EXERCISE – 2(B)

MULTIPLE CHOICE QUESTIONS

1. (AD)

Given $9(x^2 + 2x + 1) - 16(y^2 - 2y + 1) = 144$ or $\frac{(x+1)^2}{16} - \frac{(y-1)^2}{9} = 1$

Now a = 4 , b = 3 & Center : (–1, 1)

$\frac{b^2}{a^2} = e^2 - 1 \Rightarrow \frac{9}{16} = e^2 - 1$ or $e = \frac{5}{4}$

$\therefore$ focus $(-1 \pm 5, 1)$

$\therefore$ (–4, 1) & (–6, 1)

2. (ABD)

$x^2 - y^2 = \cos^2\alpha$

Vertices $\equiv (\pm\cos\alpha, 0)$

Abscissae of foci $\equiv \pm\cos\alpha\sqrt{2} - 0$

$e = \sqrt{2}$

Equation of directrics : $x = \pm\frac{\cos\alpha}{\sqrt{2}}$

3. (BCD)

For hyperbola $2a = \frac{1}{2}$ & given Ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

As the curve are confocal hence $2.\frac{1}{4}.e = 2\sqrt{a^2 - b^2} = 2$

$\Rightarrow e = 4 \to (B)$

$\therefore b^2 = a^2(e^2 - 1) = \frac{1}{16}(16 - 1) = \frac{15}{16}$

Hyperbola: $\frac{x^2}{\frac{1}{16}} - \frac{y^2}{\frac{15}{16}} = 1$

$\Rightarrow \frac{x^2}{1} - \frac{y^2}{15} = \frac{1}{16}$

Distance between directrices $= \frac{2a}{3} = \frac{\frac{1}{2}}{4} = \frac{1}{8} \quad \to (C)$

L.R. $= \frac{2b^2}{a} = \frac{2.\frac{15}{16}}{\frac{1}{4}} = \frac{15}{2} \qquad \to (D)$

4. (AB)

$\left|\sqrt{x^2 + (y-1)^2} - \sqrt{x^2 + (y+1)^2}\right| = k$

Clearly, it is of the form

$|SP - S'P| = 2a$ where $2a < SS'$

$\Rightarrow$ k < distance between (0, 1), (0, –1)

$\Rightarrow$ $k < 2$

Obviously $k > 0$

$\Rightarrow$ exhaustive values of k are (0, 2)

5. **(AB)**

$(a\cos\theta, b\sin\theta), (a\cos\phi, b\sin\phi) \& (ae, 0)$ are collinear, hence

$$\begin{vmatrix} 1 & a\cos\theta & b\sin\theta \\ 1 & a\cos\phi & b\sin\phi \\ 1 & ae & 0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & \cos\theta & \sin\theta \\ 1 & \cos\phi & \sin\phi \\ 1 & e & 0 \end{vmatrix} = 0 \Rightarrow e\sin\phi - e\sin\theta + \sin(\theta - \phi) = 0 \Rightarrow \frac{1}{e} = \frac{\sin\theta - \sin\phi}{\sin(\theta - \phi)}$$

$$\Rightarrow \frac{2\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}}{2\sin\frac{\theta-\phi}{2}\cos\frac{\theta-\phi}{2}} = \frac{1}{e} \Rightarrow \frac{\cos\frac{\theta-\phi}{2}}{\cos\frac{\theta+\phi}{2}} = e$$

$$\Rightarrow \frac{\cos\frac{\theta-\phi}{2} - \cos\frac{\theta+\phi}{2}}{\cos\frac{\theta-\phi}{2} + \cos\frac{\theta+\phi}{2}} = \frac{e-1}{e+1} \Rightarrow \tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{e-1}{e+1} \qquad \ldots\text{(B)}$$

Similarly $(a\cos\theta, b\sin\theta), (a\cos\phi, b\sin\phi)$ & (–ae, 0) are collinear, hence

$$\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{e+1}{e-1} \qquad \ldots\text{(A)}$$

6. **(AC)**

Slope of required tangent m = 3

$y = mx \pm \sqrt{a^2m^2 - b^2}$

$\Rightarrow y = 3x \pm \sqrt{1.9 - 3}$

$\Rightarrow y = 3x \pm \sqrt{6}$

7. **(CD)**

$y = mx \pm \sqrt{a^2m^2 - b^2}$

Let it pass through (h, k)

$\Rightarrow (k - mh)^2 = a^2m^2 - b^2$

$\Rightarrow m^2(h^2 - a^2) - (2hk)m + (k^2 + b^2) = 0$

Now $m_1m_2 = -1 \Rightarrow k^2 + b^2 = a^2 - b^2$

$\Rightarrow h^2 + k^2 = a^2 - b^2$

Will not have perpendicular tangent if $a^2 - b^2 < 0$ or $a^2 < b^2$

Now $e = \sqrt{1 + \frac{b^2}{a^2}} > \sqrt{2}$

8. (AB)

Let $y = mx + \frac{8}{m}$ be the tangent to $y^2 = 32x$ and

$y = mx \pm \sqrt{\frac{8}{9}}\sqrt{m^2 - 1}$ be that of $\frac{x^2}{\frac{8}{9}} - \frac{y^2}{\frac{8}{9}} = 1$

Comparing $\left(\frac{8}{m}\right)^2 = \frac{8}{9}\left(m^2 - 1\right)$

$\Rightarrow m^4 - m^2 - 72 = 0 \Rightarrow m = \pm 3$

Hence equation of common tangents are

$y = 3x + \frac{8}{3}$ & $y = -3x - \frac{8}{3}$ or $9x - 3y + 8 = 0$ & $9x + 3y + 8 = 0$

9. (ABC)

$xy = 2 \ \Rightarrow y = \frac{2}{x} \qquad \frac{dy}{dx} = \frac{-2}{x^2}$

Equation of Normal: $y - y_i = \frac{x_i^2}{2}\left(x - xi\right)$

$\Rightarrow 8x_i - 4 = 2x_i^3 - x_i^4$

$\Rightarrow x_i^4 - 3x_i^3 + 8x_i - 4 = 0$

Clearly $\sum x_i = 3 \qquad \& \qquad \pi x_i = -4$

Replacing x_i with $\frac{2}{y_1} \Rightarrow \sum y_i = 4 \ \& \prod y_i = -4$

10. (ACD)

11. (BC)

Foci of $\frac{x^2}{16} + \frac{y^2}{25} = 1$ are $\left(0, \pm 3\right)$

Also e of $\frac{x^2}{16} + \frac{y^2}{25} = 1$ is $\frac{3}{5} \Rightarrow \boxed{\frac{3}{5} \times e_{hyp} = 2}$

$\therefore$ e of hyperbola is $\frac{10}{3}$

Since hyperbola passes through foci and has axes along the coordinates axes hence let the hyperbola be $\frac{y^2}{9} - \frac{x^2}{a^2} = 1$

$\because e = \frac{10}{3} \qquad \Rightarrow a^2 = 91 \qquad \ldots$ (B)

$L.R. = \frac{2.a^2}{3} = \frac{2.91}{3} = \frac{182}{3} \qquad \ldots$(C)

12. (AB)

Confocal ellipse and hyperbola are always orthogonal

Clearly in option (A) 31 + 41 = 91 – 19

And in option (B) 71 – 17 = 31 + 23

13. **(D)**

Let $\left(t_1, \frac{1}{t}\right)$ and $\left(t_2, \frac{1}{t_2}\right)$ be the points

Now m = 4 $\Rightarrow \frac{-1}{t_1 t_2} = 4$

Given (h, k) divides the line segment in the ratio 1 : 2

$\Rightarrow (h,k) = \left(\frac{t_2 + 2t_1}{3}, \frac{t_1 + 2t_2}{3t_1 t_2}\right)$

$3h = t_2 + 2t_1$ …(1)

$\frac{-3k}{4} = t_1 + 2t_2$ …(2)

Using $t_2 = -\frac{1}{4t_1}$ we get

$3h = -\frac{1}{4t_1} + 2t_1 \;\&\; -\frac{3k}{4} = t_1 - \frac{1}{2t_1}$

or $2h + k = \frac{1}{2t_1} \;\&\; 8h + k = 4t_1$

$\Rightarrow (2h + k)(8h + k) = 2$

Required locus is $16x^2 + 10xy + k^2 = 2$

14. **(ABC)**

Let e be a root of $x^2 - ax + 2 = 0$, then

$e^2 - ae + 2 = 0$ has both the roots greater than 1.

Now let $P(e) = e^2 - ae + 2$, then

(i) $P(e) > 1 \Rightarrow a < 3$

(ii) $\frac{a}{2} > 1 \Rightarrow a > 2$

(iii) $a^2 - 8 \geq 0 \Rightarrow a \leq -2\sqrt{2}$ or $a \geq \sqrt{2}2$

Hence $2\sqrt{2} < a < 3$

15. **(AD)**

16. **(AD)**

17. **(BD)**

Given hyperbola is $\frac{x^2}{9} - \frac{y^2}{3} = 1$

Angle between Asymptotes: $2\tan^{-1}\frac{b}{a} = 2\tan^{-1}\frac{1}{\sqrt{3}} = 60^0$

$\Rightarrow$ acute angle = 60^0

$e \Rightarrow 3 = 9(e^2 - 1) \Rightarrow e = \frac{2}{\sqrt{3}}$ …(B)

L.R.: $\frac{2.3}{3} = 2$

Asymptotes: $\frac{x}{a} \pm \frac{y}{b} = 0$

Product of 1st from $\left(a\sec\theta, b\tan\theta\right) = \frac{1}{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{9}{4} > 2$ …(D)

18. **(ABCD)**

Solving $x^2 + y^2 = a^2$ and $xy = c^2$

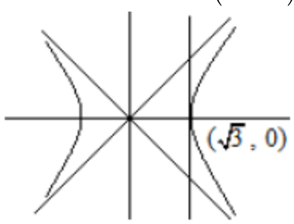
$\Rightarrow x^2 + \frac{c^4}{x^2} = a^2$

$\Rightarrow x^4 - a^2x^2 + c^4 = 0$

$\Rightarrow \sum x_i = 0 \,\&\, \prod x_i = c^4 \,\&\, \sum y_i = 0 \,\&\, \prod y_i = c^4$

19. **(BC)**

Clearly vertex is $\left(\sqrt{3}, 0\right)$



Solving with Asymptotes $x^2 - 3y^2 = 0$

$\Rightarrow \left(\sqrt{3}, 1\right)$ and $\left(\sqrt{3}, -1\right)$

$\therefore$ Triangle is formed by (0, 0), $\left(\sqrt{3}, 1\right), \left(\sqrt{3}, -1\right)$

$\Rightarrow$ Equilateral triangle

Area $= \frac{1}{2}ab\sin\theta = \frac{1}{2}.2.2.\frac{\sqrt{3}}{2} = \sqrt{3}$

20. **(ABC)**

Given $2\sqrt{2} < e_1 + e_2 < 3\sqrt{2} \,\&\, e_1e_2 = 2$

$\Rightarrow e_1^2 - 3\sqrt{2}e_1 + 2 < 0 \,\&\, e_1^2 - 2\sqrt{2}e_1 + 2 > 0$

$\Rightarrow \frac{3\sqrt{2}-\sqrt{10}}{2} < e_1 < \frac{3\sqrt{2}+\sqrt{10}}{2}$

21. **(BC)** **22.** **23.** **(BCD)**

24. **(AD)**

25. **(ACD)**

26. **(AB)** **27.** **(CD)** **28.** **(BCD)** **29.** **(ABCD)** **30.** **(AC)**

COMPREHENSION TYPE

$\frac{a}{5\left(5-b\right)}.\frac{a}{5\left(5-c\right)} = \frac{1}{2}$

$$\frac{5(5-a)(5-b)(5-c)}{5^2(5-b)(5-c)} = \frac{1}{2}$$

$2(5-9) = s \Rightarrow ab + c = 3a$

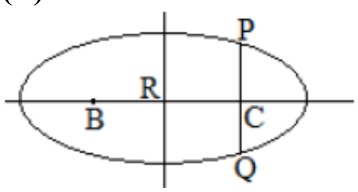
AC + AB = 3BC = 12 (BC = 4)

$\therefore$ locus of A will be ellipse with foci B(2, 4) & C(6, 4) & with length of major axis = 12

$\therefore$ 2ae = 4_1 2a = 12

$\therefore e = \frac{1}{3}$

1. **(C)**



Area of ΔPQR

$= \frac{1}{2}PQ.CR = \frac{1}{2}.\frac{2b^2}{2}.(ae)$

$= \{a^2(1-e^2)\}.\frac{1}{3}$

$= (6^2 - 2^2).\frac{1}{3} = \frac{32}{3}$

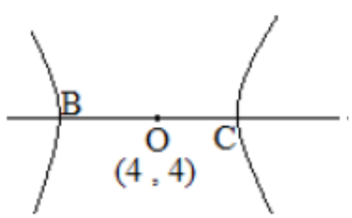
2. **(A)**

ΔPBC is right angled

$\therefore$ circum radius = $= \frac{1}{2}PB = \frac{1}{2}\sqrt{(2ae^2) + \left(\frac{b^2}{9}\right)^2} = \frac{1}{2}.\frac{20}{3} = \frac{16}{3}$

3. **(D)**

2a = BC = 4 (where 2a = length of transverse axis)



$\therefore \frac{(x-4)^2}{4} - \frac{(y-4)^2}{b^2} = 1$

Passing through (O, C)

$\therefore 4 - \frac{4}{b^2} = 1 \quad \therefore b^2 = \frac{4}{3}$

$\therefore 4(e^2 - 1) = \frac{4}{3} \therefore e = \frac{2}{\sqrt{3}}$

4. **(A)**

$$\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2e^2$$

$$G\left[ae^2\sec\theta,0\right]$$

$$g\left[0,\frac{a^2e^2\tan\theta}{b}\right]$$

$$P\left[a\sec\theta,b\tan\theta\right]$$

$$PG^2=\left(ae^2\sec\theta-a\sec\theta\right)^2+b^2\tan^2\theta$$

$$PG^2=a^2\sec^2\theta\left[\left(e^2-1^2\right)\right]+b^2\tan^2\theta$$

$$L^2=a^2\left(1+\tan^2\theta\right)\left[\left(e^2-1\right)^2+b^2\tan\theta\right]$$

$$L^2\min=a^2\left[1\times\left(e^2-1\right)\right]$$

$$=a^2\left(e^2-1\right)=\frac{b^2}{a^2}$$

$$=a^2\left[\left(\frac{b^2}{a^2}\right)^2\right]$$

$$\min=\frac{b^2}{a}$$

5. **(A)**

$$pg^2=a^2\sec^2\theta+\left(\frac{a^2e^2\tan\theta}{b}-b\tan\theta\right)^2$$

$$=a^2\sec^2\theta+\tan^2\theta\frac{\left(a^2e^2-b^2\right)^2}{b^2}$$

$$=a^2\sec^2\theta+\frac{a^4}{b^4}\tan^2\theta$$

6. **(B)**

$$PG^\theta.Pg^0=b^2$$

$\therefore$ G.M. of PG. Pg = b

7. **(A)**

$2x^2 + 3xy - 2y^2 + 5 = 0$

Clearly $2x^2 + 3xy - 2y^2 = 0$ are pair of asymptote

8. **(C)**

Given hyperbola: $x^2 + 6x - 2y^2 + 4x + 2 = 0$

$\Rightarrow$ Pain of Asymptotes: $x^2 + 6xy - 2y^2 + 4x + 2 + \lambda = 0$

$$\Rightarrow \begin{vmatrix} 1 & 3 & 2 \\ 3 & -2 & 0 \\ 2 & 0 & 2+\lambda \end{vmatrix} = 0$$

$$\Rightarrow 2(-2-\lambda) - 3(6+2\lambda) + 2(4) = 0$$

$$\lambda = \frac{-14}{11}$$

9. **(C)**

$2x^2 + 3xy - 2y^2 + 5 = 0$

$\Rightarrow$ Pair of Asymptotes: $2x^2 + 3xy - 2y^2 = 0$

Angle between Asymptotes $= 2\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{a+b}\right)$

$$\Rightarrow 2\tan^{-1}\sqrt{e^2 - 1} = \frac{\pi}{2}$$

$$\Rightarrow e = \sqrt{2}$$

10. **(A)**

Solving $x^2 + y^2 + 2gc + 2fy + k = 0$ and $\left(ct, \frac{c}{t}\right)$

$$\Rightarrow c^2t^2 + \frac{c^2}{t^2} + 2gct + \frac{2fc}{t} + k = 0$$

$$\Rightarrow c^2t^4 + c^2 + 2gct^3 + 2fct + kt^2 = 0 \qquad \ldots(1)$$

$$\sum \frac{1}{t_1} = \frac{\sum t_1t_2t_3}{t_1t_2t_3t_4} = \frac{-2fc}{c^2} = \frac{-2f}{c}$$

11. **(C)**

Form (1)

$$t_1 + t_2 + t_3 + t_4 = \frac{-29}{c} \quad (\because t_1t_2t_3t_3 = 1)$$

$$\Rightarrow t_1 + t_2 + t_3 + \frac{1}{t_1t_2t_3} = \frac{-29}{c}$$

$$\Rightarrow -g = \frac{c}{2}\left(t_1 + t_2 + t_3 + \frac{1}{t_1t_2t_3}\right) \& -f = \frac{c}{2}\left(\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + t_1t_2t_3\right)$$

12. **(B)**

$$\sum t_1 = (\text{from}(1)) = \frac{-29}{c}$$

13. **(D)**

14. **(B)**

15. **(A)**

16. **(B)**

17. **(A)**

18. **(B)**

MATRIX MATCH

1. **A-q; B-s; C-r; D-p**

(A) $e = \sqrt{\frac{a^2 + b^2}{a^2}}$

$e' = \sqrt{\frac{a^2 + b^2}{b^2}}$

$\Rightarrow \frac{1}{e^2} + \frac{1}{(e')^2} = 1$ (A – q)

(B) $e_1 = e_2 = \sqrt{2}$ (B – s)

(C) $\frac{(2y - x - 3)^2}{20} - \frac{9(2x + y - 1)}{80} = 1$

Now $a^2 = 4$ $b^2 = \frac{16}{9}$

$\Rightarrow b^2 = a^2(e^2 - 1)$

$\Rightarrow \boxed{e = \frac{\sqrt{13}}{3}}$ c – r

(D) $\frac{\sqrt{3}x - y}{4\sqrt{3}} = k$ and $\frac{4\sqrt{3}}{\sqrt{3}x + y} = k$

2. **A-r; B-r; C-q,s; D-r**

(A) angle between Asymptotes: $2\tan^{-1}\frac{b}{a} = \frac{\pi}{3}$

$2\tan^{-1}\sqrt{e^2 - 1} = \frac{\pi}{3}$

$\sqrt{e^2 - 1} = \frac{1}{\sqrt{3}}$

$\Rightarrow e^2 = \frac{4}{3}$

We know that $\frac{1}{e^2} + \frac{1}{e'^2} = 1$

$\Rightarrow \frac{3}{4} + \frac{1}{(e')^2} = 1$

$\Rightarrow e' = 2$ A – 2

(B) $x = \frac{4\sqrt{3}m^2 + 4\sqrt{3}m}{2m\sqrt{3}}$

$= 2\frac{(m^2 - 1)}{m}$

$y = \dfrac{(m^2-1)2\sqrt{5}}{m}$

(C) $x + y = k$ touches $x^2 - 2y^2 = 18$

Put $x = k - y$

$\Rightarrow x^2 = k^2 + y^2 - 2ky$

$\Rightarrow -y^2 - 2ky + (k^2 - 18) = 0$

$\Rightarrow D = 0$

$\Rightarrow 4k^2 + 4(k^2 - 18) = 0$

$k = \pm 3$ C – q, s

(D) $\dfrac{x^2}{4a^2} + \dfrac{y^2}{ab^2} = 1$ and $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ are can focal

$\Rightarrow 4a^2e_1^2 = a^2e_2^2$

$\Rightarrow 4a^2 - 4b^2 = a^2 + b^2$

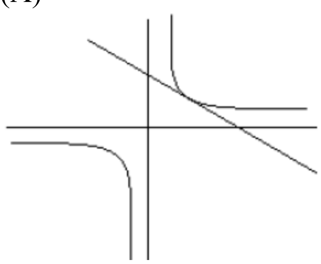
$\Rightarrow 3a^2 = 5b^2$

Now, $e_1^2 = 1 - \dfrac{b^2}{a^2} = 1 - \dfrac{3}{5} = \dfrac{2}{5}$

$e_2^2 = 1 + \dfrac{b^2}{a^2} = 1 + \dfrac{3}{5} = \dfrac{8}{5}$

3. **(A-r), (B-s), (C-r), (D-q)**

(A)



Let P be $\left(2\sqrt{2}t, \dfrac{2\sqrt{2}}{t}\right)$

Equation of tangent : $\dfrac{x}{x_1} + \dfrac{y}{y_1} = 2$

Area $= \dfrac{1}{2}(2x_1)(2y_1) = 2x_1y_1 = 16$

A – 2

(B) $\dfrac{x^2}{5} - \dfrac{y^2}{5\cos^2\theta} = 1$ $e_1 = \sqrt{1+\cos^2\theta}$

$\dfrac{x^2}{25\cos^2\theta} + \dfrac{y^2}{25} = 1$ $e_2 = \sin\theta$

Given $\sqrt{1+\cos^2\theta} = \sqrt{3}\sin\theta$

$\Rightarrow 1 + \cos^2\theta = 3\sin^2\theta$

$\Rightarrow 2 = 4\sin^2\theta$

$\Rightarrow \sin\theta = \dfrac{1}{\sqrt{2}}$

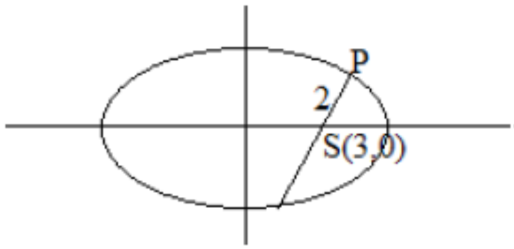
$\Rightarrow \theta = \frac{\pi}{4}$

B – s

(C) $P_1P_2 = b^2$ $\Rightarrow$ Product perpendicular = 16

C – r

(D)



$\text{L.R.} = \frac{2.16}{5}$

$\text{Semi L.R.} = \frac{16}{5}$

We know that PS, semi – L.R. and SQ are in H.P.

$\therefore \frac{16}{5} = \frac{2.PS.SQ}{PS + SQ} = \frac{2.(2)SQ}{2 + SQ}$

$\Rightarrow$ SQ = 8

$\therefore$ PQ = 10

4. **A – p; B – s; C – r; D – s**

5. **A – p; B – q; C – r; D – s**