# LIMITS
## EXERCISE 1(C)

**1.** $\lim_{x\to 1}\dfrac{x^{\frac{1}{13}}-x^{\frac{1}{7}}}{x^{\frac{1}{5}}-x^{\frac{1}{3}}}$ $\left(\dfrac{0}{0}\text{form}\right)$

Apply L'Hospital rule

$$\lim_{x\to 1}\frac{\frac{1}{13}x^{-1+\frac{1}{13}}-\frac{1}{7}x^{-1+\frac{1}{7}}}{\frac{1}{5}x^{\frac{1}{5}-1}-\frac{1}{3}x^{\frac{1}{3}-1}}$$

$$\lim_{x\to 1}\frac{\frac{1}{13}-\frac{1}{7}}{\frac{1}{5}-\frac{1}{3}}=\frac{45}{91}$$ **Ans.**

**2.** $\lim_{x\to 1}\dfrac{\sum_{k=1}^{100}x^k-100}{x-1}$

$\Rightarrow \lim_{x\to 1}\dfrac{x+1}{x-1}+\lim_{x\to 1}\dfrac{x^2-1}{x-1}+\lim_{x\to 1}\dfrac{x^3-1}{x-1}.....\lim_{x\to 1}\dfrac{x^{100}-1}{x-1}$

Applying L'Hospital rule

$\Rightarrow \lim_{x\to 1}1+\lim_{x\to 1}2x+\lim_{x\to 1}3x^2+\lim_{x\to 1}4x^3+.........\lim_{x\to 1}100x^9$

$= 1 + 2 + 3 + 4...........100$

$= \dfrac{100(100+1)}{2} = 50\times 101\left(1+2+3...........n=\dfrac{n(n+1)}{2}\right)$

$= 5050$

**3.** $\lim_{x\to 1}\left(\dfrac{p}{1-x^p}-\dfrac{q}{1-x^2}\right)$ $(\infty-\infty\text{ form})$

$$\lim_{x\to 1}\left(\frac{p(1-x^q)-q(1-x^p)}{(1-x^p)(1-x^q)}\right)$$

$$\lim_{x\to 1}\left(\frac{p-px^2-q+qx^p}{1-x^q-x^p+x^{p+q}}\right)$$ $\left(\dfrac{0}{0}\text{form}\right)$

Apply L'Hopital rule

$$\lim_{x\to 1}\left(\frac{-pqx^{q-1}+qpx^{p-1}}{-qx^{q-1}-px^{p-1}+(p+q)x^{(p+q-1)}}\right) \qquad \left(\frac{0}{0}\text{form}\right)$$

$$\lim_{x\to 1}\left(\frac{-pq(q-1)x^{q-2}+qp(p-1)x^{p-2}}{-q(q-1)x^{q-2}-p(p-1)x^{p-2}+(p+q)(p+q-1)x^{(p+q-2)}}\right)$$

$$\Rightarrow \frac{-pq^2+pq+p^2q-pq}{-q^2+q-p^2+p+p^2+pq-p+qp+q^2-q}$$

$$\Rightarrow \frac{pq(p-q)}{2pq}=\frac{p-q}{2}$$

**4.** $$\lim_{x\to 3\pi/4}\frac{1+(\tan x)^{\frac{1}{3}}}{1-2\cos^2 x}$$

$$\lim_{x\to 3\pi/4}\frac{1+(\tan x)^{\frac{1}{3}}}{-\cos 2x} \qquad \left(\frac{0}{0}\text{form}\right)$$

Applying L'Hospital rule

$$\lim_{x\to\frac{3\pi}{4}}\frac{\frac{1}{3}(\tan x)^{\frac{-2}{3}}\times\sec^2 x}{+2\sin 2x}$$

$$\Rightarrow \frac{-1}{6}\left(-\sec\frac{\pi}{4}\right)^2$$

$$\frac{-1}{6}\times 2=-\frac{1}{3}$$

**5.** $$\lim_{x\to 0}\frac{\ell n(1+\sin^2 x)}{\tan[\ell n^2(1+x)]}$$

$$\Rightarrow \lim_{x\to 0}\ell n(1+\sin^2 x)\frac{1}{\tan[\ell n^2(1+x)]}$$

$$\Rightarrow \lim_{x\to 0}\frac{(\sin x/x)^2}{\frac{\tan(\ell n^2(1+x))}{\ell n^2(1+x)}}\frac{1}{\left(\frac{1}{x^2}\right)\ell n^2(1+x)}$$

$$\Rightarrow \lim_{x\to 0} \frac{1}{\frac{\ell n(1+x)}{x}\ \frac{\ell n(1+x)}{x}} = 1$$

**6.** $$\lim_{x\to\infty} \frac{2x^{\frac{1}{2}} + 3x^{\frac{1}{3}} + 5x^{\frac{1}{5}}}{(3x-2)^{\frac{1}{2}} + (3x-3)^{\frac{1}{3}}}$$

divide numerator & denominator by $x^{1/2}$

$$\Rightarrow \lim_{x\to\infty} \frac{2 + \frac{3}{x^{\frac{1}{6}}} + \frac{5}{x^{\frac{3}{10}}}}{\left(3-\frac{2}{x}\right)^{\frac{1}{2}} + \frac{1}{x^6}\left(2-\frac{3}{x}\right)^{\frac{1}{3}}} \Rightarrow \frac{2+0+0}{\sqrt{3}+0} \Rightarrow \frac{2}{\sqrt{3}}$$

**7.** $$\lim_{x\to 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x}$$

$$\lim_{x\to 0} \frac{\frac{1}{\cos 4x} - \frac{1}{\cos 2x}}{\frac{1}{\cos 3x} - \frac{1}{\cos x}}$$

$$\lim_{x\to 0} \frac{\cos 2x - \cos 4x}{\cos x - \cos 3x} \times \frac{\cos x.\cos 3x}{\cos 4x.\cos 2x}$$

$$\lim_{x\to 0} \frac{+2\sin 3x.\sin x}{2\sin 2x.\sin x}$$

$$\lim_{x\to 0} \frac{\sin 3x}{\sin 2x} \times \frac{2x}{3x} \times \frac{3x}{2x}$$

$$\lim_{x\to 0} \frac{3x}{2x} = \frac{3}{2}$$

**8.** Let first term of on infinite G.P. is a & common ratio of infinite G.P. is r

given $a = \lim_{x\to 0} \frac{\tan x - \sin x}{\sin^3 x} \left(\frac{0}{0}\text{form}\right)$

Apply L'Hospital rule

$$\lim_{x\to 0} \frac{\sec^2 x - \cos x}{3\sin^2 x \cos x}$$

$$\lim_{x\to 0} \frac{1-\cos^3 x}{3\sin^2 x \cos 3x}$$

$$\lim_{x\to 0} \frac{1-\cos 3x}{3\sin^2 x \cos 2x} \times \lim_{x\to 0} \frac{1}{\cos x}$$

$$\lim_{x\to 0}\frac{4}{4}\times\frac{1-\cos^3 x}{3\sin^2 x\cos^2 x}\times 1$$

$$\lim_{x\to 0}\frac{4(1-\cos 3x)}{3(\sin 2x)^2} \qquad \left(\frac{0}{0}\text{form}\right)$$

Apply L'Hospital rule

$$\lim_{x\to 0}\frac{4}{3}\times\frac{(+3\cos 2x\sin x)}{2(\sin 2x)\times 2\cos 2x}$$

$$a=\lim_{x\to 0}\frac{\cos^2 x\sin x}{(2\sin x\cos x)\cos 2x}=\frac{1}{2}$$

$$r=\lim_{x\to 1}\frac{1-\sqrt{x}}{(\cos^{-1} x)^2}$$

$$\lim_{x\to 1}\frac{1\frac{1}{2\sqrt{x}}}{2(\cos^{-1} x)\times-\frac{1}{\sqrt{1-x^2}}}$$

$$\lim_{x\to 1}\frac{1\times\sqrt{1-x^2}}{4\sqrt{x}(\cos^{-1} x)}$$

$$\lim_{x\to 1}\frac{\sqrt{1-x^2}}{4}\ \cos^{-1}x \qquad \text{put } x=\cos t$$

$$r=\lim_{t\to 1}\frac{\sin t}{4t}=\frac{1}{4}$$

so sum of infinite G.P. is $\int\infty=\frac{a}{1-r}$

$$=\frac{\frac{1}{2}}{1-\frac{1}{4}}=\frac{1}{2}\times\frac{4}{3}=\frac{2}{3}$$

**9.** $\lim_{x\to\infty}(x-\ell n(\cos nx))$ $\qquad \cosh x=\frac{e^x+e^{-x}}{2}$

$$\lim_{x\to\infty}\left(x-\ell n\left(\frac{e^x+e^{-x}}{2}\right)\right)$$

$$\lim_{x\to\infty}\left(x-\ell n\left(\frac{e^{2x}+1}{2e^x}\right)\right)$$

$$\lim_{x\to\infty}\left[x-[\ell n(e^{2x}+1)-\ell n 2e^{x}]\right]$$

$$\lim_{x\to\infty}[x-\ell n(e^{2x}+1)-\ell n 2-\ell n e^{x}]$$

$$\lim_{x\to\infty}[x-\ell n(e^{2x}+1)-\ell n 2-x]$$

$$\lim_{x\to\infty}[-\ell n^{2}-\ell n(e^{2x}+1)]$$

$$-\ell n 2-\lim_{x\to\infty}\ell n\frac{(e^{2x}+1)}{e^{2x}}.e^{2x}$$

$$-\ell n 2-\lim_{x\to\infty}.\ell n\left(\frac{1+\frac{1}{e^{2x}}}{\frac{1}{e^{2x}}}\right)$$

**10.** $$\operatorname{Lim}_{x\to 0}\frac{8}{x^{8}}\left[\left(1-\cos\frac{x^{2}}{2}\right)-\cos\frac{x^{2}}{4}\left(1-\cos\frac{x^{2}}{2}\right)\right]$$

$$\operatorname{Lim}_{x\to 0}=\frac{8}{x^{8}}\left[\left(1-\cos\frac{x^{2}}{2}\right)\left(1-\cos\frac{x^{2}}{4}\right)\right]$$

$$\operatorname{Lim}_{x\to 0}=\frac{8}{x^{8}}\left[1-\left(2\cos^{2}\frac{x^{2}}{4}-1\right)\left(1-\cos\frac{x^{2}}{4}\right)\right]$$

$$\operatorname{Lim}_{x\to 0}=\frac{8}{x^{8}}\left[2-2\cos^{2}\frac{x^{2}}{4}\left(1-\cos\frac{x^{2}}{4}\right)\right]$$

$$\operatorname{Lim}_{x\to 0}=\frac{8\times 2}{x^{8}}\left[\left(1-\cos^{2}\frac{x^{2}}{4}\right)\left(1-\cos\frac{x^{2}}{4}\right)\right]$$

$$\operatorname{Lim}_{x\to 0}=\frac{16}{x^{8}}\left[\sin^{2}\frac{x^{2}}{4}\quad 2\sin^{2}\frac{x^{2}}{8}\right]$$

$$\lim_{x\to 0} = \frac{32}{x^8}\left[\sin^2\frac{x^2}{4}\sin^2\frac{x^2}{8}\right]$$

$$\lim_{x\to 0} = \frac{1}{32}\left(\frac{\sin\frac{x^2}{4}}{\left(\frac{x^2}{4}\right)}\right)\left(\frac{\sin\frac{x^2}{8}}{\left(\frac{x^2}{8}\right)}\right)^2 = \frac{1}{32}$$

**11.** $\lim_{\theta\to\frac{\pi}{4}} \frac{\sqrt{2}-\cos\theta-\sin\theta}{(4\theta-\pi)^2}$ $\left(\frac{0}{0}\text{form}\right)$

Apply L'Hospital rule

$\lim_{\theta\to\frac{\pi}{4}} \frac{\sin\theta-\cos\theta}{2(4\theta-\pi).4}$ $\left(\frac{0}{0}\text{form}\right)$

Again apply L'Hospital rule

$$\lim_{\theta\to\frac{\pi}{4}} \frac{\cos\theta+\sin\theta}{8.4} = \frac{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{32} = \frac{\sqrt{2}}{32} = \frac{1}{16\sqrt{2}}$$

**12.** $\lim_{\theta\to\frac{\pi}{2}} \frac{2^{-\cos x}-1}{x\left(x-\frac{\pi}{2}\right)}$

$\lim_{\theta\to\frac{\pi}{2}} \frac{1-2^{\cos x}}{2^{\cos x}\left(x^2-\frac{\pi}{2}x\right)}$ $\left(\frac{0}{0}\text{ form}\right)$

$\lim_{x\to\frac{\pi}{2}} \frac{1-2^{\cos x}}{x^2-\frac{\pi}{2x}}$ $\left(\frac{0}{0}\text{form}\right)$

again apply L'Hospital rule

$$\lim_{x\to\frac{\pi}{2}} \frac{-2^{\cos x}\log 2}{2x-\frac{\pi}{2}} \Rightarrow \frac{-2\log 2}{\pi}$$

**13.** $\lim_{x\to 1}\left[\ell n\left(\frac{1+x}{2}\right)\frac{1}{\sin(x-1)}\right].3.\left[\frac{4^{x-1}-x}{(7+x)^{1/3}-(1+3x)^{1/2}}\right]$

here $(1)^{\infty}$

$$=\frac{x-1}{2\sin(x-1)}.3\left[\frac{\ell n4\times(4^{x-1})-1}{\frac{1}{3}(7+x)^{-2/3}-\frac{3}{2}(1+3x)^{-1/2}}\right]\Rightarrow\frac{1}{2}.3\left[\frac{\ell n4-1}{\frac{1}{3}.\frac{1}{4}-\frac{3}{2}.\frac{1}{2}}\right]$$

$$\Rightarrow\frac{1}{2}.3.\left[\frac{\ell n4-1}{\frac{1-9}{12}}\right]\Rightarrow\boxed{\frac{-9}{4}\ell n\frac{4}{e}}$$

**14.** If $\ell=\lim_{n\to\infty}\sum_{r=2}^{n}\left((r+1)\sin\frac{\pi}{r+1}-r\sin\frac{\pi}{r}\right)$

$$\ell=\lim_{n\to\infty}\left(3\sin\frac{\pi}{3}-2\sin\frac{\pi}{2}+4\sin\frac{\pi}{4}-3\sin\frac{\pi}{3}+5\frac{\sin\pi}{5}-4\frac{\sin\pi}{4}+n\sin\frac{\pi}{4}-(n-1)\frac{\sin\pi}{n-1}\right.$$
$$\left.+(n+1)\frac{\sin\pi}{n+1}-n\frac{\sin\pi}{n}\right)$$

$$\ell=\lim_{n\to\infty}\left((n+1)\frac{\sin\pi}{n+1}-2\sin\frac{\pi}{2}\right)$$

$$\ell=\lim_{n\to\infty}\left((n+1)\frac{\frac{\sin\pi}{n+1}}{\frac{\pi}{n+1}}\times\frac{\pi}{n+1}-2\right)$$

$$\ell=\lim_{n\to\infty}(\pi-2)=3.14-2$$

$$=1.14$$

so $\{\ell\}=\ell-[\ell]$

$=1.14-1=.14$ or $\pi-3$

**15.** $\lim_{x\to-\infty}\frac{(3x^4+2x^2)\left(\sin\frac{1}{x}\right)\frac{1}{x}}{(|x|^3+|x|^2+|x|+1)\frac{1}{2}}+\frac{|x|^3+5}{|x|^3+|x|^2+|x|+1}$

$$\lim_{x\to-\infty} \frac{3x^3+2x}{-(x)^3-(x)^2-(x)+1}+\frac{-(x)^3+5}{-(x)^3+(x)^2-(x)+1}$$

$$\lim_{x\to-\infty} \frac{3+\frac{2}{x^2}}{-1-\frac{1}{x}-\frac{1}{x^2}-\frac{1}{x^3}}+\frac{-1+\frac{5}{x^3}}{-1-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}}$$

$$\lim_{x\to-\infty} \frac{-3}{1}+1=-2$$

**16.** $$\lim_{x\to3}\frac{(x^3+27)\ell n(x-2)}{(x^2-9)}$$

$$\lim_{x\to3}\frac{(x-3)\ (x^2+9-6x)\ \ell n(1+(x-2))}{(x+3)\ (x-3)}$$

$$\lim_{x\to3} x^2+9-6x\ =18-9=9$$

**17.** $$\lim_{x\to0}\frac{27^x-9^x-3^x+1}{\sqrt{2}-\sqrt{1+\cos x}}\times\frac{\sqrt{2}+\sqrt{1+\cos x}}{\sqrt{2}+\sqrt{1+\cos x}}$$

$$\lim_{x\to0}\frac{2\sqrt{2}(27^x-9^x-3^x+1)}{2-1-\cos x}$$

$$\lim_{x\to0}\frac{2\sqrt{2}(27^x-9^x-3^x+1)}{1-\cos}\times x^2$$

$$\lim_{x\to0}\frac{4\sqrt{2}(27^x-9^x-3^x+1)}{x^2}$$

$$\lim_{x\to0}4\sqrt{2}\left(\frac{9^x(3^x-1)-1(3^x-1)}{x^2}\right)$$

$$\lim_{x\to0}4\sqrt{2}\left(\frac{(9^x-1)\ (3^x-1)}{x^2}\right)\quad\Rightarrow\quad \lim_{x\to0}4\sqrt{2}\left(\frac{9^x-1}{x}\right)\times\lim_{x\to0}\frac{3^x-1}{x}\ \Rightarrow\ 8\sqrt{2}(\ell n3)^2$$

$$\Rightarrow\ 4\sqrt{2}(\ell n9)\ (\ell n3)$$

**18.** $$\lim_{x\to0}\left[\frac{(1+x)^{1/x}}{e}\right]^{1/x}\quad(1)^{\infty}$$

$$L=\boxed{e^{\lim_{x\to0}\frac{(1+x)^{1/x}-e}{e}.\frac{1}{x}}}$$

**Act.:** $\lim\limits_{x\to 0} \dfrac{e[e^{\frac{\ell n(1+x)}{x}-1}-1]}{\dfrac{e(\ell n(1+x)-x)}{x}} \cdot \left(\dfrac{\ell n(1+x)-x}{x^2}\right)$

$\lim\limits_{t\to 0} \dfrac{t-e^t+1/t^2}{(e^t-1)^2/t^2} = -\dfrac{1}{2}$

(taking commone)

$= e^{-1/2}$

**19.** $e^{\lim\limits_{n\to\infty}\left[\frac{\sqrt{n^2+n}-1-n}{n}\right](2\sqrt{n^2+n}-1)}$

$e^{\lim\limits_{x\to\infty}\frac{[(n^2+n)-(1+n)^2](2\sqrt{n^2+n}-1)}{n\left\{\sqrt{n^2+n}+(1+n)\right\}}}$

$e^{\lim\limits_{n\to\infty}\frac{(-n-1)(2\sqrt{n^2+n}-1)}{n\left\{\sqrt{n^2+n}+(1+n)\right\}}}$ [taking $n^2$ as common]

$e^{\lim\limits_{n\to\infty}\frac{\left(-1-\frac{1}{n}\right)\left(2\sqrt{1+\frac{1}{n}}-\frac{1}{n^2}\right)n^2}{n^2\left\{\sqrt{1+\frac{1}{n}}+\frac{1}{n}+1\right\}}} = e^{-1}$

**20.** $\lim\limits_{x\to 1}\left[\tan\dfrac{\pi x}{4}\right]^{\tan\frac{\pi x}{2}}$

$e^{\lim\limits_{x\to 1}\left(\tan\frac{\pi x}{4}-1\right)\tan\frac{\pi x}{2}}$

$e^{\lim\limits_{x\to 1}\frac{\sin\left(\frac{\pi x}{4}-\frac{\pi}{4}\right)}{\cos\frac{\pi}{4}.\cos\frac{\pi x}{4}}.\tan\frac{\pi x}{2}}$

$e^{\lim\limits_{x\to 1}\frac{\sin\frac{\pi}{4}(x-1)}{\frac{\pi}{4}(x-1)\cos\frac{\pi}{4}\cos\frac{\pi}{4}x}.\tan\frac{\pi x}{2}.\frac{\pi}{4}(x-1)}$

$e^{\lim\limits_{x\to 1}\frac{\pi}{2}(x-1)\cot\left(\frac{\pi}{2}-\frac{\pi}{2x}\right)} = e^{-1} = (e^{-1})$

**21.** $\lim\limits_{x\to 0}\left[\tan\left(\dfrac{\pi}{4}+x\right)\right]^{1/x} = e^{\lim\limits_{x\to 0}\frac{1}{x}\left[\tan\left(\frac{\pi}{4}+x\right)-1\right]}$

$= e^{\lim\limits_{x\to 0}\frac{1}{x}\left[\frac{2\tan x}{1-\tan x}\right]} = e^{\lim\limits_{x\to 0}\left[\frac{\tan x}{x}\right]\frac{2}{1-\tan x}} = e^2 \bullet$

**22.** $\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)=\ln 3$

**23.** $\lim_{x\to 0}\frac{\cos(\sin x)-1}{x^2}=\lim_{x\to 0}\frac{2\sin^2\left(\frac{\sin x}{2}\right)}{x^2}$

$$=\lim_{x\to 0}\frac{2\sin^2\left(\frac{\sin x}{2}\right)}{\frac{\sin^2 x}{4}}\times\frac{\sin^2 x}{4x^2}=\frac{1}{2}$$

**24.** $\lim_{x\to 0^+}\frac{e^{1/x}}{e^{\frac{1}{x}}+1}=\lim_{x\to 0^+}\frac{1}{e^{-\frac{1}{x}}+1}=1$ & $\lim_{x\to 0^-}\frac{e^{1/x}}{e^{\frac{1}{x}}+1}=0$

LHL $\neq$ RHL

Limit does not exist.

**25.** $\lim_{x\to 0}\frac{a^{\sin x}-1}{b^{\sin x}-1}=\lim_{x\to 0}\frac{\frac{a^{\sin x}-1}{\sin x}}{\frac{b^{\sin x}-1}{\sin x}}=\frac{\ln a}{\ln b}$

**26.** $\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)=1$

**27.** $\lim_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{x+1}=e^{\lim_{x\to\infty}\left(\frac{x+3}{x+1}-1\right)(x+1)}=e^{\lim_{x\to\infty}\left(\frac{2}{x+1}\right)(x+1)}=e^2$

**28.** $\lim_{x\to 0}(1-ax)^{\frac{1}{x}}=e^{\lim_{x\to 0}\frac{-ax}{x}}=e^{-a}$

**29.** By L'hospitals rule

$$\lim_{x\to 0}\frac{x^n-\sin x^n}{x-\sin^n x}=\lim_{x\to 0}\frac{nx^{n-1}\left(1-\cos x^n\right)}{1-n\sin^{n-1}x\cos x}$$

Now for any value of n greater than 1, denominator will be nonzero and numerator will be zero.

But for n = 1, limit becomes $\lim_{x\to 0}\frac{1-\cos x}{1-\cos x}=1$.

**30.** $\lim_{x\to 0}\frac{2x+ax\cos x+b\sin x}{x^3}=\lim_{x\to 0}\frac{2x+ax\left(1-\frac{x^2}{2}\right)+b\left(x-\frac{x^3}{6}\right)}{x^3}$

$=\lim_{x\to 0}\frac{(a+b+2)x-\left(\frac{a}{2}+\frac{b}{6}\right)x^3}{x^3}=2$

$\Rightarrow a+b+2=0\ \ \&\ 3a+b=-12$

$\Rightarrow a=-5, b=3$ .

**31.** $\lim_{h\to 0}\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h(\sqrt{3}\cos h-\sin h)}=\lim_{h\to 0}\frac{\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6}+h\right)-\frac{1}{2}\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\frac{\sqrt{3}}{2}\cos h-\frac{1}{2}\sin h\right)}$

$=\lim_{h\to 0}\frac{\cos\frac{\pi}{6}\sin\left(\frac{\pi}{6}+h\right)-\sin\frac{\pi}{6}\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\cos\frac{\pi}{6}\cos h-\sin\frac{\pi}{6}\sin h\right)}$

$=\lim_{h\to 0}\frac{\sinh}{\sqrt{3}h\cos\left(\frac{\pi}{6}+h\right)}=\frac{2}{3}$ .

**32.** $\lim_{x\to 0}\frac{x\cos x-\log(1+x)}{x^2}=\lim_{x\to 0}\frac{x\left(1-\frac{x^2}{2}\right)-\left(x-\frac{x^2}{2}+\frac{x^3}{3}\right)}{x^2}$

$=\lim_{x\to 0}\frac{\frac{x^2}{2}-\frac{2x^3}{3}}{x^2}=\frac{1}{2}$ .

33. $\lim_{x\to\frac{\pi}{4}}\left(\frac{1-\tan x}{1-\sqrt{2}\sin x}\right)=\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x}\left(\frac{\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x}{\frac{1}{\sqrt{2}}-\sin x}\right)$

$$= \lim_{x \to \frac{\pi}{4}} \frac{1}{\cos x} \left( \frac{\sin\left(\frac{\pi}{4} - x\right)}{\sin\frac{\pi}{4} - \sin x} \right)$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{1}{\cos x} \left( \frac{2\sin\left(\frac{\pi}{8} - \frac{x}{2}\right)\cos\left(\frac{\pi}{8} - \frac{x}{2}\right)}{2\cos\left(\frac{\pi}{8} + \frac{x}{2}\right)\sin\left(\frac{\pi}{8} - \frac{x}{2}\right)} \right)$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{1}{\cos x} \left( \frac{\cos\left(\frac{\pi}{8} - \frac{x}{2}\right)}{\cos\left(\frac{\pi}{8} + \frac{x}{2}\right)} \right) = 2$$

**34.** $$\lim_{x \to 0} \frac{3^x - 1}{\sqrt{4 + x} - 2} = \lim_{x \to 0} \frac{3^x - 1}{x}\left(\sqrt{4 + x} + 2\right) = 4\ln 3$$

**35.** $$\lim_{x \to 0} \left( \frac{\sin 2(1 + x) + \sin 2(1 - x) - 2\sin 2}{x \sin x} \right) = \lim_{x \to 0} \left( \frac{2\sin 2 \cos 2x - 2\sin 2}{x \sin x} \right)$$

$$= -2\sin 2 \lim_{x \to 0} \left( \frac{2\sin^2 x}{x \sin x} \right) = -4\sin 2 \lim_{x \to 0} \left( \frac{\sin x}{x} \right) = -4\sin 2$$

**36.** By L'hospitals rule

$$\lim_{x \to \frac{\pi}{4}} \frac{\int_{\pi/4}^{x} t^2\, dt}{\cos 2x} = \lim_{x \to \frac{\pi}{4}} \frac{x^2}{-2\sin 2x} = -\frac{\pi^2}{32}$$

**37.** By L'hospitals rule

$$\lim_{h \to 0} \frac{(x + h)\sec(x + h) - x\sec x}{\sin h} = \lim_{h \to 0} \frac{(x + h)\sec(x + h)\tan(x + h) + \sec(x + h)}{\cos h}$$

$$= x\sec x \tan x + \sec x = \sec x\left(x\tan x + 1\right)$$

Alternately

$$\lim_{h \to 0} \frac{(x + h)\sec(x + h) - x\sec x}{\sin h} = \lim_{h \to 0} \frac{(x + h)\sec(x + h) - (x + h)\sec x - x\sec x + (x + h)\sec x}{\sin h}$$

$$= \lim_{h\to 0}\frac{(x+h)\left(\sec(x+h)-\sec x\right)}{\sin h}-\lim_{h\to 0}\frac{\left(x-(x+h)\right)\sec x}{\sin h}$$

$$= \lim_{h\to 0}\frac{(x+h)\left(\cos x-\cos\left(x+h\right)\right)}{\sin h\cos x\cos\left(x+h\right)}+\lim_{h\to 0}\frac{h\sec x}{\sin h}$$

$$= \lim_{h\to 0}\frac{2(x+h)\sin\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{2\sin\frac{h}{2}\cos\frac{h}{2}\cos x\cos\left(x+h\right)}+\lim_{h\to 0}\frac{h\sec x}{\sin h}$$

$$= \frac{x\sin x}{\cos^2 x}+\sec x = x\sec x\tan x+\sec x\,.$$

**38.** $$\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(\frac{\pi}{3}-2x\right)}{2\cos 2x-1}=\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(\frac{\pi}{3}-2x\right)}{4\cos^2 x-3}$$

$$= \lim_{x\to\frac{\pi}{6}}\frac{\sin\left(\frac{\pi}{3}-2x\right)\cos x}{\cos 3x}$$

$$= \lim_{y\to 0}\frac{\sin 2y\cos\left(\frac{\pi}{6}-y\right)}{\sin 3y}\quad\left\{x=\frac{\pi}{6}-y\right\}$$

$$= \lim_{y\to 0}\frac{2\frac{\sin 2y}{2y}\cos\left(\frac{\pi}{6}-y\right)}{3\frac{\sin 3y}{3y}}=\frac{2}{3}\times\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{3}}$$

**39.** $$\lim_{x\to 0}\frac{e^{x^4}-\cos x^2}{x^4}=\lim_{x\to 0}\frac{\left(1+x^4+\frac{x^8}{2}\right)-\left(1-\frac{x^4}{2}+\frac{x^8}{4!}\right)}{x^4}=\frac{3}{2}\,.$$

**40.** $$\lim_{x\to 0}\frac{x\cos^2 x-\sin^2 x}{x+\sin x}=\lim_{x\to 0}\frac{\cos^2 x-x\frac{\sin^2 x}{x^2}}{1+\frac{\sin x}{x}}=\frac{1}{2}$$