a}\frac{(1-t^2)t^4}{t^4(1+t^2)^2}dt = -\int_a^{1/a}\frac{(1-t^2)}{(1+t^2)^2}dt = \int_a^{1/a}\frac{t^2-1}{(t^2+1)^2}dt$$

$$= -\int_{1/a}^{a}\frac{t^2-1}{(t^2+1)^2}dt = -I_1$$

$\Rightarrow$ $2I_1 = 0 \Rightarrow$ $I_1 = 0$ $\Rightarrow$ 2 is the answer.]

(B) Domain of f (x) is $(0, 1) \cup (1, \infty)$
$\ln f(x) = 1$ $\Rightarrow$ $f(x) = e$ = constant
$f'(x) = 0$, for all in $(0, \infty) - \{1\}$

(C) Clearly (1, 0) is the point of intersection of given curves.

Now, $f'(x) = \frac{2^x}{x} + 2^x (\ln 2)(\ln x)$

$\therefore$ Slope of tangent to the curve f (x) at (1, 0) = $m_1 = 2$

Similarly, $g'(x) = \frac{d}{dx}(e^{2x \ln x} - 1) = x^{2x}\left(2x \times \frac{1}{x} + 2\ln x\right)$

$\therefore$ Slope of tangent to the curve g (x) at (1, 0) = $m_2 = 2$
since $m_1 = m_2 = 2$
$\Rightarrow$ Two curves touch each other, so angle between them is 0.
Hence $\cos\theta = \cos 0 = 1$

(D) $3y^2y' - 3y - 3xy' = 0$ $\Rightarrow$ $y' = \frac{y}{y^2 - x}$
$y' = 0$ $\Rightarrow$ $y = 0$, no real x.
$y' = \infty$ $\Rightarrow$ $y^2 = x$ $\Rightarrow$ $y^3 = 1, y = 1$
The point is (1, 1)

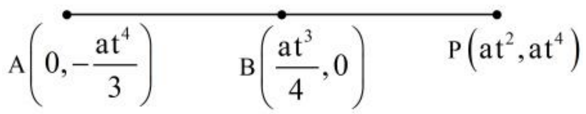
39. $(A)\rightarrow R, (B)\rightarrow Q, (C)\rightarrow P, (D)\rightarrow S$

(A) $\frac{dy}{dx} = \frac{4t}{3}$, Tangent is $y - at^4 = \frac{4t}{3}\left(x - at^3\right)$

x-intercept $= \frac{at^3}{4}$

y-intercept $= -\frac{at^4}{3}$

the point of intersection of tangent with the axes are $\left(\frac{at^3}{4}, 0\right)$ and $\left(0, -\frac{at^4}{3}\right)$



P divids AB externaly in 4 : 3

$\therefore \quad \frac{m}{n}=\frac{4}{3} \quad \Rightarrow \quad m=4 \ \& \ n=3$

as m & n are coprime to each other

$\therefore \quad m+n=7$

(B) $\frac{dx}{dy}=e^{\sin y}\cos y$: slope of normal $=-1$

equation of normal is $x+y=1$

Area $=\frac{1}{2}$

(C) $y=\frac{1}{x^2}:\frac{dy}{dx}=-\frac{1}{x^3}$: slope of tangent $=-2$

$y=e^{2-2x}:\frac{dy}{dx}=e^{2-2x}.(-2)$: slope of tangent $=-2$

$\therefore \tan^{-}=0$

(D) Length of subtangent $=\left|\frac{y}{y'}\right|=\left|\frac{be^{x/3}}{b\frac{1}{2}e^{x/3}}\right|=3$

40 $(A)\to(q),(B)\to(r),(C)\to(q),(D)\to(q)$

$y=f(x)+\frac{1}{x}$ so $y>0$

Now $f(y)f\left(f(y)+\frac{1}{y}\right)=1$ also $f(x)f(y)=1$

$\therefore f(x)=f\left(f(y)+\frac{1}{y}\right)=f\left(\frac{1}{f(x)}+\frac{1}{f(x)+1/x}\right)$

also f(x) is increasing

$\therefore x=\frac{1}{f(x)}+\frac{1}{f(x)+1/x}$

$\Rightarrow f(x)=\frac{1\pm\sqrt{5}}{2x}$

now $f(x)=\frac{1+\sqrt{5}}{2x}$ is decreasing so discarding it $f(x)=\frac{1-\sqrt{5}}{2x}$.