LIMITS

EXERCISE 1(A)

1. $LHL = \lim_{x\to 1^-} x^2 = \lim_{h\to 0^+}(1-h)^2 = \lim_{h\to 0^+}\left(1+h^2-2h\right) = 1$

$RHL = \lim_{x\to 1^+} x = \lim_{h\to 0^+}(1+h) = 1$

$\therefore LHL = RHL =$ a finite quantity.

Hence , $\lim_{x\to 1} f(x) = 1$

2. $LHL = \lim_{h\to 0^+} \frac{|2-h-2|}{(2-h)-2} = \frac{(-h)}{-h} = \frac{h}{-h} = -1$

$RHL = \lim_{h\to 0^+} \frac{|2+h-2|}{(2+h-2)} = \frac{|h|}{h} = \frac{h}{h} = 1$

$\therefore \lim_{x\to 2} \frac{|x-2|}{x-2} = 1$

3. $LHL = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}\left(\frac{2}{5-x}\right) = \lim_{h\to 0^+} \frac{2}{5-(3-h)} = \lim_{h\to 0^+} \frac{2}{2+h} = 1$

$RHL = \lim_{x\to 3^+} f(x) = \lim_{h\to 0^+}\left[5-(3+h)\right] = 2$

4. $LHL = \lim_{h\to 0^+} 3(1-h) = 3$, $\qquad RHL = \lim_{h\to 0^+} 5-3(1+h) = 2$

5. $LHL = \lim_{h\to 0^+} \frac{|h|}{h} = -1 \qquad RHL = \lim_{h\to 0^+} \frac{|h|}{h} = 1$

6. $\lim_{x\to 1}\left(3x^2+4x+5\right) = 3+4+5 = 12$

7. $\lim_{x\to 2} \frac{\left(3^{x/2}-3\right)}{\left(3^{x/2}-3\right)\left(3^{x/2}+3\right)} = \lim_{x\to 2} \frac{1}{\left(3^{x/2}+3\right)} = \frac{1}{6}$

8. $\lim_{x\to a} \frac{x^5-4^5}{x-a} = 5a^4$

9. $\lim_{h\to 0} \frac{1}{h}\left[\frac{x-(x+h)}{(x+h)x}\right] = \lim_{h\to 0} \frac{-1}{x(x+4)} = \frac{-1}{x^2}$

10. $\lim_{x\to 0} \frac{\left(\sqrt{1-x^2}-\sqrt{1+x^2}\right)\left(\sqrt{1-x^2}+\sqrt{1+x^2}\right)}{x^2\left(\sqrt{1-x^2}+\sqrt{1+x^2}\right)} = \lim_{x\to 0} \frac{\left(1-x^2\right)-\left(1+x^2\right)}{x^2\left(\sqrt{1-x^2}+\sqrt{1+x^2}\right)}$

$= \lim_{x\to 0} \frac{-2x^2}{2x^2} = -1$

11. $\lim\limits_{x\to 3}\dfrac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{(x-2)-(4-x)}=\lim\limits_{x\to 3}\dfrac{1}{2}(2)=1$

12. $\lim\limits_{x\to\infty}\dfrac{x^2}{x^2}\dfrac{\left(a+\dfrac{b}{x}+\dfrac{c}{x^2}\right)}{\left(d+\dfrac{e}{x}+\dfrac{f}{x^2}\right)}=\dfrac{a}{d}$

13. $\lim\limits_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)\dfrac{\left(\sqrt{x+\sqrt{x+x}}+\sqrt{x}\right)}{\left(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}\right)}=\lim\limits_{x\to\infty}\dfrac{\left(x+\sqrt{x+\sqrt{x}}-x\right)}{\left(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}\right)}$

$$=\lim_{x\to\infty}\left(\frac{\sqrt{x}\left(1+\sqrt{\dfrac{1}{x}}\right)^{1/2}}{\sqrt{x}\left[\left(1+\sqrt{\dfrac{1}{x}+\sqrt{\dfrac{1}{x^3}}}\right)^{1/2}+1\right]}\right)=\frac{(1)^{1/2}}{(1)^{1/2}+1}=\frac{1}{2}$$

14. $\lim\limits_{x\to 1}(1+x)^{\frac{1}{x}}=2$

$$\lim_{x\to\frac{\pi}{6}}\frac{\left(3\sin x-\sqrt{3}\cos x\right)}{6\left(x-\dfrac{\pi}{6}\right)}$$

$$=\frac{3}{6}\lim_{x\to\frac{\pi}{6}}\frac{\left(\sin x-\dfrac{1}{\sqrt{3}}\cos x\right)}{\left(x-\dfrac{\pi}{6}\right)}=\frac{1}{2}\lim$$

$$=\frac{2\sqrt{3}}{6}\lim_{x\to\frac{\pi}{6}}\frac{\left(\dfrac{\sqrt{3}}{2}\sin x-\dfrac{1}{2}\cos x\right)}{\left(x-\dfrac{\pi}{6}\right)}$$

$$\lim_{x\to\frac{\pi}{6}}\left(\frac{1}{\sqrt{3}}\right)\frac{\sin\left(x-\dfrac{\pi}{6}\right)}{\left(x-\dfrac{\pi}{6}\right)}=\frac{1}{\sqrt{3}}$$

17. $\lim\limits_{x\to 3}\dfrac{\left(x^3-27\right)-\left(x^2-9\right)}{(x-3)}=\lim\limits_{x\to 3}\dfrac{(x-3)}{(x-3)}\left[\left(x^2+3x+9\right)-(x+3)\right]=\lim\limits_{x\to 3}\left(x^2+2x+6\right)=21$

18. $\lim\limits_{x\to 2}\dfrac{(x-2)\left(x^2+2x+4\right)}{(x-2)(x+2)}=2$

19. $$\lim_{x\to 0}\frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\left(\sqrt{1+x}-\sqrt{1-x}\right)\left(\sqrt{1+2}+\sqrt{1-x}\right)}=\lim_{x\to 0}\frac{x\sqrt{1+x}-\sqrt{1-x}}{2x}=1$$

$$\lim_{x\to a}\frac{\left(\sqrt{a+2z}-\sqrt{3x}\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{\sqrt{3a+x}-2\sqrt{x}\left(\sqrt{3a+x}+2\sqrt{x}\right)\left(\sqrt{a+2x}+\sqrt{3}x\right)}=\frac{(a-x)(a+2x)+\sqrt{3x}}{(a-x)\left(\sqrt{3a+x}+2\sqrt{x}\right)}$$

20. $$\lim_{x\to a}\frac{(a-x)}{3(a-x)}\frac{\left(\sqrt{2a+x}+2\sqrt{x}\right)}{\left(\sqrt{a+2x}+\sqrt{3x}\right)}=\frac{1}{3}\frac{4\sqrt{a}}{2\sqrt{3}\sqrt{a}}=\frac{2}{3\sqrt{3}}$$

21. $$\lim_{n\to\infty}\frac{\left(n^{49}+n^{98}+\ldots\ldots.1^{99}\right)(n-1)}{n^{180}(n-1)}=\left(n^{100}-1\right)$$

22. $$\lim_{x\to 7}\frac{2-\sqrt{x-3}}{\left(x^2-49\right)}\times\frac{\left(2+\sqrt{x-3}\right)}{\left(2+\sqrt{x-3}\right)}=\lim_{x\to 7}=\frac{(7-x)}{(x-7)(x+7)\left(x+\sqrt{x+3}\right)}=\frac{-1}{6}$$

23. $$\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}=\lim_{n\to\infty}\frac{n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6n^3}=\frac{1}{6}$$

24. $$\lim_{x\to 0}\left[\left(\frac{4^x-1}{x}\right)-\left(\frac{9^x-1}{x}\right)\right]\frac{1}{\left(4^x+9^x\right)}=\frac{1}{2}(\log 4-\log 9)=\log\left(\frac{2}{3}\right)$$

25. $$\lim_{x\to 3}\frac{2}{x-3}+\frac{x-3}{x+4}-\frac{2(2x+1)}{x^2+x-12}=\lim_{x\to 3}\frac{(2x+8-4x-2)}{\left(x^2+x-12\right)}=\lim_{x\to 3}\frac{-2(x-3)}{\left(x^2+x+2\right)}=\frac{-2}{7}$$

26. $$\lim_{x\to 0}\frac{\sin x+\log(1-x)}{x^2}=\lim_{x\to 0}\frac{x-\frac{x^3}{3!}-x-\frac{x^2}{2!}-\frac{x^3}{3!}}{x^2}=\frac{1}{2}$$

27. $$\lim_{x\to\infty}\left[\frac{x^3(1-a)+x^2(-b)+x(-a)+(a-b)}{x^2+1}\right]=2$$

$$\therefore 1-a=0\Rightarrow a=1$$

$$\Rightarrow\lim_{x\to\infty}\frac{-b+\left(\frac{-a}{x}\right)+\left(\frac{1-b}{x^2}\right)}{1+\frac{a}{x^2}}=2\Rightarrow -b=2$$

28. $$\lim_{x\to\infty}\frac{x^{10}\left[\left(1+\frac{1}{x}\right)^{10}+\left(1+\frac{2}{x}\right)^{10}+\left(1+\frac{3}{x}\right)^{10}+\ldots\ldots..\left(1+\frac{100}{x}\right)^{10}\right]}{x^{10}\left[1+\left(\frac{10}{x}\right)^{10}\right]}=100$$

29. $\lim_{x\to 0}\frac{\left(xe^{x}-x\right)}{x^{2}}+\left(\frac{x-\log(1+x)}{x^{2}}\right)=L_1+L_2$

$L_1=\lim_{x\to 0}\frac{e^{x}-1}{x}=1,\ L_2=\lim_{x\to 0}\frac{x-\log(1+x)}{x^{2}}$, let $\log(1+x)=t$

$\therefore L_2=\lim_{t\to 0}\frac{e^{t}-1-t}{\left(e^{t}-1\right)^{2}}=\lim_{t\to 0}\frac{e^{t}-t-1}{t^{2}}=\frac{1}{2}$

30. $\lim_{x\to 0}\left(\frac{\sin^{-1}x-x}{x^{3}}\right)+\lim_{x\to 0}\left(\frac{x-\tan^{-1}x}{x^{3}}\right)=L_1+L_2$

Let $\sin^{-1}x=t_1\Rightarrow L_1=\lim_{t\to 0}\frac{t_1-\sin t}{\sin^{3}t}=\lim_{t_1\to 0}\frac{t_1-\sin t}{t_1^{\ 3}}=\frac{1}{6}$

Let $\tan^{-1}x=t_2\Rightarrow L_2=\lim_{t_2\to 0}\frac{\tan t_2-t_2}{\tan^{3}t_2}=\lim_{t_2\to 0}\frac{\tan t_2-t_2}{t_2^{2}}=\frac{1}{3}$

31. $\text{RHL}=\lim_{h\to 0^{+}}\frac{\sqrt{1-\sin\left(\frac{\pi}{2}+h\right)}}{2\left(\frac{\pi}{2}-\frac{\pi}{2}+h\right)}=\lim_{h\to 0^{+}}\frac{\sqrt{1-\cos h}}{2h}=\frac{\sqrt{2}\left|\ell n\frac{h}{2}\right|}{4\left(\frac{h}{2}\right)}=\frac{1}{2\sqrt{2}}$

$\text{LHL}=\lim_{h\to 0^{+}}\frac{\sqrt{2}\left|\sin\frac{h}{2}\right|}{(-2h)}=\frac{-1}{2\sqrt{2}}$

32. $\lim_{x\to 0}\left[\frac{1-\cos 2x}{(2x)^{2}}\right]\left(\frac{\sin 5x}{5x}\right)\left(\frac{3x}{\sin 3x}\right)\times\left(\frac{4\times 5}{3}\right)=\frac{10}{3}$

33. $\lim_{x\to 0}\frac{\left(x^{2}\right)}{\sin\left(x^{2}\right)}\times x=1\times 0=0$

34. $\lim_{x\to 0}3\left(\frac{\sin 3x}{3x}\right)+\lim_{x\to 0}\frac{\sin x}{x}=4$

35. $\lim_{x\to 0}\frac{(1+x)^{8}-(1+8x)}{x^{2}}=\lim_{x\to 0}\frac{\left(1+8x+\frac{8.7}{2}x^{2}+\frac{8.7.6}{3!}x^{3}+\ldots\right)-(1+3x)}{x^{2}}=28$

36. $\text{LHL}=\lim_{H\to 0^{+}}\frac{\sin(-h)}{(-h)}=1\qquad \text{RHL}=0$

37. $\lim_{x\to 0}8.\left(\frac{\sin 2x}{2x}\right)\left[\frac{1-\cos 2x}{(2x)^{2}}\right]\left(\frac{1}{\cos 2x}\right)=4$

38. $\lim_{x\to 2}f(x)=\lim_{x\to 2}\frac{\sin\left(e^{x-2}-1\right)}{\left(e^{x-2}-1\right)}\times\frac{\left(e^{x-2}-1\right)}{(x-2)}\frac{(x-2)}{\log\left[1+(x-2)\right]}=1$

39. $$\lim_{x\to\frac{\pi}{2}} \frac{a^{\cos x}\left(a^{\cot x-\cos x}-1\right)}{\cot x-\cos x}=\ell n(a)$$

40. $$\lim_{x\to 0}\frac{f(x)}{x^2}=\lim_{x\to 0}\begin{vmatrix}\sin x & \cos^2 x & \tan x\\ x & 1 & 1/x\\ 2x & 1 & 1\end{vmatrix}=\lim_{x\to 0}\left(\frac{\begin{vmatrix}\sin x & \cos x & \tan x\\ x^2 & x & 1\\ 2x & 1 & 1\end{vmatrix}}{x}\right)$$

41. $$=\begin{vmatrix}\cos x & \cot x & \tan x\\ 2x & x & 1\\ 2x & 1 & 1\end{vmatrix}+\begin{vmatrix}\sin x & -\sin x & \tan x\\ x^2 & 1 & 1\\ 2x & 0 & 1\end{vmatrix}+\begin{vmatrix}\sin x & \cos x\\ x^2 & x\\ 2x & 1\end{vmatrix}$$

42. $$\lim_{x\to 0}\left(\frac{\tan x}{x}\right)\times\frac{1}{(1+\tan x)}\times\left(\frac{1-\cos x}{x^2}\right)=1\times 1\times\frac{1}{2}=\frac{1}{2}$$

43. $$\lim_{x\to 0}\left[2(a-2)-\frac{\tan x}{x}\right]\frac{\sin 2x}{x}=0\Rightarrow 2[2a-4-1]=0\Rightarrow a=\frac{5}{2}$$

44. $$\lim_{h\to 0}\frac{\log_e\frac{(1+4h)}{(1+2h)^2}}{h^2}=\lim_{h\to 0}\frac{\ell n\left(1+\left(\frac{1+4h}{(1+2h)^2}-1\right)\right)}{\left(\frac{1+4h}{(1+2h)^2}-1\right)}\frac{\left(\frac{1+4h}{(1+2h)^2}-1\right)}{h^2}$$

$$\lim_{h\to 0}\frac{-4h^2}{h^2}=-4$$

45. $$\lim_{x\to a}\frac{\ell n(1+(x-a))}{(x-a)}=1$$

46. $$\lim_{h\to 0}\frac{\ell n(1+h)}{h}\times\log_{10}e=\log_{10}e$$

47. $$k=\lim_{x\to 0}\frac{\log\left(1+\frac{3+2x}{3-2x}-1\right)}{\left(\frac{4x}{3-2x}\right)}\times\frac{4x}{(3-2)x}=\frac{4}{3}$$

48. $$\lim_{x\to 0}\frac{(e^x-1)^2}{(x)^2}+\left(\frac{e^x-x-1}{x^2}\right)=1+\frac{1}{2}=\frac{3}{2}$$

49. $$\lim_{x\to 0}\frac{e^x-x-1}{x^2}=\lim_{x\to 0}\frac{\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.......\right)-(x+1)}{x^2}=\frac{1}{2}$$

50. $\lim_{x\to 0}\frac{\left(2^x-1\right)\left(\sqrt{1+x}+1\right)}{\left(\sqrt{1+x}-1\right)\left(\sqrt{1+x}+1\right)}=\lim_{x\to 0}2\frac{\left(2^x-1\right)}{x}=\ell n4$

51. $e\lim_{x\to 0}\left(x+3\right)\left(\frac{x+4}{x+1}-1\right)=e^{\lim_{x\to\infty}3}=e^3$

52. $e\lim_{x\to 0}\left(c+dx\right)\left(\frac{1}{a+bx}\right)=e^{\frac{d}{b}}$

53. $\lim_{x\to 0}\left(2x\right)^{3x}=1$

54. $\lim_{x\to 0}\left(1+1+\frac{\frac{1}{x}\left(\frac{1}{x}-1\right)}{2}x^2+......\right)+\left(\frac{x}{2}-1\right)\left(1+1+\frac{1}{2!}+\frac{1}{3!}+.....\right)$

55. $e^{\lim_{m\to\infty}}m\left(\cos\frac{x}{m}-1\right)=e^{\lim_{m\to\infty}}\frac{\left(\cos\frac{x}{m}-1\right)}{\left(\frac{x^2}{m^2}\right)}\times\left(\frac{x^2}{m^2}\right)\times m=1$

56. $e^{\lim_{n\to\infty}}\ n\left(n-1\right)\frac{\left(2\right)}{\left(n^2-n-1\right)}=e^2$

57. $\lim_{x\to 0}\left(\frac{1-\cos mx}{m^2x^2}\right)\times\left(\frac{n^2c^2}{1-\cos nx}\right)\times\frac{m^2}{n^2}$

$=\frac{m^2}{b^2}$

58. $\lim_{x\to\frac{\pi}{8}}\frac{\sin\left(2x-\frac{\pi}{4}\right)}{2\left(x-\frac{\pi}{8}\right)}\left(\sqrt{2}\right)=2\sqrt{2}$

59. $\lim_{x\to a}\frac{x^3-a^3}{x-a}=\lim_{x\to a}x^2+xa+a^2=3a^2$

60. $\lim_{h\to 0}\frac{2\left\{\left[1+\frac{\sin h}{4}\right]^{1/2}-1\right\}}{h}=li2\frac{\left[\frac{1}{8}\sin h+........\right]}{h}=$

61. $\lim_{\alpha\to\beta}\frac{\sin\left(\alpha+\beta\right)}{\left(\alpha+\beta\right)}\frac{\sin\left(\alpha-\beta\right)}{\left(\alpha-\beta\right)}=\frac{\sin\left(2\beta\right)}{\left(2\beta\right)}$

62. $$\lim_{x\to 0}\left(\frac{\tan 4x}{x}-2\right)\bigg/\left(6-\frac{\sin 3x}{x}\right)=(4-2)/(6-3)=2/3$$

63. $$\lim_{x\to 1}\frac{\left(5-\sqrt{26-x^2}\right)\left(5+\sqrt{26-x^2}\right)}{\left(5+\sqrt{26-x^2}\right)(x-1)}=\lim_{x\to 1}\frac{\left(x^2-1\right)}{(x-1)}\frac{1}{10}=\frac{1}{5}$$

64. $$\lim_{x\to -\infty}\frac{|x|\left(1-\frac{1}{x^2}+\sqrt{1-\frac{2}{x^2}}\right)}{x\left(1+\frac{1}{x}\right)}=(-1)\times 2=-2$$

65. $$\lim_{x\to -\infty}\frac{\sqrt{x^2-1}+\sqrt{x^2-2}}{x+1}=-\lim_{y\to 0^-}\frac{\sqrt{1-y^2}+\sqrt{1-2y^2}}{1+y}=2$$