CONTINUITY & DIFFERENTIABILITY
EXERCISE 2(A)

More than one options may be correct

Q.1 If $f(x)=\begin{cases}\dfrac{x\cdot \ell n(\cos x)}{\ell n\left(1+x^2\right)} & x\neq 0\\ 0 & x=0\end{cases}$ then :

(A) f is continuous at x = 0
(B) f is continuous at x = 0 but not differentiable at x = 0
(C) f is differentiable at x = 0
(D) f is not continuous at x = 0

Sol. **[A, C]**

$$\Rightarrow\ f'\left(0^+\right)=\lim_{h\to 0}\frac{h\ln\left(\cos h\right)}{h\ln\left(1+h^2\right)}=\lim_{h\to 0}\frac{\ln\left(\cos h\right)^{\frac{1}{h^2}}}{\dfrac{\ln\left(1+h^2\right)}{h^2}}$$

$$\Rightarrow\ \lim_{h\to 0}\frac{1}{h^2}\left(\cos h-1\right)=-\frac{1}{2}$$

$\Rightarrow$ Paralally $f'\left(0^-\right)=-\dfrac{1}{2}$

Hence f is continuous and derivable at x = 0

Q.2 Given that the derivative f ’ (a) exists. Indicate which of the following statement(s) is/are always true.

(A) $f'\left(x\right)=\lim_{h\to a}\dfrac{f\left(h\right)-f\left(a\right)}{h-a}$
(B) $f'\left(a\right)=\lim_{h\to 0}\dfrac{f\left(a\right)-f\left(a-h\right)}{h}$
(C) $f'\left(a\right)=\lim_{t\to 0}\dfrac{f\left(a+2t\right)-f\left(a\right)}{t}$
(D) $f'\left(a\right)=\lim_{t\to 0}\dfrac{f\left(a+2t\right)-f\left(a+t\right)}{2t}$

Sol. **[A, B]**

$\Rightarrow$ (C) is false and is True only if f ’ (a) = 0 limit is 2f ’ (a). In (D) same logic limit is $\dfrac{1}{2}f'\left(a\right)$

Q.3 Let [x] denote the greatest integer less than or equal to x. If $f(x)=\left[x\sin\pi x\right]$, then f (x) is:

(A) continuous at x = 0
(B) continuous in (– 1, 0)
(C) differentiable at x = 1
(D) differentiable in (– 1, 1)

Sol. **[A, B, D]**

$$\Rightarrow\ f\left(x\right)=\begin{bmatrix}0 & 0<x<1\\ 0 & x=0\text{ or }1\text{ or }-1\\ 0 & -1<x<0\end{bmatrix}$$

$\Rightarrow$ f (x) = 0 for all in [– 1, 1]

Q.4 The function, $f\left(x\right)=\left[\left|x\right|\right]-\left|\left[x\right]\right|$ where [x] denotes greatest integer function

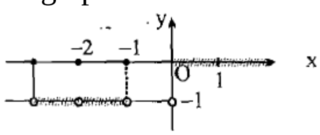
(A) is continuous for all positive integers
(B) is continuous for all non positive integers
(C) has finite number of elements in its range
(D) is such that its graph does not lie above the x – axis.

Sol. **[A, B, C, D]**

$$\Rightarrow [|x|]-|[x]| = \begin{bmatrix} 0 & x=-1 \\ -1 & -1<x<0 \\ 0 & 0\le x\le 1 \\ 0 & 1<x\le 2 \end{bmatrix}$$

$\Rightarrow$ range is {0, – 1}

The graph is



Q.5 Let f (x + y) = f (x) + f (y) for all $x, y \in R$. Then:

(A) f (x) must be continuous $\forall x \in R$
(B) f (x) may be continuous $\forall x \in R$
(C) f (x) must be discontinuous $\forall x \in R$
(D) f (x) may be discontinuous $\forall x \in R$

Sol. **[B, D]**

$\Rightarrow \lim_{h\to 0} f(x+h) = \lim_{h\to 0} f(x)+f(h)$

$\Rightarrow f(x)+\lim_{h\to 0} f(h)$

Hence if $h \to 0$

$\Rightarrow$ f (h) = 0

$\Rightarrow$ 'f' is continuous otherwise discontinuous

Q.6 The function $f(x)=\sqrt{1-\sqrt{1-x^2}}$

(A) has its domain $-1 \le x \le 1$.
(B) has finite one sided derivates at the point x = 0.
(C) is continuous and differentiable at x = 0.
(D) is continuous but not differentiable at x = 0.

Sol. **[A, B, D]**

$$\Rightarrow f'(0^+)=\frac{1}{\sqrt{2}}; f'(0^-)=-\frac{1}{\sqrt{2}}; f(x)=\frac{\sqrt{x^2}}{\sqrt{1+\sqrt{1-x^2}}}=\frac{|x|}{\sqrt{1+\sqrt{1-x^2}}}$$

Q.7 Consider the function *f(x)* = | x^3 + 1 | then

(A) Domain of f $x \in R$
(B) Range of f is R^+
(C)f has no inverse.
(D)f is continuous and differentiable for every $x \in R$.

Sol. **[A, C]**

Range is $R^+ \cup \{0\}$ $\Rightarrow$ B is not correct

f is not differentiable at x = – 1

$$\Rightarrow \text{ as } f(x) = \begin{bmatrix} x^3+1 & \text{if } x \ge -1 \\ -(x^3+1) & \text{if } x < -1 \end{bmatrix}$$

$$\Rightarrow f'(x) = \begin{bmatrix} 3x^2 & \text{if } x > -1 \\ -3x^2 & \text{if } x < -1 \end{bmatrix}$$

$$\Rightarrow f'(-1^+) = 3;$$

$$\Rightarrow f'(-1^-) = -3$$

f is not differentiable at x = – 1

also since f is not bijective hence it has no inverse

Q.8 Let $f(x) = \dfrac{\sqrt{x-2\sqrt{x-1}}}{\sqrt{x-1}-1}.x$ then:

(A) f ' (10) = 1 (B) $f'\left(\dfrac{3}{2}\right) = -1$

(C) domain of f (x) is $x \ge 1$ (D) none

Sol. **[A, B]**

$$\Rightarrow f(x) = \frac{\sqrt{\left(\sqrt{x-1}\right)^2+1-2\sqrt{x-1}}}{\sqrt{x-1}-1}\cdot x = \frac{\left|\sqrt{x-1}-1\right|}{\sqrt{x-1}-1}\cdot x = \begin{bmatrix} -x & \text{if } x \in [1,2) \\ x & \text{if } x \in (2,\infty) \end{bmatrix}$$

Q.9 f is a continuous function in [a, b]; g is a continuous function in [b, c]

A function h (x) is defined as

h (x) = f (x) for $x \in [a,b)$

= g (x) for $x \in (b,c]$

If f (b) = g (b), then

(A) h (x) has a removable discontinuity at x=b.

(B) h (x) may or may not be continuous in [a, c]

(C) $h(b^-) = g(b^+)$ and $h(b^+) = f(b^-)$

(D) $h(b^+) = g(b^-)$ and $h(b^-) = f(b^+)$

Sol. **[A, C]**

Given f is continuous in [b, c] ……..(1)

g is continuous in [b, c] ………(2)

f (b) = g (b) ………(3)

$$\Rightarrow h(x) = \left.\begin{array}{ll} f(x) & \text{for } x \in [a,b) \\ = f(b) = g(b) & \text{for } x = b \\ = g(x) & \text{for } x \in (b,c] \end{array}\right\} \quad \ldots\ldots(4)$$

$\Rightarrow$ h (x) is continuous in $[a,b)\cup(b,c]$ [using (1), (2)] also $f\left(b^-\right) = f(b);\ g(b^+) = g(b)$ ………(5) [using (1), (2)]

$\Rightarrow\ \therefore\ h\left(b^-\right) = f\left(b^-\right) = f\left(b\right) = g\left(b\right) = g\left(b^+\right) = h\left(b^+\right)$

$\Rightarrow$ now, verify each alternative. Of course! $g(b^-)$ and $f(b^+)$ are undefined.

$h(b^-) = f\left(b^-\right) = f(b) = g(b) = g(b^+)$

$\Rightarrow$ and $h(b^+) = g(b^+) = g(b) = f(b) = f\left(b^-\right)$

$\Rightarrow$ hence $h(b^-) = h(b^+) = f(b) = g(b)$

$\Rightarrow$ and $h(b)$ is not defined

Q.10 The function $f\left(x\right) = \begin{cases} |x-3| & ,\ x \ge 1 \\ \left(\dfrac{x^2}{4}\right) - \left(\dfrac{3x}{2}\right) + \left(\dfrac{13}{4}\right) & ,\ x < 1 \end{cases}$ is :

(A) continuous at x = 1 (B) differentiable at x =1

(C) continuous at x = 3 (D) differentiable at x = 3

Sol. **[A, B, C]**

$$\Rightarrow f\left(x\right) = \begin{cases} x-3 & \text{if } x \ge 3 \\ 3-x & \text{if } 1 \le x < 3 \\ \dfrac{x^2}{4} - \dfrac{3x}{2} + \dfrac{13}{4} & \text{if } x < 1 \end{cases}$$

$$\Rightarrow f'\left(1^+\right) = \underset{h\to 0}{\text{limit}} \frac{f\left(1+h\right) - f\left(1\right)}{h}$$

$$\Rightarrow \underset{h\to 0}{\text{limit}} \frac{3-\left(1+h\right)-2}{h} = -1$$

$$\Rightarrow f'\left(1^-\right) = \underset{h\to 0}{\text{limit}} \frac{\dfrac{\left(1-h\right)^2}{4} - \dfrac{3}{2}\left(1-h\right) + \dfrac{13}{4} - 2}{-h}$$

$$\Rightarrow \underset{h\to 0}{\text{limit}} \frac{\left(1-h\right)^2 - 6\left(1-h\right) + 5}{-4h}$$

$$\Rightarrow \underset{h\to 0}{\text{limit}} \frac{h^2 - 2h + 6h}{-4h} = -1$$

$\Rightarrow$ f' is continuous at x = 1

Q.11 Which of the following statements are true?

(A) If $xe^{xy} = y + \sin x$, then at y I (0) = 1.

(B)If $f(x) = a_o x^{2m+}1 + a_l x^{2m} + a_3 X^{2m-1} + \ldots\ldots + a_{2m}+1 = 0$ $(a_0 \neq 0)$ is a polynomial equation with rational co-efficients then the equation f''(x) = 0 must have a real root.($m \in N$).

(C) If (x – r) is a factor of the polynomial $f(x) = a_n x'' + a_{n\_1} x^{n-I} + a_{n-2} x^{n-2} + \ldots\ldots + a_0$ repeated m times where $1 \le m \le n$ then r is a root of the equation f' (x) = 0 repeated (m – 1) times.

(D) If y = $\sin^{-1}(\cos \sin^{-1} x) + \cos^{-1}(\sin \cos^{-1} x)$ then $\frac{dy}{dx}$ is independent on x.

Sol. **[A, C, D]**

[D] Let $\sin^{-1} x = t$

$\Rightarrow \cos^{-1} x = \frac{\pi}{2} - t$

$\Rightarrow y = \sin^{-1}(\cos t) + \cos^{-1}\left(\sin\left(\frac{\pi}{2} - t\right)\right) = \sin^{-1}(\cos t) + \cos^{-1}(\cos t)$

$\Rightarrow y = \frac{\pi}{2}$

$\Rightarrow \frac{dy}{dx} = 0$

Q.12 Let $y = \sqrt{x + \sqrt{x + \sqrt{+.........\infty}}}$ then $\frac{dy}{dx} =$

(A) $\frac{1}{2y-1}$ (B) $\frac{x}{x+2y}$ (C) $\frac{1}{\sqrt{1+4x}}$ (D) $\frac{y}{2x+y}$

Sol. **[A, C, D]**

$\Rightarrow y^2 = x + y$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y-1}$

also $y = \frac{x}{y} + 1$

$\Rightarrow \frac{dy}{dx} = \frac{y}{2x+y}$

make a quadratic in y to get explicit function

Q.13 If $\sqrt{y+x} + \sqrt{y-x} = c$ (where $c \neq 0$), then $\frac{dy}{dx}$ has the value equal to

(A) $\frac{2x}{c^2}$ (B) $\frac{x}{y+\sqrt{y^2-x^2}}$ (C) $\frac{y\sqrt{y^2-x^2}}{x}$ (D) $\frac{c^2}{2y}$

Sol. **[A, B, C]**

$\Rightarrow$ Square both sides, differentiate and rationalize

Q.14 If $f(x) = \cos\left[\frac{\pi}{x}\right] \cos\left(\frac{\pi}{2}(x-1)\right)$; where $[x]$ is the greatest integer function of x, then f(x) is continuous at

(A) x = 0 (B) x = 1 (C) x = 2 (D) none of these

Sol. **[B, C]**

$\Rightarrow$ (A) = Not defined at x = 0;

$\Rightarrow$ (B) = f (1) = cos 3; f (2) = 0 and both the limits exist

Q.15 Select the correct statements.

(A) The function f defined by f (x) = $f(x)=\begin{bmatrix} 2x^2+3 & \text{for } x \le 1 \\ 3x+2 & \text{for } x > 1 \end{bmatrix}$ is neither differentiable nor continuous at x = 1

(B) The function $f(x)=x^2|x|$ is twice differentiable at x = 0.

(C) If f is continuous at x = 5 and f (5) = 2 then $\lim\limits_{x\to 2} f\left(4x^2-11\right)$ exists.

(D) If $\lim\limits_{x\to a}\left(f(x)+g(x)\right)=2$ and $\lim\limits_{x\to a}\left(f(x)-g(x)\right)=1$ then $\lim\limits_{x\to a} f(x)\cdot g(x)$ need not exist.

Sol. **[B, C]**

Q.16 Which of the following functions has/have removable discontinuity at x = 1.

(A) $f(x)=\dfrac{1}{\ell n|x|}$

(B) $f(x)=\dfrac{x^2-1}{x^3-1}$

(C) $f(x)=2^{-2^{\left(\frac{1}{1-x}\right)}}$

(D) $f(x)=\dfrac{\sqrt{x+1}-\sqrt{2x}}{x^2-x}$

Sol. **[B, D]**

(A) $\lim\limits_{x\to 1} f(x)$ does not exist

(B) $\lim\limits_{x\to 1} f(x)=\dfrac{2}{3}$ $\therefore$ f (x) has removable discontinuity at x = 1

(C) $\lim\limits_{x\to 1} f(x)$ does not exist

(D) $\lim\limits_{x\to 1} f(x)=\dfrac{-1}{2\sqrt{2}}$ $\therefore$ f (x) has removable discontinuity at x = 1

Q.17 f (x) is an even function, x = 1 is a point of minima and x = 2 is a point of maxima for y = f (x). Further $\lim\limits_{x\to\infty} f(x)=0$, and $\lim\limits_{x\to\infty} f(x)=\infty$. f (x) is increasing in (1, – 2) & decreasing everywhere in $(0,1)\cup(2,\infty)$. Also f (1) = 3 & f (2) = 5 Then

(A) f (x) = 0 has no real roots

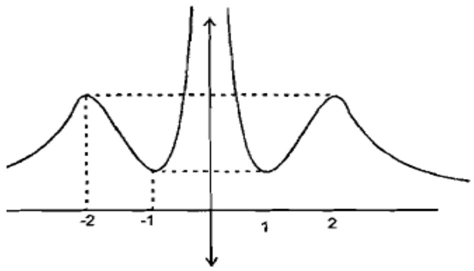
(B) y = f (x) and $y=|f(x)|$ are identical functions

(C) f ' (x) = 0 has exactly four real roots whose sum is zero

(D) f ' (x) = 0 has exactly four real roots whose sum is 6

Sol. **[A, B, C]**

$\lim\limits_{x\to 0} f(x)=\infty,$ $\lim\limits_{x\to\infty} f(x)=0$



$\Rightarrow$ f (x) is increasing in (1, 2) and decreasing in $(0,1)\cup(2,\infty)$ from the graph

Q.18

Q.19

Q.20

PASSAGE 1

A curve is represented parametrically by the equations x = f (t) = $a^{\ln(b^t)}$ and y = g (t) = $b^{-\ln(a^t)}$ a, b > 0 and $a \neq 1, b \neq 1$ where $t \in R$.

Q.21 Which of the following is not a correct expression for $\frac{dy}{dx}$?

(A) $\frac{-1}{f(t)^2}$ (B) – (g(t))^2 (C) $\frac{-g(t)}{f(t)}$ (D) $\frac{-f(t)}{g(t)}$

Sol. **[D]**

Q.22 The value of $\frac{d^2y}{dx^2}$ at the point where f (t) = g (t) is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2

Sol. **[D]**

Q.23 The value of $\frac{f(t)}{f'(t)} \cdot \frac{f'(-t)}{f'(-t)} + \frac{f(-t)}{f'(-t)} \cdot \frac{f''(t)}{f'(t)} \forall t \in R$, is equal to

(A) – 2 (B) 2 (C) – 4 (D) 4

Sol. **[B]**

$\Rightarrow$ $x = f(t) = a^{\ln(b^t)} = a^{t\ln b}$ …….(1)

$\Rightarrow$ $y = g(t) = b^{-\ln(a^t)} = (b^{\ln a})^{-t} = (a^{\ln b})^{-t} = a^{-t\ln b}$

$\Rightarrow$ $\therefore y = g(t) = a^{\ln(b^{-t})} = f(-t)$ …….(2)

From equation (1) and (2)

$\Rightarrow$ xy = 1

(i) $\because y = \frac{1}{x}$

$\Rightarrow \therefore \frac{dy}{dx} = -\frac{1}{x^2} = -\frac{1}{f^2(t)}$ **(A)** is correct

$\Rightarrow$ Also xy = 1

$\Rightarrow \frac{dy}{dx} = -\frac{1}{x^2} = -\frac{y^2}{1} = -g^2(t)$ **(B)** is correct

$\Rightarrow$ Again xy = 1 $\frac{dy}{dx} = -\frac{y}{x} = -\frac{g(t)}{f(t)}$ **(C)** is correct

(D) is incorrect

(ii) f(t) = g(t) $\Rightarrow$ f (t) = f (– t) $\Rightarrow$ t = 0
{$\because$ f (t) is one-one function}
At t = 0, x = y = 1

$\Rightarrow \therefore \frac{dy}{dx} = \frac{-1}{x^2}$ and $\frac{d^2y}{dx^2} = \frac{2}{x^3}$

$\Rightarrow$ At x = 1, $\frac{d^2y}{dx^2} = 2$

(iii) $\therefore$ xy = 1 $\therefore$ f g = 1 $\therefore$ f g’ + g f’ = 0
$\Rightarrow \therefore$ f g” + g’ f’ + g’ f’ + g f ” = 0
$\Rightarrow$ f g” + g f ” + 2 g’ f ’ = 0

$\Rightarrow \frac{fg''}{f'g'} + \frac{gf''}{g'f'} = -2$ ……(3)

from equation (2)
$\Rightarrow$ g (t) = f (– t)
$\Rightarrow \therefore$ g’ (t) = – f ’ (– t)
and g” (t) = f ” (– t)
substituting in equation (3)

$\Rightarrow \frac{f(t)}{f'(t)} \cdot \frac{f''(-t)}{-f'(-t)} + \frac{f(-t)}{-f'(-t)} \cdot \frac{f''(t)}{f'(t)} = -2$

$\Rightarrow \frac{f(t)}{f'(t)} \cdot \frac{f''(-t)}{f'(-t)} + \frac{f(-t)}{f'(-t)} \cdot \frac{f''(t)}{f'(t)} = 2$

$\Rightarrow$ 1

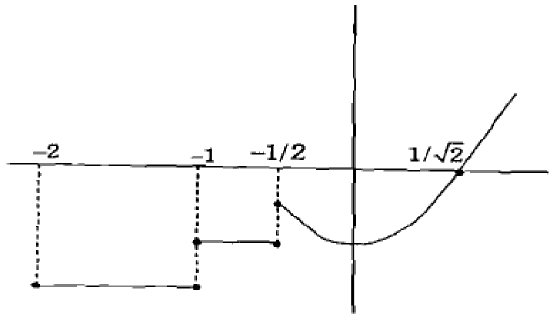
PASSAGE 2

Let a function be defined as $f(x) = \begin{cases} [x], & -2 \le x \le -\frac{1}{2} \\ 2x^2 - 1, & -\frac{1}{2} < x \le 2 \end{cases}$, where [.] denotes greatest integer function.
Answer the following question by using the above information.

Q.24 The number of points of discontinuity of f (x) is

(A) 1 (B) 2 (C) 3 (D) 0

Sol. **[B]**

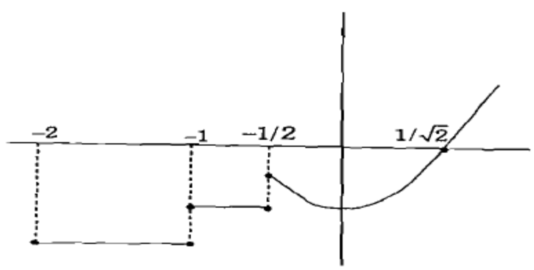


$\Rightarrow$ Two points of discontinuity $-1, -\frac{1}{2}$

Q.25 The function f (x – 1) is discontinuous at the points

(A) $-1, -\frac{1}{2}$ (B) $-\frac{1}{2}, 1$ (C) $0, \frac{1}{2}$ (D) 0, 1

Sol. **[C]**

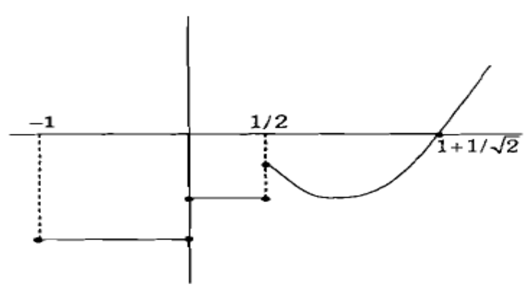


$\Rightarrow$ Discontinuous at $1, \frac{1}{2}$

Q.26 Number of points where $|f(x)|$ is not differentiable is

(A) 1 (B) 2 (C) 3 (D) 4

Sol. **[C]**



$\Rightarrow$ at $-1, -\frac{1}{2}, \frac{1}{\sqrt{2}}$ the function is not differentiable.

PASSAGE 3

Two students, A & B are asked to solve two different problem. A is asked to evaluate $\lim_{x\to 0}\frac{1-\cos(\ln(1+x))}{x^2}$ & B is asked to evaluate $\lim_{x\to\infty}\left(\frac{\sqrt{n}}{\sqrt{n^3+1}}+\frac{\sqrt{n}}{\sqrt{n^3+1}}+......+\frac{\sqrt{n}}{\sqrt{n^3+2n}}\right), n\in N.$ A provides the following solution

$$\text{Let } h=\lim_{x\to 0}\frac{1-\cos\left(\frac{\ln(1+x)}{x}\cdot x\right)}{x^2}=\lim_{x\to 0}\frac{1-\cos x}{x^2}\left(\text{As}\lim_{x\to 0}\frac{\ln(1+x)}{x}=1\right)$$

$$l_1=\frac{1}{2}$$

B provides the following solution

$$\text{Let } l_2=\lim_{n\to\infty}\left\{\sum_{r=1}^{2n}\frac{\sqrt{n}}{\sqrt{n^3+r}}\right\}=\lim_{n\to\infty}\left\{\sum_{r=1}^{2n}\frac{1}{n}\frac{\sqrt{n}}{\sqrt{n+\frac{r}{n^2}}}\right\}$$

$$\lim_{n\to\infty}\left[\frac{1}{n}\left\{\sqrt{\frac{n}{n+\frac{1}{n^2}}}+\sqrt{\frac{n}{n+\frac{2}{n^2}}}+.......+\sqrt{\frac{n}{n+\frac{2n}{n^2}}}\right\}\right]$$

$$\lim_{n\to\infty}\left[\frac{1}{n}\left(\underbrace{1+1+.......+1}_{2n\text{ times}}\right)\right]=\lim_{n\to\infty}\frac{2n}{n}=2$$

Q.27 Identify the correct statement

(A) both of them get the correct answer

(B) both of them get the incorrect answer

(C) A gets the correct answer while B gets the incorrect answer.

(D) B gets the correct answer while A gets the incorrect answer.

Sol. **[A]**

Q.28 Who has solved the problem correctly.

(A) A (B) B (C) both of them (D) no one

Sol. **[D]**

Q.29 $f(x) = \begin{cases} 4l_1\left(\dfrac{\tan x - \sin x}{x^3}\right) & x < 0 \\ k & x = 0 \\ l_2\left(\dfrac{e^x - x - 1}{1-\cos x}\right) & x > 0 \end{cases}$ where l_1 and l_2 are correct values of the corresponding limits, if is continuous at x = 0 the K is equal to:

(A) 1 (B) 2 (C) 3 (D) no value of K

Sol. **[D]**

$$\Rightarrow l_1 = \lim_{x\to 0}\frac{1-\cos(\ln(1+x))}{\ln^2(1+x)}\cdot\left(\frac{\ln(1+x)}{x}\right)^2 = \frac{1}{2}$$

A & B have made the same mistake, they used the notion of limit partly in the problem, where as once the limiting notion has been used the resulting expression must be free from the variable on which the limit has been imposed

$$\Rightarrow \lim_{n\to\infty}\frac{2n\sqrt{n}}{\sqrt{n^3+1}} < l_2 < \lim_{n\to\infty}\frac{2n\sqrt{2}}{\sqrt{n^3+1}}$$

Hence $l_2 = 2$ (sandwich theorem)

$\Rightarrow$ **Sol.1** **Hence (A)**

$\Rightarrow$ **Sol.2** **Hence (D)**

$\Rightarrow$ **Sol.3** $\lim_{x\to 0} 4\cdot\frac{1}{2}\left(\frac{\tan x - \sin x}{x^3}\right) = 4\cdot\frac{1}{2}\cdot\frac{1}{2} = 1$

$$\Rightarrow \lim_{x\to 0} l_2\left(\frac{e^x - x - 1}{x^2}\cdot\frac{x^2}{1-\cos x}\right) = 2(2\cdot 2) = 8$$

$\Rightarrow$ for no value if K

Hence (D)

PASSAGE 4

Q.30

Q.31

Q.32

Matrix match type

Q.33

Q.34

	Column – I	Column – II
(A)	$f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ \cos x & \text{if } x \ge 0 \end{cases}$, at $x = 0$ is	(P) continuous
(B)	For every $x \in R$ the function	(Q) differentiability

$$g(x)=\frac{\sin(\pi[x-\pi])}{1+[x]^2}$$ (R) discontinuous

where [x] denotes the greatest integer function is (S) non derivable

(C) $h(x)=\sqrt{\{x\}^2}$ where {x} denotes fractional part function for all $x \in I$, is

(D) $k(x)=\begin{bmatrix} x^{\frac{1}{\ln x}} \text{ if } x \neq 1 \\ e \text{ if } x = 1 \end{bmatrix}$ at $x = 1$is

Sol. **(A) $\Rightarrow$ P, S; (B) $\Rightarrow$ P, Q; (C) $\Rightarrow$ R, S; (D) $\Rightarrow$ P, Q**

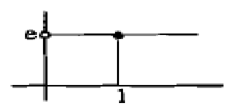
(A) $f'(0)=\lim\limits_{h\to 0}\frac{\cos h-0}{h}$ does not exist. Obviously f (0) = f (0^+) = 1

Hence continuous and not derivable

(B) g (x) = 0 for all x, hence continuous and derivable

(C) as $0 \le \{f(x)\} < 1$, hence $h(x)=\sqrt{\{x\}^2}=\{x\}$ which is discontinuous hence non derivable all $x \in I$

(D) $\lim\limits_{x\to 1} x^{\frac{1}{\ln x}} = \lim\limits_{x\to 1} x^{\log_x e} = e = f(1)$



$\Rightarrow$Hence k (x) is constant for all x > 0 hence continuous and differentiable at x = 1.

Q.35 **Column – I** **Column – II**

(A) Number of points of discontinuity of $f(x)=\tan^2 x-\sec^2 x$ in $(0,2\pi)$ is (p) 1

(B) Number of points at which $f(x)=\sin^{-1}x+\tan^{-1}x+\cot^{-1}x$ is non-differentiable in (– 1, 1) is (q) 2

(C) Number of points of discontinuity of $y=[\sin x], x\in[0,2\pi)$ where [.] represents greatest integer function (r) 0

(D) Number of points where $y=\left|(x-1)^3\right|+\left|(x-2)^5\right|+|x-3|$ is non-differentiable (s) 3

Sol. **(A) $\Rightarrow$ q; (B) $\Rightarrow$ r; (C) $\Rightarrow$ q; (D) $\Rightarrow$ p**

(A) $\tan^2 x$ is discontinuous at $x=\frac{\pi}{2},\frac{3\pi}{2}$

$\Rightarrow$ $\sec^2 x$ is discontinuous at x = $x=\frac{\pi}{2},\frac{3\pi}{2}$

$\Rightarrow$ Number of discontinuities = 2

(B) Since $f(x)=\sin^{-1}x+\tan^{-1}x+\cot^{-1}x=\sin^{-1}x+\frac{\pi}{2}$

$\Rightarrow \therefore$ f(x) is differentiable in (– 1, 1)

$\Rightarrow$number of points of non-differentiable = 0

(C) $y = [\sin x] = \begin{cases} 0 & , \ 0 \le x \frac{\pi}{2} \\ 1 & , \ x = \frac{\pi}{2} \\ 0 & , \ \frac{\pi}{2} < x \le \pi \\ -1 & , \pi < x < 2\pi \\ 0 & , \quad x = 2\pi \end{cases}$ 7t

$\Rightarrow \therefore$ Points of discontinuity are $\frac{\pi}{2}, \pi$

(D) $y = \left|(x-1)^3\right| + \left|(x-2)^5\right| + |x-3|$ is non differentiable at x = 3 only.