Ellipse

Exercise – 2(A)

Q.1 (C)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Equation of tangent at the end point of focal chord is $x=\pm\frac{a}{e}$

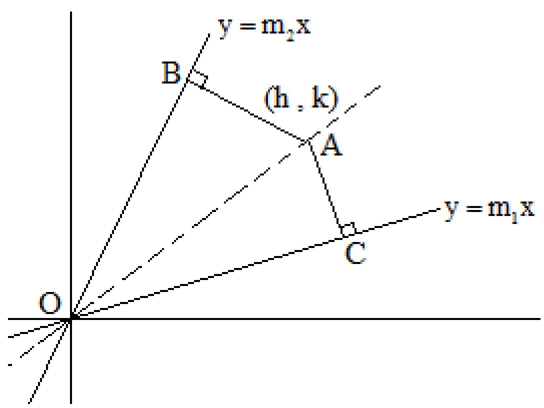
$$e=\sqrt{1-\frac{b^2}{a^2}}$$

$$\Rightarrow x=\pm\frac{a^2}{\sqrt{a^2-b^2}}$$

Q.2 (D)

OB and OC passes from origin then let the equation is $y=m_1x$ and $y=m_2x$

$$AB=\frac{|m_2h-k|}{\sqrt{1+m_2^2}}$$



Equation of $AB=\frac{y-k}{x-h}=\frac{-1}{m_2}$

$$OB=y=m_2x$$

Solving them we get the co – ordinates of B

$$B\equiv\left(\frac{h+m_2k}{1+m_2},\frac{m_2(h+m_2k)}{1+m_2^2}\right)$$

Therefore $OB=\sqrt{\frac{(h+m_2k)^2}{(1+m_2^2)}}$

Similarly $OC=\sqrt{\frac{(h+m_1k)^2}{(1+m_2^2)}}$

$$AC=\frac{|m_1h-k|}{\sqrt{1-m_1^2}}$$

Area of quad OCAB $= \frac{1}{2}\left[\frac{(m_1h-k)(m_1h+k)}{(1+m_1^2)} + \frac{(m_2h-k)(m_2h+k)}{(1+m_2^2)}\right]$

$$2c = \frac{1}{2}\left[\frac{m_1^2hk+m_1h^2-m_1k^2-kh}{(1+m_1^2)} + \frac{m_2^2hk+m_2h^2-m_2h^2-hk}{(1+m_2^2)}\right]$$

= Hyperbola

Q.3 (B)

$P \equiv (a\cos\theta\,,\, b\sin\theta)$ & $Q \equiv \left(-a\sin\theta\,,\, b\cos\theta\right)$

Let the mid- point of PQ is R (h , k), then

$$h = a\cos\theta - a\sin\theta \Rightarrow \frac{h}{a} = \cos\theta - \sin\theta$$

$$\Rightarrow \frac{h^2}{a^2} = \cos^2\theta - \sin^2\theta - 2\cos\theta\sin\theta$$

$$\Rightarrow 2\cos\theta\sin\theta = 1 - \frac{h^2}{a^2} \qquad \ldots (1)$$

$$k = b\sin\theta + b\cos\theta \Rightarrow \frac{k}{b} = \sin\theta + \cos\theta$$

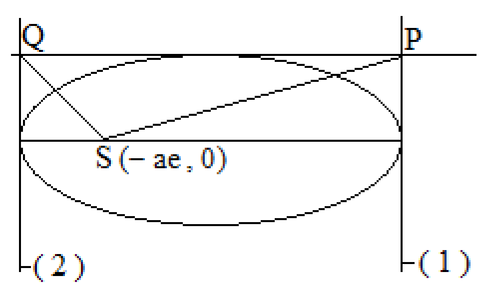
$$\Rightarrow 2\cos\theta\sin\theta = \frac{k^2}{b^2} - 1 \qquad \ldots (2)$$

From (1) and (2), $\frac{h^2}{a^2} + \frac{y^2}{b^2} = 2$

locus is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

Q.4 (A)

Let the equation of tangent be $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$



Equation of (1) is x = a

Therefore $y = b\tan\frac{\theta}{2}$

Equation of (2) is $x = -a$

Therefore $y = b\cos\frac{\theta}{2}$

slope of SQ (MSQ) $= \dfrac{b\cos\frac{\theta}{2}}{-a + ae}$

slope of SQ (MSP) $= \dfrac{b\tan\frac{\theta}{2}}{a + ae}$

MSQ $\times$ MSP $= \dfrac{-b^2}{a^2 - a^2e^2} = -1 \Rightarrow SP \perp SQ$

$\Rightarrow$ circle described on PQ as diameter will pass through the foci.

Q.5 (B)

$$\frac{x^2}{6} + \frac{y^2}{3} = 1$$

y = mx + c is the equation of tangent substituting the value of y in ellipse.

$$c^2 = a^2m^2 + b^2$$

$$\Rightarrow c^2 = 6m^2 + 3$$

Q.6 (D)

$$ax^2 + by^2 + 2gx + 2fy + c = 0$$

$$\Rightarrow a\left(x^2 + \frac{2gx}{a} + \frac{g^2}{a^2}\right) + b\left(y^2 + \frac{2fy}{b} + \frac{f^2}{b^2}\right) = -c - \frac{f^2}{b^2} - \frac{g^2}{a^2}$$

$$\Rightarrow e = \sqrt{1 - \frac{a}{b}} = \sqrt{\frac{b-a}{b}}$$

Q.7 (C)

Equation of tangent from point (α , β)

$$\frac{x\alpha}{a^2} + \frac{y\beta}{b^2} = 1$$

This touches the circle $x^2 + y^2 = c^2$

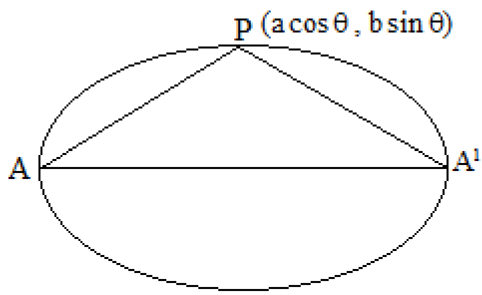
Therefore perpendicular distance from center = radius of circle.

$$\frac{|-1|}{\sqrt{\frac{\alpha^2}{a^4}+\frac{\beta^2}{b^4}}} = c$$

$$\Rightarrow \frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{c^2}$$

Q.8 (A)

Area of $\Delta APA' = \frac{1}{2} \times 2a \times b\sin\theta$



$= ab\sin\theta$

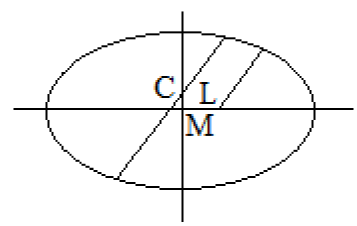
Area will be maximum when

$\sin\theta = 1$

$\therefore$ Max. Area = ab.

Q.9 (B)

Equation of normal



$ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2 - b^2$

at y = 0 we get co – ordinates of L

$$x = \frac{(a^2-b^2)\cos\theta}{a}$$

$$L \equiv \left(\frac{(a^2-b^2)\cos\theta}{a}, 0\right)$$

at x = 0 we get co – ordinate of M

$$y = -\frac{(a^2-b^2)\sin\theta}{b}$$

$$M \equiv \left(0, -\frac{(a^2-b^2)\sin\theta}{b}\right)$$

Therefore $a^2CL^2 + b^2CM^2$

$$= \frac{a^2(a^2-b^2)\cos^2\theta}{a^2} + \frac{(a^2-b^2)^2b^2\sin^2\theta}{b^2}$$

$$= (a^2-b^2)^2$$

Q.10 (C)

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

$$y_1+y_2=3$$

Let the two points be $(x_1\ ,\ y_1)\equiv(a\cos\alpha, b\sin\alpha)$ and $Q(x_2\,, y_2)\equiv(a\cos\beta\ ,\ b\sin\beta)$

Then mid – point of PQ is R (h , k) such that

$$h=\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} \qquad \text{……….(1)}$$

$$k=\frac{a\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} \qquad \text{……….(2)}$$

As $y_1+y_2=3$

$$\Rightarrow b(\sin\alpha+\sin\beta)=3$$

$$\Rightarrow \sin\alpha+\sin\beta=1 \qquad \text{……….(3)}$$

Therefore form (1) , (2) and (3) we get

$$\frac{h^2}{a^2}+\frac{k^2}{b^2}=\frac{1}{\cos^2\left(\frac{\alpha-\beta}{2}\right)}$$

$$\frac{x^2}{25}+\frac{y^2}{9}=\frac{1}{\cos^2\left(\frac{\alpha-\beta}{2}\right)}$$

From (3)

$$2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=1$$

$$\Rightarrow 2\left[\frac{k\cos\left(\frac{\alpha-\beta}{2}\right)}{a}\right]\cos\left(\frac{\alpha-\beta}{2}\right)=1$$

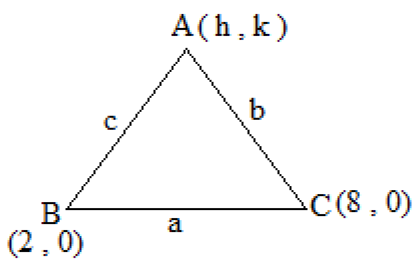
$$\Rightarrow \cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{0}{2k} = \frac{3}{2y}$$

$$\therefore \text{locus is } \frac{x^2}{25} + \frac{y^2}{9} = \frac{1}{\frac{9}{(2y)^2}}$$

$$\Rightarrow 9x^2 + 25y^2 = 150y$$

Q.11 (B)

$$4\tan\frac{B}{2}\cdot\tan\frac{C}{2} = 1$$



$$\Rightarrow \sqrt{\frac{s(s-b)}{(s-a)(s-c)}\cdot\frac{s(s-c)}{(s-a)(s-b)}} = 4$$

$$\Rightarrow \frac{s}{s-a} = 4$$

$$\Rightarrow s = 45 - 4a$$

$$\Rightarrow 3s = 4a$$

$$\Rightarrow 3a + 3b + 3c = 8a$$

$$3b + 3c = 5a$$

$$3\left(\sqrt{(x-2)^2+y^2} + \sqrt{(x-3)^2+y^2}\right) = 5\left(\sqrt{(c-2)^2+(a-0)^2}\right)$$

$$\sqrt{(x-2)^2+y^2} + \sqrt{(x-8)^2+y^2} = 10$$

$$\Rightarrow \sqrt{(x-2)^2+y^2} = 10 - \sqrt{(x-8)^2+y^2}$$

$$\Rightarrow 40 - 3x = 5\sqrt{(x-8)^2+y^2}$$

$$\Rightarrow \frac{(x-5)^2}{25} + \frac{y^2}{16} = 1$$

Q.12 (A)

Locus of point of intersection of tangents is given by

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\sec^2\left(\frac{\alpha-\beta}{2}\right)$$

Hence $\alpha-\beta=60^\circ$

$\sec^2 30^\circ=3$

$\therefore$ Locus is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{4}{3}$

Q.13 (D)

Let the equation be $y=mx+\sqrt{a^2m^2+b^2}$

m = 1 (equal angles)

$y=x+\sqrt{a^2+b^2}$

Perpendicular from origin.

$$=\sqrt{\frac{a^2+b^2}{2}}$$

Q.15 (A)

$4(x-1)^2+9(y-2)^2=36$

$$\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1$$

The circle $(x-1)^2+(y-2)^2=1$ is a point circle at center of ellipse.

So the length of common chord is 0.

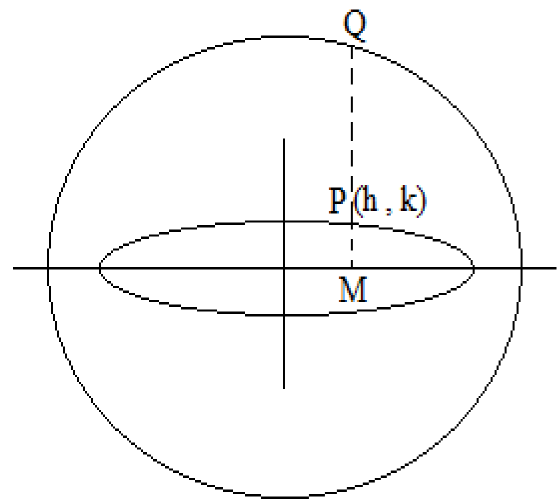
Q.16 (B)

2ae = 8 & $\frac{2a}{e}=25$

$\Rightarrow 4a^2=200$ or $2a=10\sqrt{2}$

Q.17 (C)

Let the co – ordinates P be (h, k)



Equation of auxiliary circle is $x^2 + y^2 = 25$

Therefore co – ordinates of Q $\equiv \left(\sqrt{25 - k^2}\ ,\ k\right)$

Equation of normal from P

$y - k = \frac{25k}{9h}(x - h)$ ....(1)

Equation of normal from Q

$y = \frac{k}{\sqrt{25 - y^2}}x$ ....(2)

Solving (1) and (2) we get

$h^2 + k^2 = 64$

locus is $x^2 + y^2 = 64$

Q.18 (C)

From observation only point $\left(\frac{34}{9}, \frac{11}{9}\right)$ will lie on line x + y – 5 = 0.

Alternately:

Product of perpendiculars from foci on any tangent is b^2.

Hence $\left|\frac{1-1-5}{\sqrt{2}}\right|\left|\frac{2-1-5}{\sqrt{2}}\right| = b^2 \Rightarrow b^2 = 10$

Distance between the foci = 2ae = 1

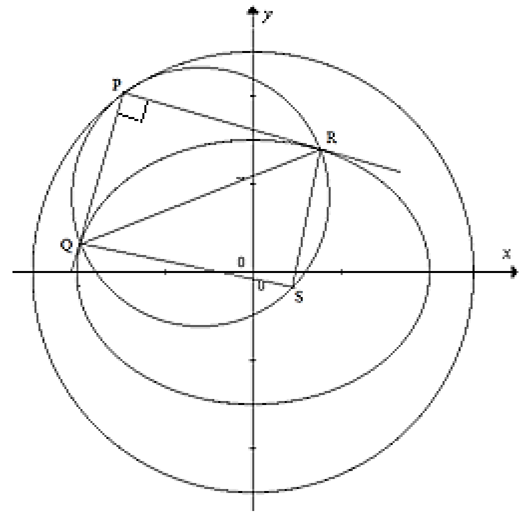
Now $a^2 = b^2 + a^2e^2 = 10 + \frac{1}{4} = \frac{41}{4}$

Also center will be midpoint of foci i.e. $\left(\frac{3}{2}, -1\right)$

Hence equation of ellipse is $\frac{4\left(x - \frac{3}{2}\right)^2}{41} + \frac{(y+1)^2}{10} = 1$.

Now find point of contact with x + y = 5.

Q.19 (C)



As PQRS is a parallelogram as well as a cyclic quadrilateral, hence it must be a rectangle, which implies $x^2 + y^2 = 25$ must be director circle of the ellipse, $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$.

$16 + b^2 = 25$ or $b^2 = 9$

$\Rightarrow e = \sqrt{\frac{16-9}{16}} = \frac{\sqrt{7}}{4}$

Q.20 (D)

Chord of contact of the tangents drawn from (8, 27) to $\frac{x^2}{4} + \frac{y^2}{9} = 1$ will be,

$\frac{8x}{4} + \frac{27y}{9} = 1$ or $2x + 3y = 1$.

Homogenizing the equation of the ellipse using this gives

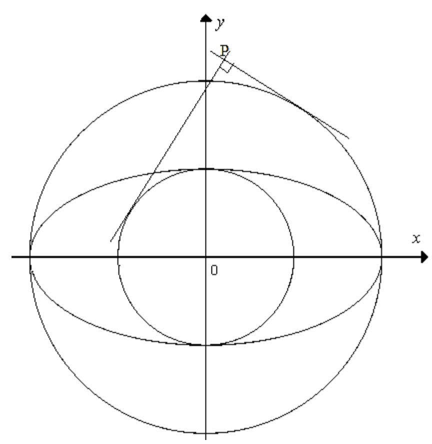
$\frac{x^2}{4} + \frac{y^2}{9} = (2x+3y)^2$ or $135x^2 + 432xy + 320y^2 = 0$

Angle between this pair of lines will be given by

$\tan\theta = \frac{2\sqrt{216^2 - 135 \times 320}}{135 + 320} = \frac{48\sqrt{6}}{455}$ or $\theta = \tan^{-1}\frac{48\sqrt{6}}{455}$.

Q.21 (B)

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then



the circle on major axis as diameter will be $x^2 + y^2 = a^2$

and

the circle on minor axis as diameter will be $x^2 + y^2 = b^2$

Any tangent with slope m to former circle will be

$y = mx + a\sqrt{1+m^2}$ or $y - mx = a\sqrt{1+m^2}$ and

a perpendicular tangent to the later circle will be

$y = -\frac{1}{m}x + b\sqrt{1+\frac{1}{m^2}}$ or $x + my = b\sqrt{1+m^2}$

From the two equations we get $m = \frac{ay - bx}{by + ax}$.

Substituting this value of m in former equation of tangent gives

$$x(by+ax)+y(ay-bx)=a\sqrt{(by+ax)^2+(ay-bx)^2}$$

or $x^2+y^2=a^2+b^2$.

Hence the required locus is the director circle.

Q.22 (A)

For circle on PF as diameter

$$\text{Center}:\left(\frac{a\cos\theta+ae}{2},\frac{b\sin\theta}{2}\right)\ \&\ \text{radius}=\frac{a-ae\cos\theta}{2}$$

For the auxiliary circle

Center : (0, 0) & radius = a

Distance between the centers =

$$\sqrt{\left(\frac{a\cos\theta+ae}{2}\right)^2+\left(\frac{b\sin\theta}{2}\right)^2}=\frac{1}{2}\sqrt{a^2\cos^2\theta+a^2e^2+b^2\sin^2\theta+2a^2e\cos\theta}$$

$$=\frac{1}{2}\sqrt{a^2e^2\cos^2\theta+a^2+2a^2e\cos\theta}$$

$$=\frac{a+ae\cos\theta}{2}$$ = difference of radii.

(Note : Circle on any focal radius as diameter touches the auxiliary circle)

Q.23 (A)

Let the midpoint of any chord of $\frac{x^2}{10}+\frac{y^2}{6}=1$ be (h, k)

then equation of chord will be $\frac{hx}{10}+\frac{ky}{6}=\frac{h^2}{10}+\frac{k^2}{6}$ $\{T=S_1\}$

Comparing this with 2x – y + 3 = 0 gives $\frac{h}{20}=-\frac{k}{6}=-\frac{\frac{h^2}{10}+\frac{k^2}{6}}{3}$

or $h=-\frac{30}{23},k=\frac{9}{23}$.

Q.24 (D)

Any tangent to given ellipse will be $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$.

Homogenizing the equation of the auxiliary circle using the equation of tangents gives

$$x^2+y^2=a^2\left(\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}\right)^2$$

$$\Rightarrow\left(b^2\sin^2\theta\right)x^2-\left(2ab\sin\theta\cos\theta\right)xy+\left(b^2-a^2\sin^2\theta\right)y^2=0$$

Now as the chord cut off by the auxiliary circle from the tangent subtends a right angle at the origin hence $b^2\sin^2\theta+b^2-a^2\sin^2\theta=0$ (coeff. of x^2 + coeff. of y^2 = 0)

$$\Rightarrow \frac{b^2}{a^2} = \frac{\sin^2\theta}{1+\sin^2\theta}$$

$$\Rightarrow e = \sqrt{1-\frac{\sin^2\theta}{1+\sin^2\theta}} = \sqrt{\frac{1}{1+\sin^2\theta}}.$$

Q.25 (B)

Let coordinates of P be $(3\cos\theta, 2\sin\theta)$, then

$PF_1 = 3-\sqrt{5}\cos\theta$ & $PF_2 = 3+\sqrt{5}\cos\theta$

Hence $(PF_1 - PF_2)^2 = 20\cos^2\theta$

Now tangent at P will be $2x\cos\theta + 3y\sin\theta = 6$

Distance of this line from the origin = $\dfrac{6}{\sqrt{4\cos^2\theta + 9\sin^2\theta}} = \dfrac{6}{\sqrt{9-5\cos^2\theta}}$

But given distance = 3.

$\Rightarrow \dfrac{6}{\sqrt{9-5\cos^2\theta}} = 3 \Rightarrow \cos^2\theta = 1$. Hence $(PF_1 - PF_2)^2 = 20$.

Q.26 (A)

By linearly combining equations of any two curves we can get equation of curve passing through their points of intersection. Hence required circle can be obtained by

$$\left(\frac{x^2}{4}+\frac{y^2}{2}\right)+\left(\frac{x^2}{2}+\frac{y^2}{4}\right) = 2 \text{ i.e. } 3x^2+3y^2 = 8.$$

Q.27 (C)

Let the midpoint of any chord of $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2} = 1$ be (h, k)

then equation of chord will be $\dfrac{hx}{a^2}+\dfrac{ky}{b^2} = \dfrac{h^2}{a^2}+\dfrac{k^2}{b^2}$ $\quad\{T = S_1\}$

As this chord passes through (0, b) hence $\dfrac{h^2}{a^2}+\dfrac{k^2}{b^2}-\dfrac{k}{b} = 0$.

Required locus is $\dfrac{x^2}{a^2/4}+\dfrac{\left(y-\frac{b}{2}\right)^2}{b^2/4} = 1$.

Q.28

Given ellipse is $\dfrac{x^2}{16}+\dfrac{y^2}{4} = 1$, hence tangent at $A\left(\dfrac{\pi}{4}\right) \equiv \left(2\sqrt{2}, \sqrt{2}\right)$ will be

$x + 2y = 4\sqrt{2}$.

Normal at $C\left(\dfrac{3\pi}{4}\right) \equiv \left(-2\sqrt{2}, \sqrt{2}\right)$ will be

$2x + y = -3\sqrt{2}$.

Solving these together we get the point B as $\left(-\frac{10\sqrt{2}}{3}, \frac{11\sqrt{2}}{3}\right)$

Now solving $2x + y = -3\sqrt{2}$ and $\frac{x^2}{16} + \frac{y^2}{4} = 1$ gives the point D as $\left(-\frac{14\sqrt{2}}{17}, -\frac{23\sqrt{2}}{17}\right)$

$$\text{Area of } \Delta ABD = \frac{1}{2}\left\|\begin{matrix} 1 & 2\sqrt{2} & \sqrt{2} \\ 1 & -\frac{10\sqrt{2}}{3} & \frac{11\sqrt{2}}{3} \\ 1 & -\frac{14\sqrt{2}}{17} & -\frac{23\sqrt{2}}{17} \end{matrix}\right\| = \frac{1024}{51}$$

Q.29 (A)

Tangent to $\frac{x^2}{4} + y^2 = 1$ at any point will be $\frac{x}{4}\cos\theta + y\sin\theta = 1$

Chord of contact of tangents drawn to $\frac{x^2}{20} + \frac{y^2}{5} = 1$ from any point P(h, k) will be $\frac{hx}{20} + \frac{ky}{5} = 1$

Comparing the two equations gives $h = 5\cos\theta, k = 5\sin\theta$.

Hence P lies on $x^2 + y^2 = 25$ i.e. the director circle of the later ellipse.

Angle between the tangents = $\frac{\pi}{2}$.

Q.30 (B)

As tangents drawn from P(α, β) are at right angles hence P lies on the director circle i.e. $x^2 + y^2 = a^2 + b^2$, hence $\alpha^2 + \beta^2 = a^2 + b^2$.

Also normals will be mutually perpendicular.

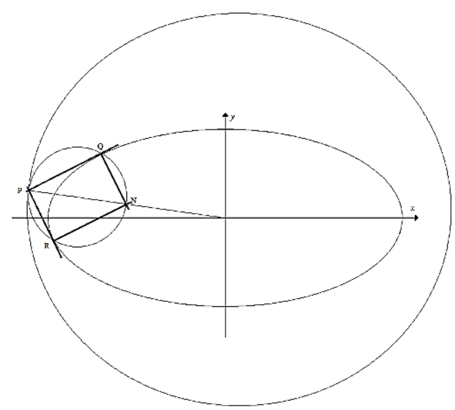
Let the point of intersection of normals be N(h, k).

Now PQNR will be a cyclic quadrilateral with circle circumscribing it will be having PN as diameter.

As this circle will be touching $x^2 + y^2 = a^2 + b^2$, hence (h, k) must lie on the line joining the origin to (α, β).

$\Rightarrow \beta h = \alpha k$

Required locus is $\beta x - \alpha y = 0$.

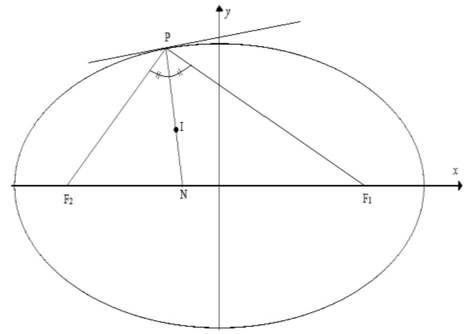


Q.31 (B)

Let P be $(a\cos\theta, b\sin\theta)$.

Also F_1 & F_2 are $(ae, 0)$ & $(-ae, 0)$

$F_1F_2 = 2ae,\ PF_1 = a(1-e\cos\theta)$ & $PF_2 = a(1+e\cos\theta)$



Now in-center will be

$$x = \frac{a(1+e\cos\theta)\times ae + a(1-e\cos\theta)\times(-ae) + 2ae\times a\cos\theta}{2a+2ae} \ \&$$

$$y = \frac{a(1+e\cos\theta)\times 0 + a(1-e\cos\theta)\times o + 2ae\times b\sin\theta}{2a+2ae}$$

$$\Rightarrow x = ae\cos\theta,\ y = \frac{be\sin\theta}{1+e} \ \text{ or } \ \cos\theta = \frac{x}{ae}, \sin\theta = \frac{y(1+e)}{be}$$

$$\Rightarrow \frac{x^2}{a^2e^2} + \frac{y^2(1+e)^2}{b^2e^2} = 1.$$

Q.32 (A)

Equation of normal in slope form : $y = mx \pm \dfrac{(a^2-b^2)m}{\sqrt{a^2+m^2b^2}}$.

For the given data, normal of slope 1 : $y = x \pm \dfrac{2\sqrt{6}}{3}$

Now coordinates of P : $\left(\mp\dfrac{2\sqrt{6}}{3}, 0\right)$ and coordinates of Q : $\left(0, \pm\dfrac{2\sqrt{6}}{3}\right)$

Also coordinates of C : (0, 0)

Now area of $\Delta CPQ = \dfrac{4}{3}$.

Q.33 (B)

Equation of PQ : $\dfrac{x}{5}\cos\dfrac{\alpha+\beta}{2} + \dfrac{y}{4}\sin\dfrac{\alpha+\beta}{2} = \cos\dfrac{\alpha-\beta}{2}$

Equation of any tangent to $\dfrac{\left(x-\dfrac{5}{2}\right)^2}{25/4} + \dfrac{y^2}{16} = 1$: $\dfrac{x-5/2}{5/2}\cos\theta + \dfrac{y}{4}\sin\theta = 1$

Comparing the two equations gives

$$\frac{\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}} = \frac{2\cos\theta}{1+\cos\theta} \ \ \& \ \ \frac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}} = \frac{\sin\theta}{1+\cos\theta}$$

$$\Rightarrow \sec\theta = \frac{2\cos\frac{\alpha-\beta}{2} - \cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha+\beta}{2}} \ \& \ \tan\theta = \frac{2\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha+\beta}{2}}$$

$$\Rightarrow \left(\frac{2\cos\frac{\alpha-\beta}{2}-\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha+\beta}{2}}\right)^2-\left(\frac{2\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha+\beta}{2}}\right)^2=1$$

$$\Rightarrow 2\cos^2\frac{\alpha-\beta}{2}-2\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}-2\sin^2\frac{\alpha+\beta}{2}=0$$

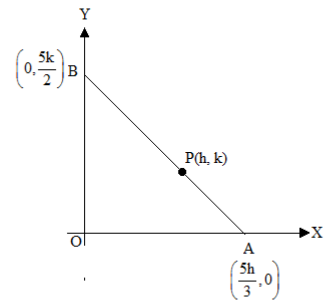
$$\Rightarrow \cos(\alpha-\beta)+\cos(\alpha+\beta)=\cos\alpha+\cos\beta$$

$$\Rightarrow 2\cos\alpha\cos\beta=\cos\alpha+\cos\beta$$

$$\Rightarrow \sec\alpha+\sec\beta=2.$$

Q.34 (C)

From adjoining figure AB = 5 gives $\frac{25h^2}{9}+\frac{25k^2}{4}=25$.



Hence the required locus is $\frac{x^2}{9}+\frac{y^2}{4}=1$

Latus rectum $=\frac{2b^2}{a}=\frac{8}{3}$

Q.35 (A)

Given $\ell^2=\frac{2(2a)^2(2b)^2}{(2a)^2+(2b)^2}$ or $\ell^2=\frac{8a^2b^2}{a^2+b^2}$

But length of diameter joining $P(a\cos\theta, b\sin\theta)$ & $Q(-a\cos\theta,-b\sin\theta)$

$$\ell=\sqrt{4a^2\cos^2\theta+4b^2\sin^2\theta}$$

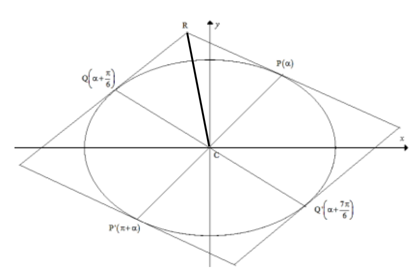
$$\Rightarrow a^2\cos^2\theta+b^2\sin^2\theta=\frac{2a^2b^2}{a^2+b^2}$$

$$\Rightarrow \cos^2\theta=\frac{b^2}{a^2+b^2} \text{ \& } \sin^2\theta=\frac{a^2}{a^2+b^2}$$

Now slope of PQ = $\frac{b\sin\theta}{a\cos\theta}=1$.

Q.36

Given $P(\alpha)\,\&\,Q\left(\alpha+\frac{\pi}{6}\right)$



Now point of intersection of tangents

|at $P(\alpha)\,\&\,Q\left(\alpha+\frac{\pi}{6}\right)$:

$$R\left(a\frac{\cos\left(\alpha+\frac{\pi}{12}\right)}{\cos\frac{\pi}{12}}, b\frac{\sin\left(\alpha+\frac{\pi}{12}\right)}{\cos\frac{\pi}{12}}\right)$$

Required area = $8\times A_{CPR}$

Q.37 (D)

Let the points be $P(\theta)$ & $Q\left(\theta+\frac{\pi}{3}\right)$, then equation of PQ will be

$$\frac{x}{3}\cos\left(\theta+\frac{\pi}{6}\right)+\frac{y}{2}\sin\left(\theta+\frac{\pi}{6}\right)=\cos\frac{\pi}{6}$$

$$\Rightarrow \frac{x}{\frac{3\sqrt{3}}{2}}\cos\left(\theta+\frac{\pi}{6}\right)+\frac{y}{\sqrt{3}}\sin\left(\theta+\frac{\pi}{6}\right)=1$$

Hence PQ will touch $\frac{4x^2}{27}+\frac{y^2}{3}=1$.

Q.38 (D)

Let the line be y = mx + c.

For being a tangent to the ellipse : $c^2=a^2m^2+b^2$

For being a tangent to the circle : $c^2=r^2\left(m^2+1\right)$

Hence $a^2m^2+b^2=r^2\left(m^2+1\right)$

$$\Rightarrow m=\pm\sqrt{\frac{r^2-b^2}{a^2-r^2}}=\tan\theta$$

Now parametric coordinates of a point at a distance p from S(ae, 0) on the line PQ will be $\left(ae+p\cos\theta, p\sin\theta\right)$

Substituting these coordinates in the equation of the circle gives

$\left(ae+p\cos\theta\right)^2+\left(p\sin\theta\right)^2=r^2$ or $p^2+\left(2ae\cos\theta\right)p+a^2e^2-r^2=0$

Roots of this equation will be SP and SQ.

As SP & SQ are measured in opposite directions from S, hence

PQ = difference of roots

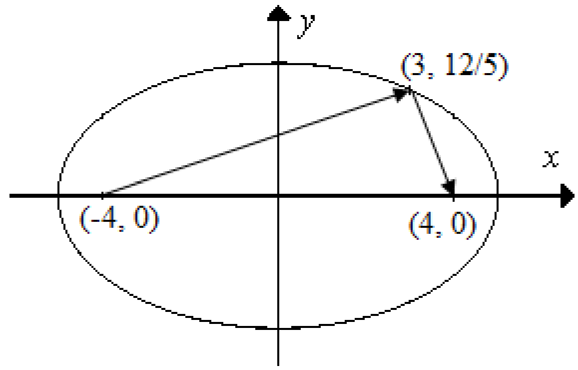
$$\Rightarrow PQ=\frac{\sqrt{4a^2e^2\cos^2\theta-4\left(a^2e^2-r^2\right)}}{2}$$

$$\Rightarrow PQ=\sqrt{r^2-a^2e^2\sin^2\theta}$$

Now $\tan^2\theta=\frac{r^2-b^2}{a^2-r^2}\Rightarrow\sin^2\theta=\frac{r^2-b^2}{a^2e^2}$

$\Rightarrow PQ=b$.

Q.39 (C)



As (-4, 0) is one focus of the given ellipse hence after reflection the line will pass through the other focus i.e. (4, 0).

Lines joining (4, 0) and $\left(3, \pm\frac{12}{5}\right)$ are

$y = \pm\frac{12}{5}(x-4)$.

Q.40 (A)

Let P be $\left(\sqrt{3}\cos\theta, \sin\theta\right)$, then A & B will be

$\left(\frac{4\sqrt{3}\cos\theta}{3}, 0\right)$ & $(0, 4\sin\theta)$ or $\left(4\sqrt{3}\cos\theta, 0\right)$ & $\left(0, \frac{4}{3}\sin\theta\right)$

Case I :

Slope of AB = $-\frac{\sqrt{3}\sin\theta}{\cos\theta}$ and slope of tangent at P = $-\frac{\cos\theta}{\sqrt{3}\sin\theta}$

$\Rightarrow -\frac{\sqrt{3}\sin\theta}{\cos\theta} = -\frac{\cos\theta}{\sqrt{3}\sin\theta}$ or $\sin\theta = \frac{1}{2}$ & $\cos\theta = \frac{\sqrt{3}}{2}$

Hence P is $\left(\frac{3}{2}, \frac{1}{2}\right)$.

Now tangent to $\frac{x^2}{3} + y^2 = 1$ at this point is x + y = 2

Case II :

Slope of AB = $-\frac{\sin\theta}{3\sqrt{3}\cos\theta}$ and slope of tangent at P = $-\frac{\cos\theta}{\sqrt{3}\sin\theta}$

$\Rightarrow -\frac{\sin\theta}{3\sqrt{3}\cos\theta} = -\frac{\cos\theta}{\sqrt{3}\sin\theta}$ or $\sin\theta = \frac{\sqrt{3}}{2}$ & $\cos\theta = \frac{1}{2}$

Hence P is $\left(\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$.

Now tangent to $\frac{x^2}{3} + y^2 = 1$ at this point is $x + 3y = 2\sqrt{3}$.