CONTINIUTY & DIFFERENTIABILITY
EXERCISE 2(B)

Q.1 If the function $f(x)=\frac{3x^2+ax+a+3}{x^2+x-2}$ is continuous at x = – 2. Find the value of a + f (– 2).

Sol. **[14]**

Since the function is conti then

$\Rightarrow$ V.F.$\big|_{x=-2}=\text{RHL}\big|_{x=-2}=\text{LHL}\big|_{x=-2}$

$\Rightarrow f(-2)=\lim_{x\to-2} f(x)$

$\Rightarrow \lim_{x\to-2}\frac{3x^2+ax+a+3}{x^2+x-2};\left(\frac{15-a}{0}\right)$form

this limit will exist when

$\Rightarrow$ 15 – a = 0

$\Rightarrow$ a = 15 ……(1)

$\Rightarrow \lim_{x\to-2}\frac{3x^2+15x+18}{x^2+x-2}$

$\Rightarrow \lim_{x\to-2}\frac{3(x+2)(x+3)}{x^2+x-2}$

$\Rightarrow \lim_{x\to-2}\frac{3(x+3)}{(x-1)}$

$\Rightarrow \frac{3}{-3}=-1$

$\Rightarrow$ hence f (– 2) = – 1

Q.2 Suppose that $f(x)=x^3-3x^2-4x+12$ and $h(x)=\begin{cases}\frac{f(x)}{x-3} & , \ x\neq 3\\ K & , \ x=3\end{cases}$ then

find the value of K that makes h continuous at x = 3

Sol. **[5]**

$\Rightarrow \lim_{x\to3^+}\frac{x^3-3x^2-4x+12}{x-3}=\lim_{x\to3^+}\frac{(x-3)(x^2-4)}{(x-3)}=\lim_{x\to3^+}(x^2-3)=5$

$\Rightarrow$ hence k = 5

Q.3 Determine the value of product of values of a & b so that f is continuous at $x=\frac{\pi}{2}.f(x)$

$$= \begin{cases} \dfrac{1-\sin^2 x}{3\cos^2 x} & \text{if} \quad x < \dfrac{\pi}{2} \\ a & \text{if} \quad x = \dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(x-2x)^2} & \text{if} \quad x > \dfrac{\pi}{2} \end{cases}$$

Sol. [2]

$$\Rightarrow \text{V.F.}\Big|_{x=\frac{\pi}{2}} = a$$

$$\Rightarrow \text{LHL}\Big|_{x=\frac{\pi}{2}} = \lim_{x\to\left(\frac{\pi}{2}\right)^-} f(x)$$

$$\Rightarrow \lim_{x\to\left(\frac{\pi}{2}\right)^-} \left(\frac{1-\sin^3 x}{3\cos^2 x}\right); \frac{0}{0} \text{ form}$$

$$\Rightarrow \text{put } x = \frac{\pi}{2} - h$$

$$\Rightarrow \lim_{h\to 0}\left(\frac{1-\sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)}\right) = \left(\lim_{h\to 0}\frac{1-\cos^3 h}{3\sin^2 h}\right) = \lim_{h\to 0}\frac{(1-\cos h)(1+\cos^2 h+\cos h)}{3(1-\cos h)(1+\cos h)} = \frac{1}{2}$$

$$\Rightarrow \text{RHL}\Big|_{x=\frac{\pi}{2}} = \lim_{x\to\frac{\pi^+}{2}} \frac{b(1-\sin x)}{(\pi-2x)^2}$$

$$\Rightarrow \text{put } x = \frac{\pi}{2} + h$$

$$\Rightarrow \lim_{h\to 0}\frac{b}{4}\frac{(1-\cos h)}{h^2} = \frac{b}{8}$$

since function is conti

$$\Rightarrow \text{V.F.}\Big|_{x=\frac{\pi}{2}} = \text{LHL}\Big|_{x=\frac{\pi}{2}} = \text{RHL}\Big|_{x=\frac{\pi}{2}}$$

$$\Rightarrow a = \frac{1}{2} = \frac{b}{8}$$

$$\Rightarrow a = \frac{1}{2}, b = 4$$

Q.4 If $f(x) = \dfrac{\sin 3x + A\sin 2x + B\sin x}{x^5}$ $(x \neq 0)$ is cont. at x = 0. Find the value of A + B + f (0)

Sol. [2]

Since the function is conti

$$\Rightarrow VF\Big|_{x=0} = RHL\Big|_{x=0} = LHL\Big|_{x=0}$$

$$\Rightarrow f(0) = \lim_{x\to 0} f(x)$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{\sin 3x + A\sin 2x + B\sin x}{x^5}\right)$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{3\cos 3x + 2A\cos 2x + B\cos x}{5x^4}\right); \left(\frac{3+2A+B}{0}\right) \text{ form}$$

$\Rightarrow$ 3 + 2A + B = 0 …..(1)

$$\Rightarrow \lim_{x\to 0}\left(\frac{-9\sin 3x + 4A\sin 2x - B\sin x}{20x^3}\right); \left(\frac{0}{0}\text{form}\right)$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{-27\cos 3x - 8A\cos 2x - B\cos x}{60x^2}\right); \left(\frac{-27-8A-B}{0}\right) \text{ form}$$

$\Rightarrow$ 27 + 8A + B = 0 …..(2)

$$\Rightarrow \lim_{x\to 0}\left(\frac{81\sin 3x + 16A\sin 2x + B\sin x}{120x}\right); \left(\frac{0}{0}\right)\text{form}$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{243\cos 3x + 32A\cos 2x + B\cos x}{120}\right)$$

$$\Rightarrow f(0) = \frac{243+32A+B}{120} \qquad …..(3)$$

using (1) & (2)

$\Rightarrow$ A = – 4, B = 5

$\Rightarrow$ then f (0) = 1

Q.5 Let $g(x) = \lim_{n\to\infty}\frac{x^n f(x) + h(x) + 1}{2x^n + 3x + 3}, x \neq 1 \text{ and } g(1) = \lim_{x\to 1}\frac{\sin^2(\pi\cdot 2^x)}{\ln(\sec(\pi\cdot 2^x))}$ be a continuous function at x = 1, find the value of 4 g (1) + 2 f (1) – h (1). Assume that f (x) and h (x) are continuous at x = 1.

Sol. [5]

$$g(x) = \lim_{x\to\infty}\frac{x^n f(x) + h(x) + 1}{2x^n + 3x + 3}$$

since g is conti

&

g (1) = 2

$$\Rightarrow g(1) = \lim_{x\to 1}\frac{\sin^2(\pi\cdot 2^x)}{\ell n(\sec(\pi\cdot 2^x))}$$

$\Rightarrow$ arg = 1 so arg – 1

$$\Rightarrow g(1) = \lim_{x\to 1}\frac{\sin^2(\pi\cdot 2^x)}{(\sec(\pi\cdot 2^x) - 1)}$$

$$\Rightarrow \lim_{x\to 1}\frac{\sin^2(\pi\cdot 2^x)\cdot\cos(\pi 2^x)}{1-\cos(\pi 2^x)}$$

$$\Rightarrow \lim_{x\to 1}\frac{\left(1-\cos \pi 2^{x}\right)\left(1+\cos \pi 2^{x}\right)}{1-\cos\left(\pi 2^{x}\right)}$$

$$\Rightarrow \lim_{x\to 1}\frac{\left(1-\cos \pi 2^{x}\right)\left(1-\cos \pi 2^{x}\right)\cos\left(\pi 2^{x}\right)}{1-\cos\left(\pi 2^{x}\right)}$$

$\Rightarrow$ g (1) = 2 ……..(1)

$$\Rightarrow g\left(x\right)=\lim_{x\to\infty}\frac{x^{n}f\left(x\right)+h\left(x\right)+1}{2x^{n}+3x+3};x\neq 1$$

$$\Rightarrow g\left(x\right)=\begin{cases}\dfrac{h\left(x\right)+1}{3\left(x+1\right)}, & 0<x<1\\ 2 \quad , & x=1\\ \dfrac{f\left(x\right)}{2} \quad , & x>1\end{cases} \qquad \text{using (1)}$$

Hint $\lim_{x\to\infty} x^{n}=\begin{cases}0 \ , & 0<x<1\\ 1 \ , & x=1\\ \infty \ , & x>1\end{cases}$

when x < 1 then $x^{n}=0$

$$\Rightarrow g\left(x\right)=\frac{h\left(x\right)+1}{3\left(x+1\right)}$$

$$\Rightarrow g\left(x\right)=\lim_{x\to\infty}\left(\frac{f\left(x\right)+\left(\dfrac{h\left(x\right)+1}{x^{n}}\right)}{2+\left(\dfrac{3x+3}{x^{n}}\right)}\right)$$

$$\Rightarrow g\left(x\right)=\frac{f\left(x\right)}{2}$$

$\Rightarrow$ h (1) = 1

$\Rightarrow$ f (1) = 4

therefore

$\Rightarrow$ 4g (1) + 2f (1) – h (1)

$\Rightarrow$ 4.2 + 2. 4 – 11

$\Rightarrow$ 8 + 8 – 11 = 5

Q.6 The function $f\left(x\right)=\left(\dfrac{2+\cos x}{x^{3}\sin x}-\dfrac{3}{x^{4}}\right)$ is not defined at x = 0.

Let L be the value of the function at x = 0 so that it is continuous at x = 0, then find the value of L^{-1}

Sol. **[60]**

Since the function is cont. At x = 0

then,

$\Rightarrow \text{V.F.}\big|_{x=0} = \text{LHL}\big|_{x=0} = \text{RHL}\big|_{x=0}$

$\Rightarrow f(x) = \left(\frac{2+\cos x}{x^3 \sin x} - \frac{3}{x^4}\right)$

$\Rightarrow L = f(0) = \lim_{x\to 0} f(x)$

$$\Rightarrow \lim_{x\to 0}\left(\frac{2x + x\cos x - 3\sin x}{x^4\left(\frac{\sin x}{x}\right).x}\right)$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{2x + x\cos x - 3\sin x}{x^5}\right)$$

$$\Rightarrow \lim_{x\to 0}\frac{1}{x^5}\left[2x + x\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}......\right) - 3\left(x - \frac{x^3}{3!} + \frac{x^5}{5!}.......\right)\right]$$

$$\Rightarrow \lim_{x\to 0}\frac{x^5}{x^5}\left[\left(\frac{1}{4!} - \frac{3}{5!}\right) + ...\right]$$

$$\Rightarrow \frac{1}{4} - \frac{3}{120} = \frac{1}{60}$$

$\Rightarrow \therefore L^{-1} = 60$

Q.7 Let $f(x) = x^3 - x^2 - 3x - 1$ and $h(x) = \frac{f(x)}{g(x)}$ where h is a rational function such that (a) it is continuous everywhere except when x = – 1, (b) $\lim_{x\to\infty} h(x) = \infty$ and (c) $\lim_{x\to -1} h(x) = \frac{1}{2}$. If $\lim_{x\to 0}(3h(x) + f(x) - 2g(x)) = -\frac{p}{q}$ where p and q are coprime then find the value of p + q

Sol. **[43]**

$\Rightarrow h(x) = \frac{f(x)}{g(x)}$

$\Rightarrow h(x) = \frac{x^3 - x^2 - 3x - 1}{(x+1)a}$

$\Rightarrow \because$ x = – 1, function will disconti; and for $x = -1 h(x) = \infty$

Now $\lim_{x\to -1} h(x) = \frac{1}{2}$

$$\Rightarrow \lim_{x\to -1}\frac{x^3 - x^2 - 3x - 1}{(x+1)a} = \frac{1}{2}$$

$$\Rightarrow \lim_{x\to -1}\frac{\left(x^2-2x-1\right)\left(x+1\right)}{\left(x+1\right)a}=\frac{1}{2}$$

$$\Rightarrow \frac{2}{a}=\frac{1}{2}$$

$\Rightarrow$ a = 4

put in (2)

$$\Rightarrow h\left(x\right)=\frac{x^3-x^2-3x-1}{\left(x+1\right)4}$$

$\Rightarrow$ g (x) = 4 (x + 1)

using (1)

$$\Rightarrow \lim_{x\to 0}\left(3.\left(\frac{x^3-x^2-3x-1}{\left(x+1\right)4}\right)+\left(x^3-x^2-3x-1\right)-2\left(4\left(x+1\right)\right)\right)$$

$$\Rightarrow \frac{3\left(-1\right)}{4}-1-8$$

$$\Rightarrow -\frac{39}{4}=-\frac{p}{q}$$

Q.8 Suppose f and g are two functions such that f, g : R $\to$ R, f (x) $\ln\left(1+\sqrt{1+x^2}\right)$ and $g\left(x\right)=\ln\left(x+\sqrt{1+x^2}\right)$ then find the value of $e^{g(x)}\left(f\left(\frac{1}{x}\right)\right)'$ + g' (x) at x = 1.

Sol. **[0]**

$$g\left(x\right)=\ln\left(x+\sqrt{1+x^2}\right)$$

$$\Rightarrow \therefore g'\left(x\right)=\frac{1}{\sqrt{1+x^2}} \qquad \ldots\ldots(1)$$

also $\left(f\left(\frac{1}{x}\right)\right)'=f'\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)$

now $f'\left(x\right)=\frac{1}{1+\sqrt{1+x^2}}\cdot\frac{x}{\sqrt{1+x^2}}$

$$\Rightarrow f'\left(\frac{1}{x}\right)=\frac{x}{x+\sqrt{1+x^2}}\cdot\frac{1}{x}\cdot\frac{x}{\sqrt{1+x^2}}$$

$$\Rightarrow f'\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)=-\frac{1}{x\sqrt{1+x^2}\left(x+\sqrt{1+x^2}\right)}$$

also $e^{g(x)}=e^{\ln\left(x+\sqrt{1+x^2}\right)}=x+\sqrt{1+x^2}$

$$\Rightarrow xe^{g(x)}=x\left(x+\sqrt{1+x^2}\right)$$

$\Rightarrow$ Hence, $e^{g(x)} \cdot x \cdot \left(f\left(\frac{1}{x}\right)\right)' = -\frac{1}{\sqrt{1+x^2}}$ ....(2)

$\Rightarrow$ From (1) and (2), $g'(x) + xe^{g(x)}\left(f\left(\frac{1}{x}\right)\right)' = 0$

Q.9 If f (x) is derivable at x = 3 & f ’ (3) = 2, then find the value of $\lim_{x\to 0} \frac{f(3+h^2) - f(3-h^2)}{2h^2}$

Sol. **[2]**

$\Rightarrow \lim_{x\to 0} \frac{f(3+h^2) - f(3h^2)}{2h^2}$

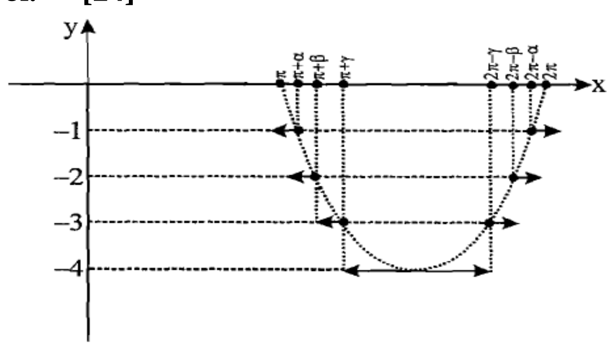
$\Rightarrow \lim_{x\to 0} \frac{2hf(3+h^2) + 2hf'(3-h^2)}{4h}$

$\Rightarrow \lim_{x\to 0} \frac{f'(3+h^2) + f(3-h^2)}{2}$ by L’ hospital

$\Rightarrow \frac{f'(3) + f'(3)}{2} = \frac{4}{2} = 2$

Q.10 Let f (x) = [3 + 4 sin x] (where [ ] denotes the greatest integer function). If sum of all the values of ‘x’ in $[\pi, 2\pi]$ where f (x) fails to be differentiable, is $\frac{k\pi}{2}$, then find the value of k.

Sol. **[24]**



$\Rightarrow$ f (x) = [3 + 4 sin x]

$\Rightarrow$ + [4 sin x]

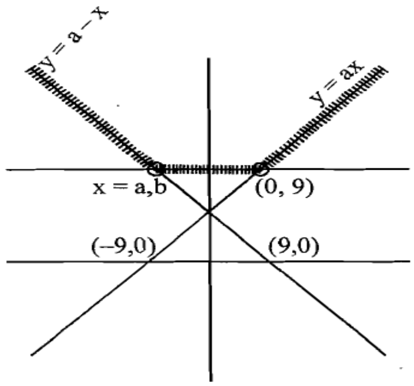
Sum of all $x = \pi + (\pi - \alpha) + (\pi + \beta) + (\pi + \gamma) + (2\pi - \beta) + (2\pi - \alpha)$

$\Rightarrow 12\pi$

$\Rightarrow \therefore k = 24$

Q.11 The number of points at which the function f (x) = max. {a – x, a + x, b}, $-\infty < x < \infty, 0 < a < b$ cannot be differentiable is

Sol. **[2]**



Q.12 Let f, g and h are differentiable functions. If f (0) = ; g (0) = 2; h (0) = 3 and the derivatives of their pair wise products at x = 0 are (f g)' (0) = 6; (gh)' (0) = 4 and (hf)' (0) = 5, then compute the value of (fgh)' (0).

Sol. **[16]**

$\Rightarrow$ (fgh)' (0) = f g h' + g h f ' + h f g' $= \dfrac{(fg)'h + (gh)'f + (hf)'g}{2}$

$$\Rightarrow \therefore (fgh)'(0) = \frac{(fg)'(0)\cdot + (gh)'(0)\cdot f(0) + (hf)'(0)\cdot g(0)}{2}$$

$$\Rightarrow \frac{6\cdot 3 + 4\cdot 1 + 5\cdot 2}{2} = 16$$

Q.13 Let $f(x) = x + \dfrac{1}{2x} + \dfrac{1}{2x} + \dfrac{1}{2x} + ...\infty$. Compute the value of $f(10)\cdot f'(10)$

Sol. **[10]**

$$\Rightarrow f(x) = x + \cfrac{1}{x + x + \cfrac{1}{2x + \cfrac{1}{2x + ...\infty}}} = x + \frac{1}{x + f(x)}$$

hence $f(x) - x = \dfrac{1}{x + f(x)}$

$\Rightarrow \therefore f^2(x) - x^2 = 1$

Differentiating w.r.t. x

$\Rightarrow 2f(x)\cdot f'(x) - 2x = 0$ or $f(x)\cdot f'(x) = x$

$\Rightarrow$ Hence $f(10)\cdot f'(10) = 10$

Q.14 If the value of the expression $y^3 \dfrac{d^2y}{dx^2}$ for the ellipse $3x^2 + 4y^2 = 12$, is $-\dfrac{p}{q}$ where p and q are co-prime then the value of p + q

Sol. **[13]**

Differentiating implicitly we have

$\Rightarrow \qquad 6x+8yy'=0$ and hence $y'=-\frac{3x}{4y};4\left[yy^{n}+(y')^{2}\right]=-3$

Differentiating again and substitute for y' we have

$\Rightarrow \qquad 3+4(y')^{2}+4yy''=0$ and hence $3+\frac{9x^{2}}{4y^{2}}+4yy''=0$

multiplying by y^2, $3y^{2}+\frac{9x^{2}}{4}+4y^{3}\frac{d^{2}y}{dx^{2}}=0$

$\Rightarrow \frac{3y^{2}}{4}+\frac{9x^{2}}{16}+y^{3}y'''=0$

$\Rightarrow \frac{3}{16}\left(3x^{2}+4y^{2}\right)+y^{3}y''=0$

$\Rightarrow$ but $3x^{2}+4y^{2}=12$ and hence $y^{3}y'''=-\frac{9}{4}=-\frac{p}{q}$ at every point on the ellipse

Q.15 If f : R $\rightarrow$R is a function such that f (x) = x^3 + x^2 f ' (1) + xf " (2) + f ''' (3) for all $x \in R$, then find the value of f (2) – f (1) + f (0).

Sol. **[0]**

Given that f{x) = x^3 +x^2 f '(1) = 1+ f '(1) + xf "(2) + f '''(3) ....(1)

Putting x = 0 and x = 1 in (1), we get

f (0) = f ''' (3) and f (1) = 1 + f ' (1) + f " (2) + f ''' (3)

$\therefore$ f (1) – f (0) = 1 + f ' (1) + f ''' (2) ….(2)

Differentiating both sides of (1) w.r.t. x, we get

$\Rightarrow$ f ' (x) = $3x^2$ + 2x f ' (1) + f " (2) …..(3)

$\Rightarrow$ and f " (x) = 6x + 2f ' (1) …..(4)

$\Rightarrow$ also f ''' (x) = 6 …..(5)

Putting x = 1, 2, 3 in (3), (4), (5) respectively then

$\Rightarrow$ f ' (1) = 3 + 2f ' (1) + f " (2)

or f ' (1) = f " (2) = – 3 ……(6)

$\Rightarrow$ f " (2) = 12 + 2 f ' (1) or 2f ' (1) – f " (2) = – 12 ……(7)

$\Rightarrow$ and f ''' (3) = 6 ……(8)

Solving (6) and (7), we get f ' (1) = – 5 and f " (2) = 2

Hence f (1) – f (0) = 1 – 5 + 2 = – 2

Also from (1), f (2) = 8 + 4f ' (1) + 2 f " (2) + f " (3)

$\Rightarrow$ 8 – 20 + 4 + 6 = – 2 ……(10)

Hence from (9) and (10), we get f (2) = f (1) – f (0).

$\Rightarrow \therefore$ f (2) – f (1) + f (0) = 0

Q.16 Let y = x sin kx. Find the sum of possible values of k for which the differential equation $\frac{d^{2}y}{dx^{2}}+y=2k\cos kx$ holds true for all $x \in R$

Sol. **[0]**

y = x sin kx ....(1)

$\Rightarrow y_1$= k x cos kx+sin kx

$\Rightarrow y_2$ = k [cos kx – kx sin kx] + k cos kx

$\Rightarrow \frac{d^2y}{dx^2} = 2k\cos kx - k^2 x \sin kx$ ……..(2)

$\Rightarrow$ Now, given $\frac{d^2y}{dx^2} + y = 2\cos kx$ (substituting the values of y and $\frac{d^2y}{dx^2}$)

$\Rightarrow$ Hence, 2k cos hx – k^2x sin kx + x sin kx = 2k cos kx

$\Rightarrow$ x sin kx [1 – k^2] = 0,

$\Rightarrow$ hence k = 1, – 1 or 0

Q.17 The function f : R $\rightarrow$ R satisfies f (x^2) $\cdot$ f ” (x) = f ’ (x) $\cdot$ f ’ (x^2) for all real x. Given that f (1) = 1 and f ’” (1) = 8, compute the value of f ’ (1) + f ” (1).

Sol. **[6]**

Given, f (x^2) $\cdot$ f ” (x) = f ’(x) $\cdot$ f ’ (x^2)

Put x = 1 in the given relation

$\Rightarrow$ f (1) $\cdot$ f ” (1) = (f ’ (1))^2

let f ’ (1) = a …..(1) (as f (1) = 1)

to find (a + b) = ?

differentiating the given relation

we get f (x^2) $\cdot$ f ’” (x) + f ” (x) $\cdot$ 2x + f ’ (x^2) $\cdot$ f ” (x)

put x = 1

$\Rightarrow$ 8 + 2ba = 2ab + ab

$\Rightarrow$ ab = 8 ……(2)

from (1) and (2)

$\Rightarrow a^3 = 8$

$\Rightarrow$ a = 2 and b = 4

$\Rightarrow$ a + b = 2 + 4 = 6

Q.18 A polynomial function f (x) is such that f (2x) = f ’ (x) f ” (x), then find the value of f (3)

Sol. **[12]**

Suppose degree of f (x) = n.

$\Rightarrow$ Then deg f ’ = n – 1 and deg f ” = n – 2, so n = n – 1 + n – 2.

Hence n = 3.

$\Rightarrow$ So put f (x) = $ax^3 + bx^2 + cx + d$. ($a \neq 0$). Now using f (2x) = f ’ (x) $\cdot$ f ” (x)

Then we have $8ax^3 + 4bx^2 + 2cx + d = (3ax^2 + 2bx + c)(6ax + 2b)$

$18a^2x^3 + 18\,abx^2 + (6ac + 4b^2)x + 2bc$.

$\Rightarrow$ Comparing x^3, $18a^2 = 8a$ ….(1) $\Rightarrow$ $a = \frac{4}{9}$

$\Rightarrow$ Comparing x^2, 18ab = 4b ….(2) $\Rightarrow$ b = 0

$\Rightarrow$ Comparing x, $2c = 6ac + 4b^2$ ….(3) $\Rightarrow$ c = 0

$\Rightarrow$ Comparing constant term, a = 2bc $\Rightarrow$ d = 0

$\Rightarrow f(x) = \frac{4x^3}{9} \therefore f(3) = 12.$

Q.19

Sol. **[6]**

$$f\left(x+y^n\right)=f\left(x\right)+\left\{f\left(y\right)\right\}^n, \forall \ x,y\in R$$

Differentiating w.r.to x gives

$$f'\left(x+y^n\right)=f'\left(x\right)$$

Hence $f'\left(x\right)=\text{constant}$

$$\Rightarrow f\left(x\right)=ax+b$$

for $x=1, y=0$

$$f\left(1\right)=f\left(1\right)+\left\{f\left(0\right)\right\}^n \Rightarrow f\left(0\right)=0$$

for $x=0, y=1$

$$f\left(1\right)=f\left(0\right)+\left\{f\left(1\right)\right\}^n \Rightarrow f\left(1\right)=0 \text{ or } 1$$

But $f\left(1\right)=0\Rightarrow f\left(x\right)=0$ for all x.

Hence $f\left(x\right)=x$

$$\Rightarrow f'\left(10\right)+f\left(5\right)=6$$

Q.20

$$f\left(x\right)=\begin{cases}\dfrac{1-a^x+x a^x \ln a}{a^x.x^2} & , \quad x<0\\ \dfrac{2^x a^x - x\ln 2 - x\ln a - 1}{x^2} & , \quad x>0\end{cases}$$

$$\Rightarrow \ell=\lim_{x\to 0^-} f\left(x\right)=\lim_{x\to 0^-}\frac{1-a^x+x a^x \ln a}{a^x x^2}$$

Replace x with 2x to get $\ell=\lim_{x\to 0^-}\dfrac{1-a^{2x}+2x a^{2x}\ln a}{4a^{2x}x^2}$

$$\Rightarrow \ell=\lim_{x\to 0^-}\frac{\left(1-a^x+x a^x \ln a\right)\left(a^x-x a^x \ln a+1\right)+x^2 a^{2x}\left(\ln a\right)^2}{4a^{2x}x^2}$$

$$\Rightarrow \ell=\frac{\left(\ln a\right)^2}{4}+\frac{\ell}{4}\lim_{x\to 0^-}\frac{\left(a^x-x a^x \ln a+1\right)}{a^x}$$

$$\Rightarrow \ell=\frac{\left(\ln a\right)^2}{4}+\frac{\ell}{2}$$

$$\Rightarrow \ell=\frac{\left(\ln a\right)^2}{2}$$

Further $r=\lim_{x\to 0^+} f\left(x\right)=\lim_{x\to 0^-}\dfrac{(2a)^x - x\ln 2a - 1}{x^2}$

Replace x with 2x to get

$$\Rightarrow r = \lim_{x \to 0^-} \frac{(2a)^{2x} - 2x \ln 2a - 1}{4x^2}$$

$$\Rightarrow r = \lim_{x \to 0^-} \frac{\left((2a)^x - x \ln 2a - 1\right)\left((2a)^x + x \ln 2a + 1\right) + x^2 (\ln 2a)^2}{4x^2}$$

$$\Rightarrow r = \frac{r}{4} \times \lim_{x \to 0^-} \left((2a)^x + x \ln 2a + 1\right) + \frac{(\ln 2a)^2}{4}$$

$$\Rightarrow r = \frac{(\ln 2a)^2}{4} + \frac{r}{2}$$

$$\Rightarrow r = \frac{(\ln 2a)^2}{2}$$

For f(x) to be continuous,

$\ell = r = f(0)$