LIMITS
EXERCISE 1(B)

1 If $f(x)=\begin{cases} x^2+2, x\ge 1 \\ 2x+1, x<1 \end{cases}$, then $\lim\limits_{x\to 1} f(x)$ equals -

(A) 1 (B) 2
(C) 3 (D) Does not exist

Sol. $\lim\limits_{x\to 1-0} f(x)\ \lim\limits_{h\to 0} = [2(1–h)+1]= 3$

$\lim\limits_{x\to 1+0} f(x) = \lim\limits_{h\to 0}\ [(1+h)^2 + 2] = 3$

$\therefore$ LHL = RHL, so $\lim\limits_{x\to 1} f(x) = 3$. **Ans.[C]**

2 $\lim\limits_{x\to 0} \dfrac{1+e^{-1/x}}{1-e^{-1/x}}$ is equal to -

(A) 1 (B) –1
(C) 0 (D) Does not exist

Sol. LHL $= \lim\limits_{h\to 0} \dfrac{1+e^{1/h}}{1-e^{1/h}}$

$= \lim\limits_{h\to 0} = \dfrac{e^{-1/h}+1}{e^{-1/h}-1} -– 1$

RHL $= \lim\limits_{h\to 0} = \dfrac{1+e^{-1/h}}{1-e^{-1/h}} = \dfrac{1+0}{1-0} = 1$ **Ans.[D]**

LHL ≠ RHL, so given limit does not exist.

3 If $f(x)=\begin{cases} x-1, x<0 \\ 1/4, x=0 \\ x^2, x>0 \end{cases}$ then $\lim\limits_{x\to 0} f(x)$ equals -

(A) 0 (B) 1
(C) –1 (D) Does not exist

Sol. Here $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} x^2 = 0$

and $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} (x–1) = –1$

$\because \lim\limits_{x\to 0^+} f(x) \neq \lim\limits_{x\to 0^-} f(x)$

$\therefore \lim\limits_{x\to 0} f(x)$ does not exist. **Ans.[D]**

4 $\lim\limits_{x\to 3} \dfrac{x-3}{|x-3|}$, is equal to -

(A) 1 (B) –1
(C) 0 (D) Does not exist

Sol. LHL $= \lim\limits_{h\to 0} \dfrac{(3-h)-3}{|(3-h)-3|}$

$= \lim\limits_{h\to 0} \dfrac{-h}{|-h|} = -–1$

$$\text{RHL} = \lim_{h\to 0} \frac{(3+h)-3}{|(3+h)-3|}$$

$$= \lim_{h\to 0} \frac{h}{|h|} = 1$$

LHL $\neq$ RHL, so limit does not exist. **Ans.[D]**

5 $\lim_{x\to\infty} \dfrac{2x^2+3x}{3x^2+4}$ equals -

(A) 1/2 (B) 2/3

(C) 3/4 (D) 0

Sol. $= \lim_{x\to\infty} \dfrac{2+(3/x)}{3+(4/x^2)} = \dfrac{2}{3}$ **Ans.[B]**

6 $\lim_{x\to\infty} \left(\sqrt{x^2+1}-x\right)$ equals -

(A) –1 (B) 0

(C) 1 (D) None of these

Sol. Limit $= \lim_{x\to\infty} x\left[\left(1+\dfrac{1}{x^2}\right)^{1/2} - 1\right]$

$$= \lim_{x\to\infty} x\left[1+\frac{1}{2x^2}-\frac{1}{8x^4}+\ldots-1\right]$$

$$= \lim_{x\to\infty} \left[\frac{1}{2x}-\frac{1}{8x^3}+\ldots\right] = 0.$$ **Ans.[B]**

7 If $f(x) = \sqrt{\dfrac{x-\sin x}{x+\cos^2 x}}$, then $\lim_{x\to\infty} f(x)$ equals -

(A) 0 (B) ∞

(C) 1 (D) None of these

Sol. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \sqrt{\dfrac{\{1-(\sin x/x)\}}{\{1+(\cos^2 x/x)\}}}$

$= \sqrt{\dfrac{1-0}{1+0}} = 1.$ **Ans.[C]**

8 $\lim_{x\to a} \left[\dfrac{x^2-(a+1)x+a}{x^3-a^3}\right]$ is equal to -

(A) $\dfrac{a-1}{3a^2}$ (B) a – 1

(C) a (D) 0

Sol. $\lim_{x\to a} \left[\dfrac{x^2-(a+1)x+a}{x^3-a^3}\right]$ $\left(\dfrac{0}{0}\text{ form}\right)$

$$= \lim_{x\to a} = \frac{2x-a-1}{3x^2} = \frac{a-1}{3a^2}$$

(D.L.Hospital rule) **Ans.[A]**

9 $\lim_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$ is equal to -

(A) 1/2 (B) 2

(C) 1 (D) 0

Sol. Limit = $\lim_{x\to 0} \frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{(1+x)-(1-x)}$

$= \lim_{x\to 0} \frac{\sqrt{1+x}+\sqrt{1-x}}{2} = 1.$ **Ans.[C]**

10 $\lim_{x\to 0} \frac{\tan 2x - x}{3x - \sin x}$ equals-

(A) 2/3 (B) 1/3

(C) 1/2 (D) 0

Sol. The given limit is in the form , therefore applying L 'Hospital's rule, we get

Limit $= \lim_{x\to 0} \frac{2\sec^2 2x - 1}{3 - \cos x} = \frac{2-1}{3-1} = \frac{1}{2}$ **Ans.[C]**

11 $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$ is equal to -

(A) e^3 (B) $e^{1/3}$ (C) 1 1 (D) e

Sol. Limit = $\lim_{x\to 0} \left(\frac{x + x^3/3 + ...}{x}\right)^{1/x^2}$

$= \lim_{x\to 0} \left(1 + \frac{x^3}{3}\right)^{1/x^2}$

[$\because$ x $\to$ 0, so neglecting higher powers of x]

$= \lim_{x\to 0} \left[\left(1 + \frac{x^2}{3}\right)^{3/x^2}\right]^{1/3} = e^{1/3}$ **Ans.[B]**

12 $\lim_{x\to 0} \frac{\sin x^o}{x}$ is equal to -

(A) 1 (B) π (C) x (D) π/180

Sol. Limit = $\lim_{x\to 0} \frac{\sin(\pi/180)x}{x}$

$= \lim_{x\to 0} \frac{(\pi/180)\cos(\pi/180)x}{1}$

$= \frac{\pi}{180}$ **Ans.[D]**

13 $\lim_{x\to\infty} \left(\frac{\pi}{2} - \tan^{-1} x\right)^{1/x}$ equals -

(A) 0 (B) 1 (C) ∞ (D) –1

Sol. Let $y = \lim_{x\to\infty}\left(\frac{\pi}{2} - \tan^{-1} x\right)^{1/x}$

$= \lim_{x\to\infty} (\cot^{-1}x)^{1/x}$

$\therefore \log y = \lim_{x\to\infty} \frac{\log \cot^{-1} x}{x}$ $\left(\frac{\infty}{\infty}\text{form}\right)$

$= \lim_{x\to\infty} -\frac{1}{(1+x^2)\cos^{-1} x}$ (0x ∞ form)

$= -\lim_{x\to\infty} \frac{(1+x^2)^{-1}}{\cot^{-1} x}$ $\left(\frac{0}{0}\text{form}\right)$

$= -\lim_{x\to\infty} \frac{\frac{-2x}{(1+x^2)^2}}{\frac{-1}{1+x^2}}$ $= -2\lim_{x\to\infty} \frac{x}{1+x^2}$

$= -2\lim_{x\to\infty} \frac{1}{2x} = 0$ $\therefore y = e^0 = 1.$ **Ans.[B]**

14 If $G(x) = -\sqrt{25-x^2}$,

then $\lim_{x\to 1} \frac{G(x)-G(1)}{x-1}$ equals -

(A) 1/24 (B) 1/5

(C) $-\sqrt{24}$ (D) None of these

Sol. Here $G(1) = -\sqrt{25-x^2} = -\sqrt{24}$

$\therefore$ Given limit

$= \lim_{x\to 1} \frac{-\sqrt{25-x^2}+\sqrt{24}}{x-1}$ $\left(\frac{0}{0}\text{form}\right)$

$= \lim_{x\to 1} \frac{x}{\sqrt{25-x^2}}$ (By L Hospital ruel)

$= \frac{1}{\sqrt{24}}$ **Ans.[D]**

15 If f(9) = 9 and f'(9) = 4, then $\lim_{x\to 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ is equal to -

(A) 1 (B) 3 (C) 4 (D) 9

Sol. Given limit is in 0/0 form, so using Hospital rule, we get

$$\text{Limit} = \lim_{x\to 9} \frac{\frac{1}{2\sqrt{f(x)}}.f(x)}{\frac{1}{2\sqrt{x}}}$$

$= \frac{f(9).\sqrt{9}}{\sqrt{f(9)}} = \frac{4.3}{3} = 4$ **Ans.[C]**

16 By L'hospital's rule

$$\lim_{x\to 1} \frac{f(1)\,g(x)-f(1)-g(1)\,f(x)+g(1)}{g(x)-f(x)} = \lim_{x\to 1} \frac{f(1)\,g'(x)-g(1)\,f'(x)}{g'(x)-f'(x)} = k = 4$$ **Ans.[A]**

17 By L'hospital's rule

$$\lim_{x\to 0}\left(\frac{\int_0^{2x^2}\sec^2 2t\,dt}{x\sin x}\right)=\lim_{x\to 0}\left(\frac{4x\sec^2 4x^2}{x\cos x+\sin x}\right)=\lim_{x\to 0}\left(\frac{4\sec^2 4x^2}{\cos x+\frac{\sin x}{x}}\right)=2$$

Ans.[B]

18 $$\lim_{x\to 0}\frac{\sin^{-1}\left(3x-4x^3\right)}{\ln(1+2x)}=\lim_{x\to 0}\frac{3\sin^{-1}x}{\ln(1+2x)}=\lim_{x\to 0}\frac{3\frac{\sin^{-1}x}{x}}{2\frac{\ln(1+2x)}{2x}}=\frac{3}{2}.$$

Ans.[C]

19 $$\lim_{x\to 2}\frac{\left(x^2+5\right)^{\frac{1}{2}}-\left(x^3+1\right)^{\frac{1}{2}}}{x^2-4}=\lim_{x\to 2}\frac{\left(x^2+5\right)-\left(x^3+1\right)}{\left(x^2-4\right)\left(\left(x^2+5\right)^{\frac{1}{2}}+\left(x^3+1\right)^{\frac{1}{2}}\right)}$$

$$=\lim_{x\to 2}\frac{-(x-2)\left(x^2+x+2\right)}{(x-2)(x+2)\left(\left(x^2+5\right)^{\frac{1}{2}}+\left(x^3+1\right)^{\frac{1}{2}}\right)}=\lim_{x\to 2}\frac{-\left(x^2+x+2\right)}{(x+2)\left(\left(x^2+5\right)^{\frac{1}{2}}+\left(x^3+1\right)^{\frac{1}{2}}\right)}=-\frac{1}{3}$$

Ans.[C]

20 $$\lim_{x\to 0}\frac{\tan 2x-\sin x}{x}=2\lim_{x\to 0}\frac{\tan 2x}{2x}-\lim_{x\to 0}\frac{\sin x}{x}=1$$

Ans.[A]

21 $$\lim_{x\to 0}\frac{\sin 4x-\sin 2x+\sin x}{x\cot x+1}=\lim_{x\to 0}\frac{\sin 4x-\sin 2x+\sin x}{\left(\frac{x}{\tan x}+1\right)}=\frac{0}{2}=0$$

22 $$\lim_{x\to 0}\frac{\ln(1+\sin x)}{e^{\tan 2x}-1}=\lim_{x\to 0}\frac{\ln(1+\sin x)}{\sin 2x}\times\frac{\tan 2x}{e^{\tan 2x}-1}\times\frac{\frac{\sin 2x}{2x}}{\frac{\tan 2x}{2x}}=1$$

Ans.[A]

23 $$\lim_{x\to\frac{\pi}{4}}(1+\cos 2x)^{\frac{1}{1-\tan x}}=e^{\lim_{x\to\frac{\pi}{4}}\frac{\cos 2x}{1-\tan x}}$$

$$=e^{\lim_{x\to\frac{\pi}{4}}\frac{1-\tan^2 x}{1+\tan^2 x}\times\frac{1}{1-\tan x}}=e^{\lim_{x\to\frac{\pi}{4}}\frac{1+\tan x}{1+\tan^2 x}}=e.$$

Ans.[A]

24 $$\lim_{x\to 0}\frac{a\sin x+b\cos x+ce^x}{x^2}=\lim_{x\to 0}\frac{a\left(x-\frac{x^3}{6}\right)+b\left(1-\frac{x^2}{2}\right)+c\left(1+x+\frac{x^2}{2}\right)}{x^2}$$

$$=\lim_{x\to 0}\frac{(b+c)+(a+c)x+(c-b)\frac{x^2}{2}-a\frac{x^3}{6}}{x^2}$$

$\Rightarrow b+c=0, a+c=0$ & $c-b=4$

$\Rightarrow a=b=-2, c=2.$

25 $$\lim_{x\to\infty}\left(\frac{ax+1}{ax+2}\right)^{2x}=e^{\lim_{x\to\infty}2x\left(\frac{ax+1}{ax+2}-1\right)}$$

$$= e^{\lim_{x\to\infty}\left(\frac{-2x}{ax+2}\right)} = e^{-\frac{2}{a}} = e^{\frac{1}{2}} \Rightarrow a = -4$$ **Ans.[C]**

26 In RHL $[x] = 0$ hence limit is not defined. **Ans.[D]**

27 $\lim_{x\to 0^-}(1+[x])^{\frac{1}{x}}$ not defined as $1+[x]=0$ & $\frac{1}{x}\to -\infty$ **Ans.[D]**

$\lim_{x\to 0^+}(1+[x])^{\frac{1}{x}} = 1$ as $[x]=0$

28 $$\lim_{x\to\infty}\left(\sqrt{x^2+8x+3}-\sqrt{x^2+4x+3}\right) = \lim_{x\to\infty}\left(\frac{4x}{\sqrt{x^2+8x+3}+\sqrt{x^2+4x+3}}\right)$$

$$= \lim_{x\to\infty}\left(\frac{4}{\sqrt{1+\frac{8}{x}+\frac{3}{x^2}}+\sqrt{1+\frac{4}{x}+\frac{3}{x^2}}}\right) = 2$$ **Ans.[C]**

29 $$\lim_{x\to 0}\frac{2x\left(3^x-1\right)}{1-\cos 2x} = \lim_{x\to 0}\frac{2x\left(3^x-1\right)}{2\sin^2 x} = \lim_{x\to 0}\frac{\left(\frac{3^x-1}{x}\right)}{\frac{\sin^2 x}{x^2}} = \ln 3$$ **Ans.[A]**

30 $$\lim_{n\to\infty}\frac{1^5+2^5+3^5+........+n^5}{n^6} = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^5$$

$$= \int_0^1 x^5 dx = \frac{1}{6}.$$ **Ans.[D]**

31 $$\lim_{n\to\infty}\left[\frac{3}{1+n^3}+\frac{12}{8+n^3}+\frac{27}{27+n^3}+....+n \text{ terms}\right] = 3\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{\left(\frac{r}{n}\right)^2}{\left(\frac{r}{n}\right)^3+1}$$

$$= 3\int_0^1\frac{x^2}{1+x^3}dx = \int_1^2\frac{1}{t}dt \quad \{\text{by substitution } 1+x^3=t\}$$

= ln 2. **Ans.[B]**

32 $$\lim_{x\to 0}\frac{\sqrt{4+x}-\sqrt{4-x}}{\sin^{-1}2x} = \lim_{x\to 0}\frac{2x}{\sin^{-1}2x}\times\frac{1}{\sqrt{4+x}+\sqrt{4-x}} = \frac{1}{4}.$$ **Ans.[D]**

33 $$\lim_{x\to\infty}\left(\frac{1^x+3^x+5^x+...+(2n-1)^x}{n}\right)^{\frac{1}{x}}$$

$$= \lim_{x\to\infty}(2n-1)\left(\frac{\left(\frac{1}{2n-1}\right)^x+\left(\frac{3}{2n-1}\right)^x+\left(\frac{5}{2n-1}\right)^x+...+\left(\frac{2n-3}{2n-1}\right)^x+1}{n}\right)^{\frac{1}{x}}$$

$$= 2n - 1 \quad \left\{ \text{as all } \frac{1}{2n-1}, \frac{3}{2n-1}, \ldots, \frac{2n-3}{2n-1} < 1 \right\}$$ **Ans.[A]**

34 $$\lim_{x \to 0} \left(\frac{1^x + 2^x + 3^x + \ldots n^x}{n} \right)^{\frac{1}{x}} = e^{\lim_{x\to 0} \frac{1}{x}\left(\frac{1^x + 2^x + 3^x + \ldots n^x}{n} - 1\right)}$$

$$= e^{\lim_{x\to 0} \frac{1}{n}\left(\frac{1^x - 1 + 2^x - 1 + 3^x - 1 + \ldots n^x - 1}{x}\right)} = e^{\frac{\ln 2 + \ln 3 + \ldots + \ln n}{n}} = (n!)^{\frac{1}{n}}$$ **Ans.[B]**

35 $$\lim_{x \to 0} \frac{8\left(2^x - 3^x\right)\tan x}{1 - \cos 4x} = \lim_{x \to 0} \frac{8\left(\frac{2^x - 1}{x} - \frac{3^x - 1}{x}\right)\frac{\tan x}{x}}{8\frac{\sin^2 2x}{4x^2}}$$

$$= \ln 2 - \ln 3 = \ln \frac{2}{3}$$ **Ans.[D]**