# ELLIPSE

## EXERCISE 2(B)

**Q.1 (A)(B)**

Given $\frac{x^2}{8}+\frac{y^2}{2}=1 \quad ---(i)$ & $y^2=4x \quad ---(ii)$

Equation of the tangent to the parabola will be $y=\frac{x}{t}+t \quad ---(iii)$

Equation of the tangent to the ellipse will be $\frac{x}{2\sqrt{2}}\cos\theta+\frac{y}{\sqrt{2}}\sin\theta=1 \quad ---(iv)$

Comparing (iii) & (iv) we get $\frac{\cos\theta}{2\sqrt{2}}=-\frac{1}{t^2}$ , $\frac{\sin\theta}{\sqrt{2}}=\frac{1}{t}$

$\Rightarrow \frac{8}{t^2}+\frac{2}{t^2}=1$ $i.e.\ t=\pm 2$

Hence equations of tangents are $y=\pm\frac{x}{2}\pm 2$

**Q.2 (A)(B)**

Equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, so equation of Latus Rectum will be $x=\pm ae$ and end points of L.R. will be $\left(\pm ae, \pm\frac{b^2}{a}\right)$

Also $b^2=a^2-a^2e^2$

Now $x^2=a^2e^2$ & $\pm ay=b^2$ gives $x^2\pm ay=a^2$.

**Q.3 (A)(C)(D)**

(i) $a^2=9$ & $b^2=5$.

Hence equation of director circle is $x^2+y^2=14$

(ii) $a^2=25$ & $b^2=36,\ (a<b)$.

Sum of focal distances of any point = length of major axis i.e. 12.

(iii) General equation of tangent to the parabola is $y=\frac{x}{t}+at$.

Equation of perpendicular line from focus ($a$, 0) is $y + t\,(x - a) = 0$.

So point of intersection is $(0, at)$.

(iv) Points on the ellipse are

$P(\cos\theta, b\sin\theta)$, $Q(a\cos(\theta+\alpha), b\sin(\theta+\alpha))$, $O(0,0)$

Hence area of $\Delta POQ = \frac{1}{2}\left|a\cos\theta\, b\sin(\theta+\alpha) - a\cos(\theta+\alpha)\, b\sin\theta\right|$ *i.e.* $ab\sin\alpha$.

**Q.4 (A)(C)**

Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Equations of tangents at the points $\left(\pm ae, \pm a\left(1-e^2\right)\right)$ will be

$\pm\frac{ea}{a^2}x \pm \frac{a(1-e^2)}{b^2}y = 1 \ or\ \pm ex \pm y = a$

As $b$ is variable hence $e$ will also be variable and $a$ is constant hence the tangent always passes through (0, $a$) or (0, $-a$)

**Q.5 (A)(C)**

For the ellipse $\frac{x^2}{25} + \frac{4y^2}{25} = 1$ at y = 2 , $x = \sqrt{\left(1 - \frac{16}{25}\right)25} = \pm 3$

Hence coordinates of P are ($\pm$ 3, 2)

Now Equation of tangent are $\pm 3x + 8y = 25$.

**Q.6 (B)(C)**

For the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, $a = 5 \,\&\, b = 4 \Rightarrow e = \sqrt{\frac{a^2 + b^2}{b^2}} = \frac{3}{5}$

Hence focus $\equiv (3,0)$ & $(-3,0)$

Now let equation of the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

As it passes through (3,0) , (–3,0) hence $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$.

Given that eccentricity of the hyperbola, $e_2 = \frac{5}{3}$, hence $b^2 = 9\left(\frac{25}{7} - 1\right) = 16$ .

Therefore equation is $\frac{x^2}{9}-\frac{y^2}{25}=1$ and coordinates of its focus are $(\pm 5, 0)$.

**Q.7 (A)(B)**

As the line cuts equal intercept hence slope must be 1 or –1.

Equation of the ellipse is $\frac{x^2}{25}+\frac{y^2}{16}=1$ thus equation of tangent will be $\frac{x}{5}\cos\theta+\frac{y}{4}\sin\theta=1$

Now $-\frac{4}{5}\cot\theta=\pm 1$

$\Rightarrow \tan\theta=\pm\frac{4}{5}$.

$\Rightarrow \sin\theta=\frac{4}{\sqrt{41}}$ & $\cos\theta=\pm\frac{5}{\sqrt{41}}$

So, equation of tangent is $\pm x \pm y=\sqrt{41}$.

**Q.8 (B)(C)**

Let equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

As it passes through (4, –1) hence $\frac{16}{a^2}+\frac{1}{b^2}=1$ ----(1)

So equation of ellipse becomes $\frac{x^2}{a^2}+y^2\left(1-\frac{16}{a^2}\right)=1$.

putting $x=10-4y$ in the equation we get

$$\frac{(10-4y)^2-16y^2}{a^2}+y^2=1$$

Given that $x + 4y – 10 = 0$ is a tangent hence roots of above equation must be real

$\Rightarrow 90^2 – 4a^2(100 – a^2) = 0$ Or $4a^4 – 400a^2 + 6400 = 0$.

Therefore $a^2 = 80$ or $a^2 = 20$ and $b^2=\left(\frac{1}{1-\frac{16}{20}}\right)=5$ or $b^2=\left(\frac{1}{1-\frac{16}{80}}\right)=\frac{5}{4}$

So required equation is $\frac{x^2}{80}+\frac{4y^2}{5}=1$ or $\frac{x^2}{20}+\frac{y^2}{5}=1$.

**Q.9 (A)(B)**

Slope of the tangent = – 4.

For the ellipse $\frac{x^2}{3}+y^2=1$, $a^2 = 3$ & $b^2 = 1$.

Equation of tangent is slope from is $y=mx\pm\sqrt{m^2a^2+b^2}$

So, Equation of tangent is $y=-4x\pm\sqrt{16\times3+1}$

Or $y+4x\pm7=0$.

**Q.10 (B)(D)**

Equation of ellipse is $4x^2+9y^2=1$.

Slope of tangent = $\frac{8}{9}$.

Equation of tangent will be $y=\frac{8}{9}x\pm\sqrt{\left(\frac{8}{9}\right)^2\times\frac{1}{4}+\left(\frac{1}{9}\right)}$ or $9y-8x=\pm5$ ---(i).

Equation of tangent at $(x_1, y_1)$ is $4x_1x+9y_1y=1$ ---(ii)

Comparing (i) & (ii) we get, $x_1=\frac{(-8)}{\pm5\times4}=\pm\frac{2}{5}$ & $y_1=\mp\frac{1}{5}$.

**Q.11 (A)(B)(D)**

Equation of ellipse is $\frac{x^2}{6}+\frac{y^2}{2}=1$. Let the point be $\left(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta\right)$

Hence by distance formula $6\cos^2\theta+2\sin^2\theta=4$ or $\cos\theta=\pm\frac{1}{\sqrt{2}}$

So, $\theta=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$.

**Q.12 (A)(D)**

Equation of the ellipse is $\frac{x^2}{9}+\frac{y^2}{4}=1$, hence foci are $\left(\pm\sqrt{5},0\right)$

Let the point P be $(3\cos\theta, 2\sin\theta)$, then $\Delta_{PF_1F_2}=\left|\frac{1}{2}\times 2\sqrt{5}\times 2\sin\theta\right|$.

Now $\left|\frac{2\sqrt{5}}{3}\sin\theta\right|=\sqrt{10}\Rightarrow \sin^2\theta=\frac{1}{2}$.

Coordinates of the point P are $\left(\frac{3}{\sqrt{2}},\sqrt{2}\right)$ or $\left(-\frac{3}{\sqrt{2}},-\sqrt{2}\right)$.

**Q.13 (A)(C)**

$x^2\tan^2\alpha+y^2\sec^2\alpha=1\Rightarrow \frac{x^2}{\cot^2\alpha}+\frac{y^2}{\cos^2\alpha}=1$

Hence $b^2=\cos^2\alpha$ & $a^2=\cot^2\alpha$

Latus Rectum = $\frac{2\text{b}^2}{\text{a}}$ therefore $\frac{2\cos^2\alpha}{|\cot\alpha|}=\frac{1}{2}$

$\Rightarrow 4\cos^2\alpha.\sin^2\alpha=\frac{1}{4}$ or $\sin 2\alpha=\frac{1}{2}$.

Hence $\alpha=\frac{\pi}{12},\frac{5\pi}{12},\frac{7\pi}{12},\frac{11\pi}{12}$

**Q.14 (A)(B)(D)**

Given $\frac{x^2}{\tan^2\alpha}+\frac{y^2}{\sec^2\alpha}=1$, hence $a^2=\sec^2\alpha$ & $b^2=\tan^2\alpha$.

Now $\tan^2\alpha=\sec^2\alpha(1-e^2)$ gives

$e^2=\frac{\sec^2\alpha-\tan^2\alpha}{\sec^2\alpha}$ *or* $e=|\cos\alpha|$.

vertex $:\left(0,\pm\sec\alpha\right)$.

Focus $:\left(0,\pm\sec\alpha\cos\alpha\right)$ *or* $\left(0,\pm1\right)$.

length of Latus Rectum $=\frac{2\times\tan^2\alpha}{\sec^2\alpha}$ *or* $2\sin^2\alpha$.

Hence eccentricity, ordinate of vertex and latus rectum vary with $\alpha$.

**Q.15 (A)(B)**

Equation of tangent of slope m is $y = mx \pm \sqrt{a^2m^2 + b^2}$.

Slope is given as $\frac{1}{3}$, hence equation will be $y = \frac{x}{3} \pm \sqrt{\frac{a^2}{9} + b^2}$ *or* $3y = x \pm \sqrt{a^2 + 9b^2}$.

For this equation be normal to the circle it must pass through (–1,–1)

So, $-3 = -1 \pm \sqrt{a^2 + 9b^2}$ *or* $a^2 + 9b^2 = 4$

Now by $A.M. \geq G.M.$, $\frac{a^2 + 9b^2}{2} \geq \sqrt{a^2 \times 9b^2} \Rightarrow 3ab \leq 2$.

Further $a^2 + 9a^2(1 - e^2) = 4$ or $e^2 = \left(\frac{10a^2 - 4}{9a^2}\right)$.

As $0 < e^2 < 1$ hence $0 < \frac{10a^2 - 4}{9a^2} < 1$ or $a^2 > \frac{2}{5}$.

Also as $a^2 + 9b^2 = 4$, hence $a^2 < 4$.

Range of values of a, $\left(\sqrt{\frac{2}{5}}, 2\right)$.

**Q.16 (A)(C)**

Equation of tangent to ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$ at $P(a\cos\phi, b\sin\phi)$ is

$\frac{x}{4}\cos\phi + \frac{y}{2}\sin\phi = 1$.

It passes through (4,2) hence $\cos\phi + \sin\phi = 1$ or $\sin\left(\phi + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\phi + \frac{\pi}{4} = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ $\Rightarrow \phi = 0, \frac{\pi}{2}$.

**Q.17 (A)(D)**

Equation of tangent to $16x^2 + 11y^2 = 256$ at $\left(4\cos\phi, \frac{16}{\sqrt{11}}\sin\phi\right)$ is

$$\frac{x}{4}\cos\phi + \frac{y\sqrt{11}}{16}\sin\phi = 1.$$

It is also tangent to circle $x^2 + y^2 - 2x - 15 = 0$ hence its distance from center (1,0) must be equal to radius 4.

$$\Rightarrow \frac{\left|\frac{\cos\phi}{4} - 1\right|}{\sqrt{\left(\frac{\cos\phi}{4}\right)^2 + \left(\frac{\sqrt{11}\sin\phi}{16}\right)^2}} = 4 \text{ or } \frac{4\left|\cos\phi - 4\right|}{\sqrt{16\cos^2\phi + 11\sin^2\phi}} = 4$$

Or $\cos^2\phi - 8\cos\phi + 16 = 16\cos^2\phi + 11\sin^2\phi$

$\Rightarrow 4\cos^2\phi + 8\cos\phi - 5 = 0$ or $\cos\phi = \frac{1}{2}$

Therefore $\phi = \pm\frac{\pi}{3}$ .

**Q.18 (A)(B)**

If it is an ellipse , sum of the distance of the foci from the origin = 2a

Thus $\sqrt{12^2 + 5^2} + \sqrt{24^2 + 7^2} = 2a \Rightarrow a = 19$.

Distance between the foci, $2ae = \sqrt{(24-5)^2 + (12-7)^2} \Rightarrow ae = \frac{\sqrt{386}}{2}$ .

Hence $e = \frac{\sqrt{386}}{38}$ .

If it is a hyperbola, diff of distance of the foci from the origin = 2a

Thus $\left|\sqrt{12^2 + 5^2} - \sqrt{24^2 + 7^2}\right| = 2a \Rightarrow a = 6$

Now $2ae = \sqrt{386} \Rightarrow e = \frac{\sqrt{386}}{12}$ .

**Q.19 (A)(B)(C)(D)**

Equation of tangent to $y^2 = 4x$ at P(t) is $x - ty + t^2 = 0$ .

Equation of normal at $\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$ for ellipse $\frac{x^2}{5} + \frac{y^2}{4} = 1$ is

$$\frac{x\sin\theta}{2}-\frac{y\cos\theta}{\sqrt{5}}=\left(\frac{\sqrt{5}}{2}-\frac{2}{\sqrt{5}}\right)\sin\theta\cos\theta \quad \text{or} \quad x-\frac{2\cot\theta}{\sqrt{5}}y-\frac{\cos\theta}{\sqrt{5}}=0$$

So, $t=\frac{2\cot\theta}{\sqrt{5}}$ & $2t^2=-\frac{\cos\theta}{\sqrt{5}}$ or $\left(\frac{2}{\sqrt{5}}\cot\theta\right)^2=-\frac{\cos\theta}{\sqrt{5}}$.

$$4\cos^2\theta+\sqrt{5}\cos\theta\left(1-\cos^2\theta\right)=0.$$

Hence $\cos\theta=0,\ -\frac{2}{\sqrt{5}}$.

Therefore $\theta=\frac{\pi}{2},\frac{3\pi}{2}\ \&\ t=0,\frac{1}{\sqrt{5}}$.

**Q.20 (B)(C)**

Equation of ellipse is $\frac{x^2}{4}+y^2=1$ hence $e=\frac{\sqrt{3}}{2}$.

End points of latus rectum are $(x_1,y_1)\equiv\left(-\sqrt{3},-\frac{1}{2}\right)$ & $(x_2,y_2)\equiv\left(\sqrt{3},-\frac{1}{2}\right)$

Same are the end points of latus rectum of parabola, hence focus of the parabola is $\left(0,-\frac{1}{2}\right)$

and $a=\frac{\sqrt{3}}{2}$. Now vertex $\left(0,-\frac{1}{2}\pm\frac{\sqrt{3}}{2}\right)$ hence equation is

$$(x-2)^2=-4\times\frac{\sqrt{3}}{2}\left(y-\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\right) \quad \text{or} \quad x^2+2\sqrt{3}y=3-\sqrt{3}.$$

And $(x-0)^2=\frac{4\sqrt{3}}{2}\left(y-\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)\right)$ or $x^2-2\sqrt{3}y=\sqrt{3}+\sqrt{3}$.

**PASSAGE - I**

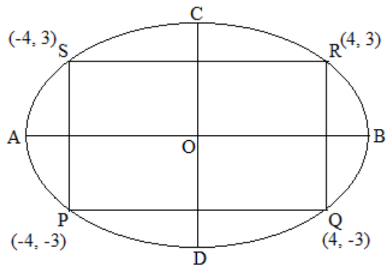
**Q.21 (D)**

Given PS = 8 & RS = 6

Co-ordinates of R are (4,3)

Distance of R from center = 5 units

Vertex could not be (5,0)




**Q.22 (B)**

If four point are co-normal the sum of eccentric angles = $(2n-1)\pi$ .

Eccentric angle of A,B,C **&** D are $0, \frac{\pi}{2}, \pi \; \& \; \frac{3\pi}{2}$

If eccentric angle of R is $\theta$, then

Eccentric angles of Q, R,S will be $\pi-\theta, \; \pi+\theta \; \& \; 2\pi-\theta$

Now set of co-normal points are (A,C,R,S),(A,C,Q,P), (B,D,Q,R) **&** (B,D,S,P)

**Q.23 (A)**

Let ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Substituting (4, 3) gives $\frac{4^2}{a^2}+\frac{3^2}{b^2}=1$ …(i)

Given eccentricity = $\frac{1}{2}$, hence $b^2=a^2\left(1-\frac{1}{4}\right)$ …(ii)

$\Rightarrow a=\sqrt{28} \;\; b=\sqrt{21}$

Area = $\pi ab = 14\pi\sqrt{3}$

<u>**PASSAGE – 2**</u>

**Q.24 (C)**

Since product of length of perpendiculars, $p_1p_2 = b^2$ (geometrical property of ellipses)

$\therefore$ By A.M. $\geq$ G.M. , $\frac{p_1+p_2}{2} \geq \sqrt{p_1p_2}$

Hence $p_1+p_2 \geq 2b$ .

**Q.25 (B)**

Product of length of perpendiculars = $b^2$

$$\Rightarrow b^2 = \left|\frac{255-0+2\sqrt{10}}{\sqrt{5}}\right| \times \left|\frac{-255-0+2\sqrt{10}}{\sqrt{5}}\right|$$

Or $b^2 = 4$.

Also distance between foci = $2ae = 2\sqrt{5}$

$\Rightarrow a^2 - b^2 = 5$

$\because b^2 = 4$ hence $a^2 = 9$.

Equation of the ellipse is $\frac{x^2}{9} + \frac{y^2}{4} = 1$

**Q.26 (D)**

By property that Normal at any point P bisect the angle subtended by P at focal segment.

**PASSAGE - 3**

**Q.27 (B)**

Equation of the ellipse is $2(x-2)^2 + (y-1)^2 = 8$

Or $\frac{(x-2)^2}{4} + \frac{(y-1)^2}{8} = 1$

$a^2 = 4$ & $b^2 = 8 \Rightarrow e^2 = \sqrt{1-\frac{4}{8}}$ or $e = \frac{1}{\sqrt{2}}$ (a < b).

**Q.28 (A)**

Center is (2, 1) and be = 2 gives the Foci as

$(2, 1 \pm 2)$ or $(2, 3)$ & $(2, -1)$

**Q.29 (D)**

Length of latus rectum = $\frac{2a^2}{b}$

Or L.R. = $2\sqrt{2}$

**PASSAGE - 4**

General equation of tangent to $y^2 = 4x$ at $P_1\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ is $y = mx + \frac{1}{m}$.

If it is tangent to ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$, then $\left(\frac{1}{m}\right)^2 = 8m^2 + 2$ or $8m^2 + 2m^2 - 1 = 0$

Hence $m = \pm\frac{1}{2}$.

Now $P_1\left(\frac{1}{(1/2)^2}, \frac{2}{1/2}\right)$ or $(4, 4)$ & $E_1(-2, 1)$

Similarly $P_2(4, -4)$ & $E_2(-2, -1)$.

**Q.30 (B)**

$$P_1E_2 = \sqrt{(4+2)^2 + (4+1)^2} = \sqrt{61}$$

**Q.31 (A)**

Equation of chord $P_1P_2$ is $x = 4$.

Let M is (h, k)

Equation of chord of contact yk = 2(x + h) … by T = 0.

or ky - 2x = 2h

On comparison with x = 4 we get M as (–4, 0).

$$\frac{ME_1}{MP_1} = \frac{\sqrt{(-4+2)^2 + 1}}{\sqrt{(-4-4)^2 + (0-4)^2}} = \frac{\sqrt{5}}{\sqrt{80}} = \frac{1}{4}$$

**Q.32 (A)**

Area of Quadrilateral $P_1E_1E_2P_2$ (trapezium)

Hence Area $= \frac{1}{2} \times (E_1E_2 + P_1P_2) \times$ (distance between $P_1P_2$ & $E_1E_2$)

$= \frac{1}{2} \times (2+8) \times (6) = 30.$

## ASSERTION AND REASONS

**Q.33 (A)**

A line y = mx + c, touches ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $c^2=a^2m^2+b^2$.

$\therefore y=-\frac{l}{m}x-\frac{n}{m}$ is tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\left(-\frac{n}{m}\right)^2=a^2\left(-\frac{l}{m}\right)^2+b^2$

or $n^2=a^2l^2+b^2m^2$.

**Q.34 (C)**

$\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha \Rightarrow \cos^{-1}\frac{x}{a}=\cos^{-1}(\cos\alpha)-\cos^{-1}\frac{y}{b}$

Or $\frac{x^2}{a^2}-\frac{2xy\cos\alpha}{ab}+\frac{y^2}{b^2}=\sin^2\alpha$

This equation always represent a conic only if $\sin\alpha\neq 0$.

**Q.36 (B)**

equation of normal at $\left(ae,\frac{b^2}{a}\right)$ is $\frac{a^2x}{ae}-\frac{b^2y}{b^2/a}=a^2-b^2$.

or $x-ey=ae^3$

Given that normal passes through (0, –b) hence $ab=a^2e^2$

or $e^2=\frac{b}{a}$ & $e^4=\frac{b^2}{a^2}$

$e^4=1-e^2$

Hence $e=\sqrt{\frac{\sqrt{5}-1}{2}}$

Q.37 (C)

By reflection property of ellipse.

## MATRIX MATCH

**Q.38** $(A)\rightarrow(r),(B)\rightarrow(q),(C)\rightarrow(p),(D)\rightarrow(s)$

(A) Equation of chord with mid – point (h, k), by T = $S_1$,

$$\frac{xh}{a^2}+\frac{ky}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

This chord passes through then, (0, b)

$$\frac{kb}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}.$$

Replacing (h, k) by (x, y) gives required locus as $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{y}{b}=0$.

$\frac{x^2}{a^2}+\frac{\left(y-\frac{b}{2}\right)^2}{b^2}=\frac{1}{4}$ which is an ellipse.

(B) Length of major axis 2a is Given. Let $h=ae,\ k=\frac{b^2}{a}$

$k=\frac{b^2}{a}(1-e^2)$ or $\frac{k}{a}=1-\frac{h^2}{a^2}$

Required locus is $\frac{x^2}{a^2}=1-\frac{y}{a}$ which is a parabola.

(C) Locus is Auxiliary circle.

(D) Let equation of line be $\frac{x}{a}+\frac{y}{b}=1$

This line meets x – axis at A(a, 0) & y – axis at B(0, b)

Also let mid – point be (h, k), then A(2h, 0) & B(0, 2k), hence a = 2h & b = 2k.

$\frac{x}{2h}+\frac{x}{2k}=1$ *or* $\frac{\alpha}{2x}+\frac{\beta}{2y}=1$ which is a hyperbola.

**Q.39** $(A)\rightarrow(s),(B)\rightarrow(q),(C)\rightarrow(p),(D)\rightarrow(r)$

(A) points (7, 0) & (0, –5) lie on ellipse, then

$$\frac{7^2}{a^2}+\frac{0}{b^2}=1\ \&\ \frac{0}{a^2}+\frac{25^2}{b^2}=1$$

$\Rightarrow a^2 = 49, \ b^2 = 25$

$e^2 = 1 - \frac{25}{49} \quad or \ \ e = \frac{2}{7}\sqrt{6}$

(B) Given $\angle SBS' = 90^0$

$\Rightarrow \tan 45^0 = \frac{OB}{OS'}$, hence $\frac{b}{ae} = 1$

or $e^2 = \frac{b^2}{a^2} = 1 - e^2$, therefore $e = \frac{1}{\sqrt{2}}$.

(C) we know that $m_1 m_2 = \frac{-b^2}{a^2}$

Hence $1 \times \frac{-2}{3} = \frac{-b^2}{a^2} \quad \left(1 - \frac{b^2}{a^2} = e^2\right)$

$\Rightarrow e^2 = \frac{1}{3}$ or $e = \frac{1}{\sqrt{3}}$

(D) Given $2ae = 6$ & $2b = 8$

Hence $a^2e^2 = 9, \ b^2 = 16 \Rightarrow a^2 = 16 + 9 = 25 \Rightarrow e = \frac{3}{5}$

**Q.40** $(A) \to (r), (B) \to (s), (C) \to (q), (D) \to (p)$

Given ellipse is $\frac{x^2}{5} + \frac{y^2}{4} = 1$.

a = $\sqrt{5}$, b = 2, $e^2 = 1 - \frac{4}{5} \ \Rightarrow e = \frac{1}{\sqrt{5}}$

Directrices are $x = \pm \frac{a}{e}$ or $x = \pm 5$

Equation of minor Axis : $x = 0$

Tangent at end of Major Axis : $x = \pm\sqrt{5}$

Equation of latus rectum : $x = \pm ae$ or $x = \pm 1$.