EXERCISE – 1(B)

1. **(B)**

$\Rightarrow \cos^2 h\theta - \sin^2 h\theta = 1$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

$\Rightarrow e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{16}{9}} = \frac{5}{3}$

2. **(D)**

We know $\cos^2 h\theta - \sin^2 h\theta = 1$

$\Rightarrow \cos h\,\theta = \frac{x+y}{a}\ \&\ \sin h\,\theta = \frac{x-y}{a}$

$\Rightarrow \cos^2 h\theta - \sin^2 h\theta = 1$

$\Rightarrow (x+y)^2 - (x-y)^2 = a^2$

$\Rightarrow xy = \frac{a^2}{4}$

3. **(A)**

Let mid – point is (h, k). Equation of chord is T = Q!. So xh – yk – 4 = h^2 – k^2 – 4. Comparing with x + 2y + 3 = 0

$\Rightarrow \frac{h}{1} = \frac{-k}{2} = \frac{k^2-h^2}{3}$

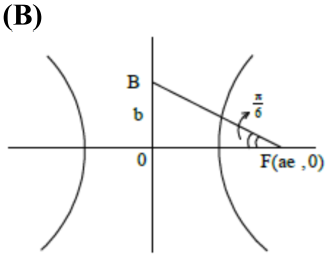
On solving h = 1, k = –2

4. **(A)**

Let P is (h, k) equation of chord with P as middle points is T = S'. Slope obtained is $\frac{3h+2}{2k+3}$ which is equal to 2.

So, 3h – 4k = 4

5. **(B)**



$\Rightarrow \frac{\pi}{6} = \frac{b}{ae}\ \&\ b^2 = a^2\left(e^2-1\right)$

On solving we get $e = \sqrt{\frac{3}{2}}$

6. **(C)**

Let midpoint of a chord be P(h, k) then by 'T = S_1' its equal will be

$\frac{hx}{a^2} - \frac{ky}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$

As it passes through (α, β) hence

$$\frac{\alpha h}{a^2} - \frac{\beta k}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$$

Required locus is

$$\frac{\alpha x}{a^2} - \frac{\beta y}{b^2} = \frac{x^2}{a^2} - \frac{y^2}{b^2} \text{ or } \frac{\left(x - \frac{\alpha}{2}\right)^2}{a^2} - \frac{\left(y - \frac{\beta}{2}\right)^2}{b^2} = \frac{\alpha^2}{4a^2} - \frac{\beta^2}{4b^2}$$

Which is hyperbola having center at $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$.

7. **(A)**

Area of triangle formed by any tangent and the asymptotes is (ab)

Now $ab = a^2 \tan\lambda \Rightarrow \tan\lambda = \frac{b}{a}$

Hence $e = \sqrt{1 + \frac{b^2}{a^2}} \Rightarrow e = \sqrt{1 + \tan^2\lambda}$ or $e = |\sec\lambda|$

8. **(D)**

Locus of feet of perpendicular from foci on any tangent is the auxiliary circle.

Hence required locus is $x^2 + y^2 = \frac{1}{16}$

9. **(D)**

Let A (1, –1) & B (2, 1) be two fixed points and P (x, y) be a moving point, then

$|Z - 1 + i| - |Z - 2 - i| = 3 \Rightarrow PA - PB = 3$

Hence locus will be no real curve as $AB = \sqrt{5} < 3$

10. **(B)**

Eccentricity of $\frac{x^2}{5} - \frac{y^2}{5\cos^2\alpha} = 1, e_1 = \sqrt{1 + \cos^2\alpha}$

Eccentricity of $\frac{x^2}{25\cos^2\alpha} - \frac{y^2}{25} = 1, e_2 = \sqrt{1 - \cos^2\alpha}$

Given $e_1 = \sqrt{3}e_2$ hence

$1 + \cos^2\alpha = 3\sin^2\alpha \text{ or } \sin^2\alpha = \frac{1}{2}$

A value of α is $\frac{\pi}{4}$

11. **(B)**

Equation of tangents with slope m to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $y = mx \pm \sqrt{a^2m^2 - b^2}$

Slope of tangent perpendicular to y = x is –1

Hence equation of tangents with slope -1 to $\frac{x^2}{18} - \frac{y^2}{9} = 1$ are $x + y = \pm 3$.

12. **(B)**

For the hyperbola $\frac{x^2}{3}-y^2=1,\left(\sqrt{3},0\right)$ is one the vertices, hence tangent at this point will be equally inclined to the asymptotes.

Also the asymptotes are $y=\frac{1}{\sqrt{3}}x\ \&\ y=-\frac{1}{\sqrt{3}}x$ hence angle between the asymptotes is 60^0.

The tangent and the asymptotes must from an equilateral triangle.

13. **(D)**

Any tangent to $xy = c^2$ is $x + t^2y = 2ct$

Now foot of perpendicular on this tangent from (0, 0) will be given by

$\frac{x-0}{1}=\frac{y-0}{t^2}=-\frac{0+0-2ct}{1+t^2}$ or $x=\frac{2ct}{1+t^4}\ \&\ y=\frac{2ct^3}{1+t^4}$

Eliminating t between x & y gives the required locus as $\left(x^2+y^2\right)^2=4c^2xy$

14. **(C)**

Standard result in geometrical properties.

15. **(B)**

Equation of asymptotes $bx - ay = 0$ & $x + ay = 0$

Any point P on hyperbola (a sec t, b tan t).

Product of perpendicular from P on asymptotes $\left|\frac{ab\left(\sec t-\tan t\right)}{\sqrt{a^2+b^2}}\right|\times\left|\frac{ab\left(\sec t+\tan t\right)}{\sqrt{a^2+b^2}}\right|$

i.e., $\frac{a^2b^2}{a^2+b^2}=6$, but given $e^2=\frac{a^2+b^2}{a^2}=3$, hence $b^2 = 18$ & $a^2 = 9$.

16. **(B)**

For a rectangular hyperbola, eccentricity is $\sqrt{2}$ & independent of 'c'.

Hence $e_1+e_2=\sqrt{2}+\sqrt{2}$ i.e., $2\sqrt{2}$

17. **(A)**

$\sqrt{3}x-y-4\sqrt{3}t=0\Rightarrow t=\frac{\sqrt{3}x-y}{4\sqrt{3}}$

Now $\sqrt{3}\,tx+ty-4\sqrt{3}=0\Rightarrow\left(\sqrt{3}x-y\right)\left(\sqrt{3}x+y\right)=48$ or $\frac{x^2}{16}-\frac{y^2}{48}=1$

Hence $e^2=\frac{a^2+b^2}{a^2}\Rightarrow e^2=\frac{16+48}{16}$ i.e. $e = 2$.

18. **(C)**

Any tangent of $y^2 = 8x$: $y=mx+\frac{2}{m}$

If this is a tangent to $xy=-1$ as well then $x\left(mx+\frac{2}{m}\right)=-1$ must have real & equal roots.

Now discriminant of $m^2x^2+2x+m=0, 4-4m^3=0\Rightarrow m=1$.

19. **(C)**

Tangent to $xy = c^2$ at P(h, k) : $kx + hy = 2c^2$.

x – intercept, $x_1 = \frac{2c^2}{k}$, y – intercept, $y_1 = \frac{2c^2}{h}$.

Normal to $xy = c^2$ at P(h, k) : $hx - ky = h^2 - k^2$.

x – intercept, $x_2 = \frac{h^2 - k^2}{h}$, y – intercept, $y_2 = \frac{k^2 - h^2}{k}$.

Clearly $\frac{x_2}{y_1} + \frac{y_2}{x_1} = 0$ or $x_1x_2 + y_1y_2 = 0$

20. **(B)**

$xy = hx + ky \Rightarrow (x - k)(y - h) = hk$

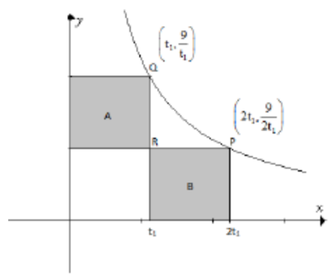
Hence the center is (k, h).

21. **(C)**

22. **(D)**

23. **(A)**

Coordinates of R : $\left(t_1, \frac{9}{2t_1}\right)$



Area $A = \left(\frac{9}{2t_1} - \frac{9}{t_1}\right) \times t_1$ i.e. $\frac{9}{2}$ &

Area $B = \frac{9}{2t_1}(2t_1 - t_1)$ i.e. $\frac{9}{2}$

Hence A = B

24. **(A)**

Polar of a pole is chord of contact from the given point. Let pole is (h, k) equation of polar is T = 0

$\Rightarrow 3hx - 5ky - 15 = 0$

Comparing with 2x +5y – 5 = 0 we get,

$\Rightarrow \frac{3h}{2} = \frac{-5k}{5} = \frac{-15}{-5}$

$\Rightarrow$ h = 2, k = –3

25. **(D)**

Let pole of 3x – y + 1 = 0 is (h, k) on comparing it with (5h) x – (6k) y – 15 = 0.

We get, $\Rightarrow \frac{5h}{3} = \frac{6k}{1} = -15$

$\Rightarrow h = -9; k = \frac{-5}{2}$

This $\left(-9, \frac{-5}{2}\right)$ satisfies 2x – ky + 3 = 0

So, $2(-9) - k\left(\frac{-5}{2}\right) + 3 = 0$

$\Rightarrow$ k = 6

26. **(A)**

Let mid points is (h, k). Equation of chord is T = Q !.

So $xh - yk - 4 = h^2 - k^2 - 4$.
Comparing with $x + 2y + 3 = 0$
$\Rightarrow \frac{h}{1} = \frac{-k}{2} = \frac{k^2 - h^2}{3}$
On solving $h = 1, k = -2$

27. **(B)**
Centre of hyperbola is
$\Rightarrow \frac{\delta s}{\delta y} = 0 \Rightarrow 6x - 5y + 17 = 0$
$\Rightarrow \frac{\delta s}{\delta y} = 0 \Rightarrow -5x - 4y + 1 = 0$
On solving : we get centre $\left(\frac{-9}{7}, \frac{13}{7}\right)$
Equation of asymptotes(s) differ from that of hyperbola by a constant. Let the Asymptotes(s) are $3x^2 - 5xy - 2y^2 + 17x + y + \lambda = 0$. It satisfies $\left(\frac{-9}{7}, \frac{13}{7}\right)$
On solving we get $\lambda = 10$

28. **(D)**
Let other is $2x - y + \lambda = 0$ (Equation of hyperbola & asymptote differ by a constant)
So, $(2x - y + \lambda)(x + 2y - 3) = 2x^2 + 3xy - 2y^2 - 7x + y + \lambda$
Compare co – efficient of $x \Rightarrow \lambda - 6 = -7 \Rightarrow \lambda = -1$
So equation is $2x - y - 1 = 0$

29. **(B)**
Let other is $x - y + \lambda = 0$
So, $(x + y + 1)(x - y + \lambda) = x^2 - y^2 + x - y + \lambda$
Comparing coefficient of $y \Rightarrow \lambda - 1 = -1 \Rightarrow \lambda = 0$
So equation is $x - y = 0$

30. **(D)**

31. **(D)**
$\Rightarrow y = mx + \frac{2}{m}; y = mx + \sqrt{m^2 - 3}$
Comparing $\frac{4}{m^2} = m^2 - 3 \Rightarrow m = \pm 2$
So tangents are $2x - y + 1 = 0$ & $2x + y + 1 = 0$

32. **(B)**
A point (a, b) can be taken on $x^2 - y^2 = a^2 - b^2$. A tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with slope 'm' is $y = mx \pm \sqrt{a^2m^2 + b^2}$. If it passes through (a, b) then we have $b = am \pm \sqrt{a^2m^2 + b^2}$
$\Rightarrow 2amb = 0; m = 0$

33. **(A)**
Any tangent to $x^2 + y^2 = a^2$ is $x\cos\theta + y\sin\theta = a$

Let P is its pole w.r.t. $x^2 - y^2 = a^2$
So comparing $xh - yk = a^2$
$\Rightarrow h = a\cos\theta, k = -a\sin\theta \Rightarrow x^2 + y^2 = a^2$

34. **(B)**
Tangent to $4x^2 - 3y^2 = a^2$ is $(2\sec\theta)x - y(\sqrt{3}\tan\theta) = a$
Let P(h, k) is its pole w.r.t. $y^2 = 4ax$
So polar is yk = 2x (x + h)
On comparing we get $h = \frac{-a\cos\theta}{2}$ and $k = a\sqrt{3}\sin\theta$
So we have $2h^2 + k^2 = 3a^2$

35. **(B)**
Given hyperbola are $\frac{x^2}{9} - \frac{y^2}{16} = 1$ … (i) and $\frac{y^2}{9} - \frac{x^2}{16} = 1$ …(ii)
Any tangent to (i) having slope m is $y = mx \pm \sqrt{9m^2 - 16}$ … (iii)
Putting in (ii), we ger $16\left[mx \pm \sqrt{9m^2 - 16}\right]^2 - 9x^2 = 144$
$(16m^2 - 9)x^2 \pm 32m\left(\sqrt{9m^2 - 16}\right)x + 144m^2 - 256 - 144 = 0$
$\Rightarrow (16m^2 - 9)x^2 \pm 32m\left(\sqrt{9m^2 - 16}\right)x + (144m^2 - 400) = 0$ …(iv)
If (iii) is a tangent to (ii), then the roots of (iv) are real and equal.
$\therefore$ Discriminant = 0; $32 \times 32m^2(9m^2 - 16) = 0(16m^2 - 9)(144m^2 - 400) = 64(16m^2 - 9)(9m^2 - 25)$
$16m^2(9m^2 - 16) = (16m^2 - 9)(9m^2 - 25) \Rightarrow 144m^4 - 256m^2 = 144m^4 - 481m^2 + 225$
$\Rightarrow 225m^2 = 225 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$

36. **(A)**
Let the point of intersection of tangent be $P(x_1, y_1)$.
Then the equation of pair of tangents from $P(x_1, y_1)$ to the given hyperbola is
$(4x^2 - 9y^2 - 36)(4x_1^2 - 9y_1^2 - 36) = [4x_1x - 9y_1y - 36]^2$ …(i)
From $SS_1 = T^2$ or $x^2(y_1^2 + 4) + 2x_1y_1xy + y^2(x_1^2 - 9) + ... = 0$ …(ii)
Since angle between the tangents is $\pi/4$
$$\therefore \tan(\pi/4) = \frac{2\sqrt{\left[x_1^2y_1^2 - (y_1^2 + 4)(x_1^2 - 9)\right]}}{y_1^2 + 4 + x_1^2 - 9}$$
Hence locus of $P(x_1, y_1)$ is $(x^2 + y^2 - 5)^2 = 4(9y^2 - 4x^2 + 36)$

37. **(A)**
The equation of normal at $(a\sec\phi, b\tan\phi)$ to the given hyperbola is $ax\cos\phi + by\cot\phi = (a^2 + b^2)$
This meet the transverse axis i.e., x – axis at G.
So the co – ordinates of the vertices A and A' are A(a, 0) and $A'(-a, 0)$ respectively.

$$\therefore AG.A'G = \left(-a + \left(\frac{a^2+b^2}{a}\right)\sec\phi\right)\left(a + \left(\frac{a^2+b^2}{a}\right)\sec\phi\right)$$

$$= \left(\frac{a^2+b^2}{a}\right)\sec^2\phi - a^2 = \left(ae^2\right)^2\sec^2\phi - a^2 = a^2\left(e^4\sec^2\phi - 1\right)$$

38. **(A)**

Let (x_1, y_1) be the required point.

Then the equation of the chord of contact of tangents drawn from (x_1,y_1) to the given hyperbola is

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1 \qquad \ldots(i)$$

The given line is $lx + my + n = 0$ $\qquad \ldots(ii)$

Equation (i) and (ii) represent the same line

$$\therefore \frac{x_1}{a^2 l} = -\frac{y_2}{b^2 m} = \frac{1}{-h} \Rightarrow x_1 = \frac{-a^2 l}{n}, y_1 = \frac{b^2 m}{n};$$

Hence the required point is $\left(-\frac{a^2 l}{n}, \frac{b^2 m}{n}\right)$

39. **(A)**

The given hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$ $\qquad \ldots(i)$

Any tangent to (i) is $y = mx + \sqrt{16m^2 - 9}$ $\qquad \ldots(ii)$

Let (x_1, y_1) be the midpoint of the chord of the circle $x^2 + y^2 = 16$

Then equation of the chord is $T = S_1$ i.e., $xx_1 + yy_1 - \left(x_1^2 + y_1^2\right) = 0$ $\qquad \ldots(iii)$

Since (ii) and (iii) represents the same line.

$$\therefore \frac{m}{x_1} = \frac{-1}{y_1} = \frac{\sqrt{16m^2 - 9}}{-\left(x_1^2 + y_1^2\right)}$$

$$\Rightarrow m = -\frac{x_1}{y_1} \text{ and } \left(x_1^2 + y_1^2\right)^2 = y_1^2\left(16m^2 - 9\right)$$

$$\Rightarrow \left(x_1^2 + y_1^2\right)^2 = 16.\frac{x_1^2}{y_1^2}y_1^2 - 9y_1^2 = 16x_1^2 - 9y_1^2$$

$$\therefore \text{Locus of } \left(x_1, y_1\right) \text{ is } \left(x^2 + y^2\right) = 16x^2 - 9y^2$$

40. **(A)**

Let (x_1, y_1) be the given point.

Its polar w.r.t. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

i.e., $y = \frac{b^2}{y_1}\left(1 - \frac{xx_1}{a^2}\right) = -\frac{b^2 x_1}{a^2 y_1}x + \frac{b^2}{y_1}$

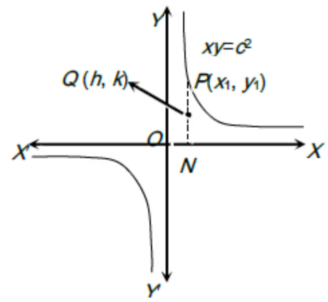
This touches $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $\left(\frac{b^2}{y_1}\right) = a^2.\left(\frac{b^2 x_1}{a^2 y_1}\right) - b^2$

$$\Rightarrow \frac{b^4}{y_1^2} = \frac{a^2 b^4 x_1^2}{a^4 y_1^2} - b^2 \Rightarrow \frac{b^2}{y_1^2} = \frac{b^2 x_1^2}{a^2 y_1^2} - 1 \Rightarrow \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

$\therefore$ Locus of (x_1, y_1) is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
Which is the same hyperbola.

41. **(B)**
Coordinates of P and D are $(a\sec\phi, b\tan\phi)$ and $(a\tan\phi, b\sec\phi)$ respectively.
Then, $(CP)^2-(CD)^2=a^2\sec^2\phi+b^2\tan^2\phi-a^2\tan^2\phi-b^2\sec^2\phi$
$=a^2(\sec^2\phi-\tan^2\phi)-b^2(\sec^2\phi-\tan^2\phi)$
$=a^2(1)-b^2(1)=a^2-b^2$

42. **(D)**
Let $xy = c^2$ be the rectangular hyperbola, and let $P(x_1, y_1)$ be a point on it.
Let Q(h, k) be the mid – point of PN. Then the coordinates of Q are $\left(x_1, \frac{y_1}{2}\right)$



$\therefore x_1 = h$ and $\frac{y_1}{2}=k \Rightarrow x_1 = h$ and $y_1 = 2k$
But (x_1, y_1) lies on $xy = c^2$.
$\therefore h.(2k)=c^2 \Rightarrow hk \Rightarrow c^2/2$
Hence, the locus of (h, k) is $xy=c^2/2$, which is hyperbola

43. **(C)**
Let the hyperbola be $xy = c^2$.
Tangent at any point t is $x + yt^2 - 2ct = 0$
Putting y = 0 and then x = 0 intercept on the axes are $a_1 = 2ct$ and $b_1=\frac{2c}{t}$
Normal is $xt^3 - yt - ct^4 + c = 0$
Intercepts as above are $a_2=\frac{c(t^4-1)}{t^3}, b^2=\frac{-c(t^4-1)}{t}$
$\therefore a_1a_2+b_1b_2=2ct\times\frac{c(t^4-1)}{t^3}+\frac{2c}{t}\times\frac{-c(t^4-1)}{t}=\frac{2c^2}{t^2}(t^4-1)-\frac{2c^2}{t^2}(t^4-1)=0$;
$\therefore a_1a_2+b_1b_2=0$

44. **(B)**
Let t_1, t_2, t_3, t_4 be the parameter of the points P, Q, R and S respectively.
Then the coordinates of P, Q, R and S are
$\left(ct_1, \frac{c}{t_1}\right), \left(ct_2, \frac{c}{t_2}\right), \left(ct_3, \frac{c}{t_3}\right)$ and $\left(ct_4, \frac{c}{t_4}\right)$ respectively

Now, $PQ \perp RS \Rightarrow \frac{\frac{c}{t_2}-\frac{c}{t_1}}{ct_2-ct_1}\times\frac{\frac{c}{t_4}-\frac{c}{t_3}}{ct_4-ct_3}=-1$

$\Rightarrow -\frac{1}{t_1t_2}\times-\frac{1}{t_3t_4}=-1 \Rightarrow t_1t_2t_3t_4=-1$ …(i)
$\therefore$ Product of the slopes of CP, CQ, CR and CS

$$\frac{1}{t_1^2}\times\frac{1}{t_2^2}\times\frac{1}{t_3^2}\times\frac{1}{t_4^2}=\frac{1}{t_1^2t_2^2t_3^2t_4^2}=1 \qquad \text{[Using (i)]}$$

45. **(B)**

Let the equation of circle be $x^2+y^2=a^2$ … (i)

Parametric equation of rectangular hyperbola is $x=ct, y=\frac{c}{t}$

Put the values of x and y in equation (i) we get $c^2t^2+\frac{c^2}{t^2}=1$