# Applications Of Derivatives

## Exercise 1(A)

**Q.1** (b)

Given $x^2 + y^2 = 2c^2$

Differentiating *w.r.t. x*, $2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow 2y\frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(c,c)} = -1$

**Q.2** (a)

Given curve $x^2 = 3 - 2y$

diff. *w.r.t. x*, $2x = -\frac{2dy}{dx}$ ; $\frac{dy}{dx} = -x$

Slope of the line = – 1

$\frac{dy}{dx} = -x = -1$ ; $x = 1$

$\therefore y = 1$ point (1, 1)

**Q.3** (b)

Given $y = 2x^2 - x + 1$

Let the co-ordinate of *P* is (*h*, *k*) then $\left(\frac{dy}{dx}\right)_{(h,k)} = 4h - 1$

Clearly 4h – 1 = 3.

h = 1$\Rightarrow$ k = 3. *P* is (1, 2).

**Q.4** (d)

$x^2 = -4y \Rightarrow 2x = -4\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{2} \Rightarrow \left(\frac{dy}{dx}\right)_{(-4,-4)} = 2$.

We know that equation of tangent is

$(y - y_1) = \left(\frac{dy}{dx}\right)_{(x_1,y_1)}(x - x_1)$

$\Rightarrow y + 4 = 2(x + 4)$

$\Rightarrow 2x - y + 4 = 0$.

**Q.5** (b)

$y = \sin\frac{\pi x}{2} \Rightarrow \frac{dy}{dx} = \frac{\pi}{2}\cos\frac{\pi}{2}x$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 0$

$\therefore$ Equation of normal is $y - 1 = \frac{1}{0}(x - 1)$

$\Rightarrow x = 1$ .

**Q.6** (d)

Curve is $y = be^{-x/a}$

Since the curve crosses $y$-axis (*i.e.*, $x = 0$) $\therefore$ $y = b$

Now $\frac{dy}{dx} = \frac{-b}{a}e^{-x/a}$ .

At point (0, $b$), $\left(\frac{dy}{dx}\right)_{(0,b)} = \frac{-b}{a}$

$\therefore$ Equation of tangent is $y - b = \frac{-b}{a}(x - 0)$

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$ .

**Q.7** (d)

Slope of the normal $= \frac{-1}{dy/dx}$

$\Rightarrow \tan\frac{3\pi}{4} = \frac{-1}{\left(\frac{dy}{dx}\right)_{(3,4)}}$

$\therefore \left(\frac{dy}{dx}\right)_{(3,4)} = 1$ ; $f'(3) = 1$ .

**Q.8** (d)

$y^3 + 3x^2 = 12y \Rightarrow 3y^2.\frac{dy}{dx} + 6x = 12.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}(3y^2 - 12) + 6x = 0$

$\Rightarrow \frac{dy}{dx} = \frac{6x}{12 - 3y^2}$

$\Rightarrow \frac{dx}{dy} = \frac{12 - 3y^2}{6x}$

Tangent is parallel to $y$-axis, $\frac{dx}{dy} = 0$

$\Rightarrow 12 - 3y^2 = 0$ or $y = \pm 2$.

Then $x = \pm\frac{4}{\sqrt{3}}$ , for $y = 2$

$y = -2$ does not satisfy the equation of the curve,

$\therefore$ The point is $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

**Q.9** (c)

Let the point be $(x_1, y_1)$

$\therefore y_1 = be^{-x_1/a}$ ......(i)

Also, curve $y = be^{-x/a} \Rightarrow \frac{dy}{dx} = \frac{-b}{a}e^{-x/a}$

$$\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{-b}{a}e^{-x_1/a} = \frac{-y_1}{a} \qquad \text{(by (i))}$$

Now, the equation of tangent of given curve at point $(x_1, y_1)$ is

$$y - y_1 = \frac{-y_1}{a}(x - x_1) \;\Rightarrow\; \frac{x}{a} + \frac{y}{y_1} = \frac{x_1}{a} + 1$$

Comparing with $\frac{x}{a} + \frac{y}{b} = 1$, we get, $y_1 = b$ and $1 + \frac{x_1}{a} = 1 \Rightarrow x_1 = 0$

Hence, the point is (0, *b*).

**Q.10** (d)

$y = x^3 - 3x^2 - 9x + 5 \Rightarrow \frac{dy}{dx} = 3x^2 - 6x - 9$.

We know that this equation gives the slope of the tangent to the curve.

The tangent is parallel to *x*-axis $\frac{dy}{dx} = 0$

Therefore, $3x^2 - 6x - 9 = 0$

$\Rightarrow x = -1, 3$.

**Q.11** (b)

Given curve $y^2 = x$ and $x^2 = y$

Differentiating *w.r.t. x*, $2y\frac{dy}{dx} = 1$ and $2x = \frac{dy}{dx}$

$\left(\frac{dy}{dx}\right)_{(1,1)} = \frac{1}{2}$ and $\left(\frac{dy}{dx}\right)_{(1,1)} = 2$

Angle between the curve

$$\Rightarrow \tan\phi = \frac{2 - \frac{1}{2}}{1 + \frac{1}{2}.2}$$

$\Rightarrow \tan\phi = \frac{3}{4} \Rightarrow \phi = \tan^{-1}\frac{3}{4}$.

**Q.12** (a)

Clearly the point of intersection of curves is (0, 1)

Now, slope of tangent of first curve, $m_1 = \frac{dy}{dx} = a^x \log a \Rightarrow \left(\frac{dy}{dx}\right)_{(0,1)} = m_1 = \log a$

Slope of tangent of second curve, $m_2 = \frac{dy}{dx} = b^x \log b \Rightarrow m_2 = \left(\frac{dy}{dx}\right)_{(0,1)} = \log b$

$\therefore \tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\log a - \log b}{1 + \log a \log b}$.

**Q.13** (b)

The equation of two curves are $xy = 6$ and $x^2 y = 12$ from (i) we obtain $y = \frac{6}{x}$

putting this value of $y$ in equation (ii) to obtain $x^2\left(\frac{6}{x}\right) = 12 \Rightarrow 6x = 12 \Rightarrow x = 2$

Putting $x = 2$ in (i) or (ii) we get, $y = 3$.

Thus, the two curves intersect at *P*(2, 3)

Differentiating (i) *w.r.t. x*, we get $x\frac{dy}{dx}+y=0 \Rightarrow \frac{dy}{dx}=\frac{-y}{x} \Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)}=-\frac{3}{2}=m_1$

Differentiating (ii) *w.r.t. x*, we get $x^2\frac{dy}{dx}+2xy=0 \Rightarrow \frac{dy}{dx}=\frac{-2y}{x}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)}=-3=m_2$

$$\Rightarrow \tan\theta=\frac{m_1-m_2}{1+m_1m_2}=\frac{\left(\frac{-3}{2}+3\right)}{\left(1+\left(\frac{-3}{2}\right)(-3)\right)}=\frac{3}{11}$$

$\Rightarrow \theta=\tan^{-1}\frac{3}{11}$.

**Q.14** (c)

Equation of the curve $x^2y^2=a^4$.

Differentiating the given equation,

$x^2 2y\frac{dy}{dx}+y^2 2x=0 \Rightarrow \frac{dy}{dx}=\frac{-y}{x}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(-a,a)}=-\left(\frac{a}{-a}\right)=1$

Therefore, sub-tangent = $\frac{y}{\left(\frac{dy}{dx}\right)}=a$.

**Q.15** (a)

$y^n=a^{n-1}x \Rightarrow ny^{n-1}\frac{dy}{dx}=a^{n-1}$

$\Rightarrow \left(\frac{dy}{dx}\right)=\frac{a^{n-1}}{ny^{n-1}}$

$\therefore$ Length of the subnormal = $y\frac{dy}{dx}=\frac{ya^{n-1}}{ny^{n-1}}=\frac{a^{n-1}y^{2-n}}{n}$

We also know that if the subnormal is constant,

then $\frac{a^{n-1}}{n}.y^{2-n}$ should not contain *y*.

Therefore, $2-n=0$ or $n=2$.

**Q.16** (a)

$\sqrt{x}+\sqrt{y}=\sqrt{a} \Rightarrow \frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$

$\Rightarrow \therefore \frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}$

Hence tangent at (*x*, *y*) is $Y-y=-\frac{\sqrt{y}}{\sqrt{x}}(X-x)$

or $X\sqrt{y}+Y\sqrt{x}=\sqrt{xy}(\sqrt{x}+\sqrt{y})=\sqrt{axy}$

or $\frac{X}{\sqrt{a}\sqrt{x}}+\frac{Y}{\sqrt{a}\sqrt{y}}=1$ .

Clearly its intercepts on the axes are $\sqrt{a}\sqrt{x}$ and $\sqrt{a}\sqrt{y}$ .

Sum of the intercepts = $\sqrt{a}(\sqrt{x}+\sqrt{y})=\sqrt{a}\,.\sqrt{a}=a$ .

**Q.17** (c)

Differentiating the given equation *w.r.t.* $x$ , $2y\frac{dy}{dx}=4$

at point (2, 4) $\frac{dy}{dx}=\frac{1}{2}$

$$P=\frac{y_1-x_1\left(\frac{dy}{dx}\right)}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}$$

$$=\frac{4-2\left(\frac{1}{2}\right)}{\sqrt{1+\frac{1}{4}}}=\frac{6}{\sqrt{5}}.$$

**Q.18** (b)

$f(x)=3x+\frac{2}{x} \Rightarrow f'(x)=3-\frac{2}{x^2}$

Clearly $f'(x)>0$ on the interval (1, 3)

$\therefore$ $f(x)$ is strictly increasing.

**Q.19** (c)

$f(x)=(x-1)^2-1$

Hence decreasing in $x<1$

**Alternative method:**

$f'(x)=2x-2=2(x-1)$

To be decreasing, $2(x-1)<0$

$\Rightarrow (x-1)<0 \Rightarrow x<1$ .

**Q.20** (a)

$f'(x)=6x^2+36x-96>0$ , for increasing

$\Rightarrow f'(x)=6(x+8)(x-2)\geq 0$

$\Rightarrow x\geq 2, x\leq -8$ .

**Q.21** (a)

Let $y=x^x \Rightarrow \frac{dy}{dx}=x^x(1+\log x)$;

For $\frac{dy}{dx}>0$ , $x^x(1+\log x)>0$

$\Rightarrow 1+\log x>0$

$\Rightarrow \log_e x>\log_e \frac{1}{e}$

For this to be positive, $x$ should be greater than $\frac{1}{e}$.

**Q.22** (b)

$f(x)$ will be monotonically decreasing, if $f'(x) < 0$.

$\Rightarrow f'(x) = -\sin x - 2p < 0$

$\Rightarrow \frac{1}{2}\sin x + p > 0 \Rightarrow p > \frac{1}{2}$ $[\because -1 \le \sin x \le 1]$

**Q.23** (c)

$f'(x) = 5x^4 - 60x^2 + 240$

$= 5(x^4 - 12x^2 + 48) = 5[(x^2 - 6)^2 + 12]$

$\Rightarrow f'(x) > 0 \ \forall \ x \in R$

*i.e.*, $f(x)$ is monotonically increasing everywhere.

**Q.24** (d)

If $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$ decreases monotonically for all $x \in R$,

then $f'(x) \le 0$ for all $x \in R$

$\Rightarrow 3(a+2)x^2 - 6ax + 9a \le 0$ for all $x \in R$

$\Rightarrow (a+2)x^2 - 2ax + 3a \le 0$ for all $x \in R$

$\Rightarrow a + 2 < 0$ and discriminant $\le 0$

$\Rightarrow a < -2$ and $-8a^2 - 24a \le 0$

$\Rightarrow a < -2$ and $a(a+3) \ge 0 \Rightarrow a < -2$

and $a \le -3$ or $a \ge 0 \Rightarrow a \le -3$

$\Rightarrow -\infty < a \le -3$

**Q.25** (d)

The function is monotonic increasing if, $f'(x) > 0$

$$\Rightarrow \frac{(2\sin x + 3\cos x)(\lambda\cos x - 6\sin x)}{(2\sin x + 3\cos x)^2} - \frac{(\lambda\sin x + 6\cos x)(2\cos x - 3\sin x)}{(2\sin x + 3\cos x)^2} > 0$$

$\Rightarrow 3\lambda(\sin^2 x + \cos^2 x) - 12(\sin^2 x + \cos^2 x) > 0$

$\Rightarrow 3\lambda - 12 > 0 \Rightarrow \lambda > 4$.

**Q.26** (b)

Let $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$

$$\therefore \quad f'(x) = \frac{\ln(e+x) \times \frac{1}{\pi + x} - \ln(\pi + x)\frac{1}{e + x}}{\{\ln(e + x)\}^2}$$

$$= \frac{(e+x)\ln(e+x) - (\pi + x)\ln(\pi + x)}{\{\ln(e+x)\}^2 \times (e+x)(\pi + x)}$$

$\Rightarrow f'(x) < 0$ for all $x \ge 0$ $\{\because \pi > e\}$.

Hence, $f(x)$ is decreasing in $[0, \infty)$.

**Q.27** (c)

Obviously, here $\cos 3x$ in not decreasing in $\left(0, \frac{\pi}{2}\right)$ because $\frac{d}{dx} \cos 3x = -3 \sin 3x$.

But at $x = 75^o$, $-3 \sin 3x > 0$.

Hence the result.

**Q.28** (b)

We have $f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right) \Rightarrow f'(x) = 1 - e^x$

For $f(x)$ to be increasing, we must have $f'(x) > 0$

$\Rightarrow 1 - e^x > 0 \Rightarrow e^x < 1$

$\Rightarrow x < 0 \Rightarrow x \in (-\infty, 0)$

**Q.29** (a)

$f'(x) = e^{x(1-x)} + x . e^{x(1-x)} .(1 - 2x)$

$= e^{x(1-x)}\{1 + x(1 - 2x)\}$

$= e^{x(1-x)}.(-2x^2 + x + 1)$

Now by the sign-scheme for $-2x^2 + x + 1$

$f'(x) \ge 0$, if $x \in \left[-\frac{1}{2}, 1\right]$, because $e^{x(1-x)}$ is always positive.

So, $f(x)$ is increasing on $\left[-\frac{1}{2}, 1\right]$.

**Q.30** (b)

$f(x) = x \sin x + \cos x + \cos^2 x$

$\therefore f'(x) = \sin x + x \cos x - \sin x - 2 \cos x \sin x$

$= \cos x(x - 2 \sin x)$

Hence $x \to 0$ to $\pi$, then $f'(x) \le 0$, *i.e.*,

$f(x)$ is decreasing function.

**Q.31** (b)

Let $f(x) = x^2 e^{-x}$

$\Rightarrow \frac{dy}{dx} = 2xe^{-x} - x^2 e^{-x} = e^{-x}(2x - x^2)$

Hence $f'(x) \ge 0$ for every $x \in [0, 2]$,

therefore it is non-decreasing in [0, 2].

**Q.32** (b)

$f(x) = \sin^4 x + \cos^4 x$

$= (\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x$

$= 1 - \frac{4 \sin^2 x \cos^2 x}{2} = 1 - \frac{\sin^2 2x}{2} = 1 - \frac{1}{4}(2 \sin^2 2x)$

$= 1 - \left(\frac{1 - \cos 4x}{4}\right) = \frac{3}{4} + \frac{1}{4} \cos 4x$

Hence function $f(x)$ is increasing when $f'(x) > 0$

$f'(x) = -\sin 4x > 0 \Rightarrow \sin 4x < 0$

Hence $\pi < 4x < \frac{3\pi}{2}$ or $\frac{\pi}{4} < x < \frac{3\pi}{8}$.

**Q.33** (c)

From mean value theorem $f'(c) = \frac{f(b) - f(a)}{b - a}$

$a = 0, f(a) = 0 \Rightarrow b = \frac{1}{2}, f(b) = \frac{3}{8}$

$f'(x) = (x-1)(x-2) + x(x-2) + x(x-1)$,

$f'(c) = (c-1)(c-2) + c(c-2) + c(c-1)$

$= c^2 - 3c + 2 + c^2 - 2c + c^2 - c$, $f'(c) = 3c^2 - 6c + 2$

According to mean value theorem

$\Rightarrow f'(c) = \frac{f(b) - f(a)}{b - a}$

$$\Rightarrow 3c^2 - 6c + 2 = \frac{\left(\frac{3}{8}\right) - 0}{\left(\frac{1}{2}\right) - 0} = \frac{3}{4} \Rightarrow 3c^2 - 6c + \frac{5}{4} = 0$$

$$c = \frac{6 \pm \sqrt{36 - 15}}{2 \times 3} = \frac{6 \pm \sqrt{21}}{6} = 1 \pm \frac{\sqrt{21}}{6}.$$

**Q.34** (a)

$f'(x_1) = \frac{-1}{x_1^2}$,

$$\therefore \frac{-1}{x_1^2} = \frac{\frac{1}{b} - \frac{1}{a}}{b - a} = -\frac{1}{ab} \Rightarrow x_1 = \sqrt{ab}.$$

**Q.35** (a)

Given that equation of curve $y = x^3 = f(x)$

So $f(2) = 8$ and $f(-2) = -8$

Now $f'(x) = 3x^2 \Rightarrow f'(x) = \frac{f(2) - f(-2)}{2 - (-2)}$

$$\Rightarrow \frac{8 - (-8)}{4} = 3x^2; \therefore x = \pm \frac{2}{\sqrt{3}}.$$

**Q.36** (c)

To determine '$c$' in Rolle's theorem, $f'(c) = 0$

Here $f'(x) = (x^2 + 3x)e^{-(1/2)x} . \left(-\frac{1}{2}\right) + (2x + 3)e^{-(1/2)x}$

$= e^{-(1/2)x}\left\{-\frac{1}{2}(x^2 + 3x) + 2x + 3\right\}$

$= -\frac{1}{2}e^{-(x/2)}\left\{x^2 - x - 6\right\}$

$\therefore f'(c) = 0 \Rightarrow c^2 - c - 6 = 0 \Rightarrow c = 3, -2$.

But $c = 3 \notin [-3, 0]$, Hence $c = -2$.

**Q.37** (a)

$f(x)=x^3-6x^2+ax+b \Rightarrow f'(x)=3x^2-12x+a$

$\Rightarrow f'(c)=0 \Rightarrow f'\left(2+\frac{1}{\sqrt{3}}\right)=0$

$\Rightarrow 3\left(2+\frac{1}{\sqrt{3}}\right)^2-12\left(2+\frac{1}{\sqrt{3}}\right)+a=0$

$\Rightarrow 3\left(4+\frac{1}{3}+\frac{4}{\sqrt{3}}\right)-12\left(2+\frac{1}{\sqrt{3}}\right)+a=0$

$\Rightarrow 12+1+4\sqrt{3}-24-4\sqrt{3}+a=0$

**Q.38** (a)

$y=x^5-5x^4+5x^3-10$

$\therefore \frac{dy}{dx}=5x^4-20x^3+15x^2$

$=5x^2(x^2-4x+3)$

$=5x^2(x-3)(x-1)$

$\frac{dy}{dx}=0$, gives $x=0,1,3$ ......(i)

Now, $\frac{d^2y}{dx^2}=20x^3-60x^2+30x=10x(2x^2-6x+3)$

and $\frac{d^3y}{dx^3}=10(6x^2-12x+3)$

For $x=0: \frac{dy}{dx}=0, \frac{d^2y}{dx^2}=0, \frac{d^3y}{dx^3}\neq 0$, $\therefore$ Neither minimum nor maximum

For $x=1$, $\frac{d^2y}{dx^2}=-10$ =negative, $\therefore$ Maximum value $y_{max.}=-9$

For $x=3$, $\frac{d^2y}{dx^2}=90$ =positive, $\therefore$ Minimum value $y_{min.}=-37$ .

**Q.39** (c)

$y=\sin x(1+\cos x)=\sin x+\frac{1}{2}\sin 2x$

$\therefore \frac{dy}{dx}=\cos x+\cos 2x$ and $\frac{d^2y}{dx^2}=-\sin x-2\sin 2x$

On putting $\frac{dy}{dx}=0, \cos x+\cos 2x=0$

$\Rightarrow \cos x=-\cos 2x=\cos(\pi-2x)$

$\Rightarrow x=\pi-2x$

$\therefore x=\frac{\pi}{3}$, $\therefore \left(\frac{d^2y}{dx^2}\right)_{x=\pi/3}=-\sin\left(\frac{1}{3}\pi\right)-2\sin\left(\frac{2}{3}\pi\right)$

$=\frac{-\sqrt{3}}{2}-2.\frac{\sqrt{3}}{2}=\frac{-3\sqrt{3}}{2}$ which is negative.

$\therefore$ at $x=\frac{\pi}{3}$ the function is maximum.

**Q.40** (d) $\frac{dy}{dx} = \frac{a}{x} + 2bx + 1 \Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = a + 2b + 1 = 0$

$\Rightarrow a = -2b - 1$

and $\left(\frac{dy}{dx}\right)_{x=2} = \frac{a}{2} + 4b + 1 = 0 \Rightarrow \frac{-2b-1}{2} + 4b + 1 = 0$

$\Rightarrow -b + 4b + \frac{1}{2} = 0$

$\Rightarrow 3b = \frac{-1}{2} \Rightarrow b = \frac{-1}{6}$ and $a = \frac{1}{3} - 1 = \frac{-2}{3}$.

**Q.41** (b)

$f(x) = \left(\frac{1}{x}\right)^x \Rightarrow f'(x) = \left(\frac{1}{x}\right)^x \left(\log\frac{1}{x} - 1\right)$

$f'(x) = 0 \Rightarrow \log\frac{1}{x} = 1 = \log e$

$\Rightarrow \frac{1}{x} = e \Rightarrow x = \frac{1}{e}$.

Therefore, maximum value of function is $e^{1/e}$.

**Q.42** (b)

$y = f(x) = -x^3 + 3x^2 + 9x - 27$

The slope of this curve $f'(x) = -3x^2 + 6x + 9$

Let $g(x) = f'(x) = -3x^2 + 6x + 9$

Differentiate with respect to *x*, $g'(x) = -6x + 6$

Put $g'(x) = 0 \Rightarrow x = 1$

Now, $g''(x) = -6 < 0$ and hence at $x = 1, g(x)$

(Slope) will have maximum value.

$\therefore [g(1)]_{\text{max.}} = -3 \times 1 + 6 + 9 = 12$.

**Q.43** (c)

$f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-3)^5\, dt$, $\therefore$ $f'(x) = x(e^x - 1)(x-1)(x-2)^3(x-3)^5$

For local minima, slope *i.e.*, $f'(x)$ should change sign from – *ve* to +*ve*

$f'(x) = 0 \Rightarrow x = 0, 1, 2, 3$

If $x = 0 - h$, where *h* is a very small number, then $f'(x) = (-)(-)(-1)(-1)(-1) = -ve$

If $x = 0 + h$, $f'(x) = (+)(+)(-)(-1)(-1) = -ve$

Hence at $x = 0$ neither maxima nor minima.

If $x = 1 - h$, $f'(x) = (+)(+)(-)(-1)(-1) = -ve$

If $x = 1 + h$, $f'(x) = (+)(+)(+)(-1)(-1) = +ve$

Hence, at $x = 1$ there is a local minima.

If $x = 2 - h$, $f'(x) = (+)(+1)(+)(-)(-) = +ve$

If $x = 2 + h$, $f'(x) = (+)(+)(+)(+)(-1) = -ve$

Hence at $x = 2$ there is a local maxima.

If $x = 3 - h$, $f'(x) = (+)(+)(+)(+)(-) = -ve$

If $x = 3 + h$, $f'(x) = (+)(+)(+)(+)(+) = +ve$

Hence at $x=3$ there is a local minima.

**Q.44** (c)

$f(x)=2x^3-9ax^2+12a^2x+1$

$f'(x)=6x^2-18ax+12a^2$

$f''(x)=12x-18a$

For maximum and minimum, $6x^2-18ax+12a^2=0$

$\Rightarrow x^2-3ax+2a^2=0$

$x=a$ or $x=2a$ at $x=a$ maximum and at $x=2a$ minimum

$\because p^2=q$

$a^2=2a \Rightarrow a=2$ or $a=0$ but $a>0$, therefore $a=2$.

**Q.45** (c)

$\phi(x)=\int_1^x e^{-t^2/2}(1-t^2)dt \Rightarrow \phi'(x)=e^{-x^2/2}(1-x^2)$

Now $\phi'(x)=0 \Rightarrow 1-x^2=0 \Rightarrow x=\pm 1$

Hence, $x=\pm 1$ are points of extrema of $\phi(x)$.

**Q.46** (c)

Let $y=x^3-18x^2+96x \Rightarrow \frac{dy}{dx}=3x^2-36x+96=0$

$\therefore x^2-12x+32=0 \Rightarrow (x-4)(x-8)=0, x=4,8$

Now, $\frac{d^2y}{dx^2}=6x-36$ at $x=4, \frac{d^2y}{dx^2}=24-36=-12<0$

$\therefore$ at $x=4$ function will be maximum and $[f(x)]_{\max.}=64-288+384=160$

at $x=8\ \frac{d^2y}{dx^2}=48-36=12>0$

$\therefore$ at $x=8$ function will be minimum and $[f(x)]_{\min.}=128$.

**Q.47** (d)

$y=2\cos 2x-\cos 4x$

$= 2\cos 2x(1-\cos 2x)+1$

$=4\cos 2x\sin^2 x+1$

Obviously, $\sin^2 x \geq 0$

Therefore, to be least value of *y*, cos 2*x* should be least *i.e.*, – 1.

Hence least value of *y* is – 4 + 1 = –3.

**Q.48** (a)

$f(x)=x^2\log x \Rightarrow f'(x)=(2\log x+1)x$

Now $f'(x)=0 \Rightarrow x=e^{-1/2},0$

$\because 0<e^{-1/2}<1$, $\because$ None of these critical points lies in the interval $[1,e]$

$\therefore$ So we only compare the value of $f(x)$ at the end points 1 and *e*.

We have $f(1)=0, f(e)=e^2$

$\therefore$ greatest value = $e^2$

**Q.49** (a)

$xy = 1 \Rightarrow y = \frac{1}{x}$ and let $z = x + y$

$z = x + \frac{1}{x} \Rightarrow \frac{dz}{dx} = 1 - \frac{1}{x^2}$

$\Rightarrow \frac{dz}{dx} = 0 \Rightarrow 1 - \frac{1}{x^2} = 0$

$\Rightarrow x = -1, +1$ and $\frac{d^2z}{dx^2} = \frac{2}{x^3}$

$\left(\frac{d^2z}{dx^2}\right)_{x=1} = \frac{2}{1} = 2 = +ve$ ,

$\therefore$ $x = 1$ is point of minima.

$x = 1, y = 1$ ,

$\therefore$ minimum value = $x + y = 2$ .

**Q.50** (c)

Let $x + y = 4$ or $y = 4 - x$

$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$ or $f(x) = \frac{4}{xy} = \frac{4}{x(4-x)}$

$f(x) = \frac{4}{4x - x^2}$ , $f'(x) = \frac{-4}{(4x - x^2)^2}.(4 - 2x)$

Put $f'(x) = 0 \Rightarrow 4 - 2x = 0 \Rightarrow x = 2$ and $y = 2$

$\therefore$ min. $\left(\frac{1}{x} + \frac{1}{y}\right) = \frac{1}{2} + \frac{1}{2} = 1$ .

**Q.51** (b)

Let number = $x$, then cube = $x^3$

Now $f(x) = x - x^3$ (Maximum) $\Rightarrow f'(x) = 1 - 3x^2$

Put $f'(x) = 0 \Rightarrow 1 - 3x^2 = 0 \Rightarrow x = \pm\frac{1}{\sqrt{3}}$

Because $f''(x) = -6x = -ve$ . when $x = +\frac{1}{\sqrt{3}}$ .

**Q.52** (c)

$2x + 2y = 100 \Rightarrow x + y = 50$ .....(i)

Let area of rectangle is $A$, then $A = xy \Rightarrow y = \frac{A}{x}$

From (i), $x + \frac{A}{x} = 50 \Rightarrow A = 50x - x^2$

$\Rightarrow \frac{dA}{dx} = 50 - 2x$

for maximum area $\frac{dA}{dx} = 0$

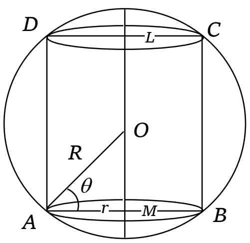
$\therefore$ $50 - 2x = 0 \Rightarrow x = 25$ and $y = 25$

$\therefore$ adjacent sides are 25 *cm* and 25 *cm*.

**Q.53** (b)

If $r$ be the radius and $h$ the height, the from the figure,

$$r^2 + \left(\frac{h}{2}\right)^2 = R^2 \Rightarrow h^2 = 4(R^2 - r^2)$$




Now, $V = \pi r^2 h = 2\pi r^2 \sqrt{R^2 - r^2}$

$$\therefore \ \frac{dV}{dr} = 4\pi r\sqrt{R^2 - r^2} + 2\pi r^2 . \frac{1}{2} \frac{(-2r)}{\sqrt{R^2 - r^2}}$$

For max. or min., $\frac{dV}{dr} = 0$

$$\Rightarrow \ 4\pi r\sqrt{R^2 - r^2} = \frac{2\pi r^3}{\sqrt{R^2 - r^2}} \Rightarrow 2(R^2 - r^2) = r^2$$

$$\Rightarrow \ 2R^2 = 3r^2 \ \Rightarrow \ r = \sqrt{\frac{2}{3}}R \Rightarrow \frac{d^2V}{dr^2} = -ve \ .$$

Hence $V$ is max. when $r = \sqrt{\frac{2}{3}}R$ .

**Q.54** (a)

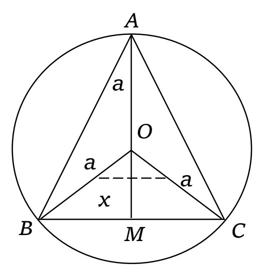
Let $OM = x$

Then height of cone *i.e.*, $h = x + a$ (where $a$ is radius of sphere)

Radius of base of cone = $\sqrt{a^2 - x^2}$

Therefore, volume $V = \frac{1}{3}\pi(a^2 - x^2)(x + a) \Rightarrow \frac{dV}{dx} = \frac{\pi}{3}(a + x)(a - 3x)$

Now, $\frac{dV}{dx} = 0 \ \Rightarrow \ x = -a, \frac{a}{3}$




But $x \neq -a$, So, $x = \frac{a}{3}$

The volume is maximum at $x = \frac{a}{3}$

Height of a cone $h = a + \frac{a}{3} = \frac{4}{3}a$

Therefore ratio of height and diameter = $\frac{\frac{4}{3}a}{2a} = \frac{2}{3}$ .