Ellipse

Exercise – 1(A)

Q.1 (C)

Let the locus be an ellipse

Then PA + PB = 4 = 2a

Where A and B are the foci

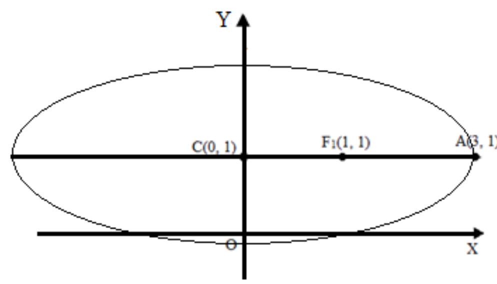
$\Rightarrow$ a = 2 & 2ae = 3

Thus $e = \frac{3}{4} < 1$, hence our assumption is right.

Q.2 (A)

CA = a = 3

$CF_1 = ae = 1$



$\Rightarrow e = \frac{1}{3}$

$b^2 = a^2(1 - e^2)$

$= 9\left(1 - \frac{1}{8}\right) = 8$

$\Rightarrow b = 4\sqrt{2}$

Q.3 (C)

Let the point P be $(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$

Then, $OP = 6\cos^2\theta + 2\sin^2\theta = (2)^2$

$\Rightarrow 4\cos^2\theta = 2$

$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}$

Q.4 (A)

$S_1 \equiv (3, 1)$, $S_2 \equiv (1, 1)$, $P \equiv (1, 3)$

$2a = PS_1 + PS_2$

$\Rightarrow 2a = 2\sqrt{2} + 2 = 2(\sqrt{2} + 1)$

$S_1S_2 = 2ae \Rightarrow 2ae = 2$

$\Rightarrow e = \frac{1}{\sqrt{2}+1} \therefore e = \sqrt{2} - 1$

Q.5 (B)

$16x^2 + 9y^2 + 32x + 36y - 164 = 0$

$16(x+1)^2 + 9(y+2)^2 = 216$

Then centre is $(-1, -2)$

Q.6 (A)

Point P is on ellipse then

a = 5 & b = 4

$b^2 = a^2(1 - e^2)$

If S_1 and S_2 are focus then, $S_1P + S_2P = 2a = 10$

Q.7 (A)

Normal at any point $(x_1\ y_1)$ to the ellipse $bx^2 + a^2y^2 = a^2b^2$ is

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$$

Normal at $\left(ae, \frac{b^2}{a}\right)$ is

$$\frac{a^2x}{ae} - \frac{b^2y}{\frac{b^2}{a}} = a^2 - b^2$$

$$\Rightarrow \frac{a}{c}x - ay = a^2 - b^2$$

It passes through (0 , b), then

$$-ab = a^2 - b^2$$

$$\Rightarrow b^2 - a^2 = ab$$

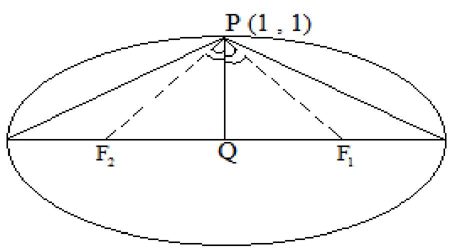
$$\Rightarrow a^2(1-e^2) - a^2 = a^2\sqrt{1-e^2}$$

$$\Rightarrow -a^2 = \sqrt{1-e^2}$$

$$\Rightarrow e^4 + e^2 - 1 = 0$$

Q.8 (D)

F_1P is gets reflected along F_2P. so PQ is normal



Normal at P (1 , 1) is

$$\left(\frac{1}{2}\right)\frac{x}{1} - \left(\frac{1}{3}\right)\left(\frac{y}{1}\right) = \frac{1}{2} - \frac{1}{3}$$

$$\Rightarrow 3x - 2y = 1$$

So $Q \equiv \left(\frac{1}{3}, 0\right)$

Q.9 (C)

$$4x^2 + 6y^2 = 12$$

$$\frac{x\cos\theta}{\sqrt{3}} + \frac{y\sin\theta}{\sqrt{2}} = 1$$

$$A \rightarrow \left(\frac{\sqrt{3}}{\cos\theta}, \theta\right) \quad \& \quad B \rightarrow \left(0, \frac{\sqrt{2}}{\sin\theta}\right)$$

Mid – point of A and B is C(h , k), then

$$h = \frac{\sqrt{3}}{2\cos\theta}, \quad k = \frac{\sqrt{2}}{2\sin\theta}$$

$$\Rightarrow 3k^2 + 2h^2 = 4h^2k^2$$

$$\Rightarrow 3y^2 + 2x^2 = 4x^2 + k^2$$

Q.10 (D)

$$\frac{x^2}{4} + \frac{y^2}{2} = 1$$

Let P is $(2\cos\theta, \sqrt{2}\sin\theta)$

Slope of OP $= \tan^{-1}\frac{\pi}{4} = \frac{\tan\theta}{\sqrt{2}}$

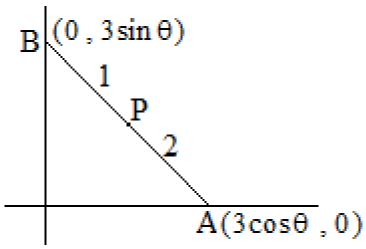
$$\Rightarrow \tan\theta = \sqrt{2}$$

$$r = \sqrt{4\cos^2\theta + 2\sin^2\theta} \;\&\; \sin^2\theta = \frac{2}{3}, \; \cos^2\theta = \frac{1}{3}$$

$$r = \sqrt{\frac{4}{3} + \frac{4}{3}} = \frac{2\sqrt{2}}{\sqrt{3}}$$

Q.11 (A)

AB = 3 & $\frac{AP}{BP} = \frac{2}{1}$



So P is $(\cos\theta, 2\sin\theta)$

Let $h = \cos\theta$; $k = 2\sin\theta$

Ellipse is $x^2 + \frac{y^2}{4} = 1$

$a^2 = 1$ $b^2 = 4$

$e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

Q.12 (A)

PA + PB = 5

A = (2 , 1) & B = (5 , 5) give AB = 5

Then from the property of ellipse, s = 2a

Q.13 (A)

$x^2 + \tan^2\alpha + y^2 \sec^2\alpha = 1$

$\frac{x^2}{\cot^2\alpha} + \frac{y^2}{\cos^2\alpha} = 1$

$\Rightarrow a^2 = \cot^2\alpha$ $b^2 = \cos^2\alpha$

$\frac{2b}{a} = \frac{2\cos^2\alpha}{\cot\alpha} = \frac{1}{2}$

$\Rightarrow 2\cos\alpha\sin\alpha = \frac{1}{2}$

$\Rightarrow \sin 2\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{12}$

Q.14 (C)

lx + my + n = 0 cuts the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at two points whose eccentric angles differ by $\frac{\pi}{2}$.

Let the points be

$A \equiv (a\cos\theta\ ,\ b\sin\theta) \qquad B \equiv (-a\sin\theta, b\cos\theta)$

$$\Rightarrow \frac{x}{a}\cos\left(\theta + \frac{\pi}{4}\right) + \frac{y}{b}\sin\left(\theta + \frac{\pi}{4}\right) - \cos\frac{\pi}{4} = 0$$

$$\frac{a\,l}{\cos\left(\theta + \frac{\pi}{4}\right)} = \frac{mb}{\sin\left(\theta + \frac{\pi}{4}\right)} = -n\sqrt{2}$$

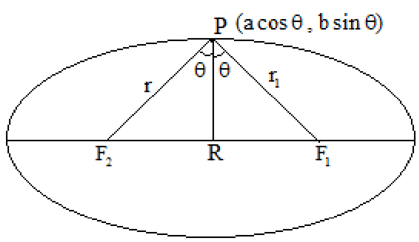
$$\Rightarrow \frac{a^2l^2}{2n^2} + \frac{m^2b^2}{2n^2} = 1 \qquad \left\{\cos^2\left(\theta + \frac{\pi}{4}\right) + \sin^2\left(\theta + \frac{\pi}{4}\right) = 1\right\}$$

$$\Rightarrow a^2l^2 + m^2b^2 = 2n^2$$

Q.15 (C)

Let P is $(a\cos\theta\ ,\ b\sin\theta)$ Normal at P

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$$



Therefore R is $\left(\frac{a^2 - b^2}{a}\cos\theta\ ,\ 0\right)$

$$PR = \frac{b}{a}\sqrt{b^2\cos^2\theta + a^2\sin^2\theta} = \frac{b}{a}\sqrt{r\,r_1}$$

Q.16 (D)

$$x^2 + 2y^2 - 2 = 0$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{1} = 1$$

Tangent to this ellipse is $y = mx \pm \sqrt{2m^2 + 1}$

As this is equally inclined to axis

So, $m = \pm 1$

Thus tangents are

$$y = \pm m\sqrt{2+1} = \pm x \pm \sqrt{3}$$

Distance of any of three tangents from origin is equal to $\sqrt{\frac{3}{2}}$

Q.17 (C)

Let at point P (h , k) line lx + my + n = 0 touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\Rightarrow \frac{h}{a^2 l} = \frac{k}{b^2 m} = \frac{-1}{n}$$

$$\Rightarrow h = \frac{-a^2 l}{n}$$

$$k = \frac{-b^2 m}{n}$$

So P is $\left(\frac{-a^2 l}{n}, \frac{-b^2 m}{n}\right)$

Q.18 (C)

Let chord joins (a , 0) and $(a\cos\theta, b\sin\theta)$. Let the mid – point is P (h , k)

$$h = \frac{a(1+\cos\theta)}{2} \qquad k = \frac{b\sin\theta}{2}$$

Eliminating a will get the locus.

$$\left(\frac{2x}{a}-1\right)^2+\left(\frac{2k}{b}\right)^2=1$$

This is the equation of an ellipse.

Q.19 (A)

x – 2y + 4 = 0 is the common tangent to $y^2 = 4x$ and $\frac{x^2}{4}+\frac{y^2}{b^2}=0$

So put x = 2y – 4 in equation of ellipse

$$\frac{(2y-x)^2}{4}+\frac{y^2}{b^2}=1$$

$$\Rightarrow (y-2)^2+\frac{y^2}{b^2}=1$$

$$\Rightarrow y^2\left(1+\frac{1}{b^2}\right)-4y+3=0$$

$$D = 0 \Rightarrow 16-12\left(1+\frac{1}{b^2}\right)=0$$

$$\Rightarrow \frac{12}{b^2}=4$$

$$\Rightarrow b=\sqrt{3}$$

By symmetry other tangent will be of slope $= -\frac{1}{2}$

$$\therefore \text{ Equation is } y=-\frac{x}{2}\pm\sqrt{4\left(\frac{1}{4}\right)+3}$$

$$2y=-x\pm 4$$

$$2y + x + 4 = 0$$

Q.20 (D)

$$S\equiv\frac{x^2}{9}+\frac{y^2}{4}-1$$

$$S_1(1,2) = \frac{1}{9} + \frac{4}{4} - 1 > 0$$

$$S_1(2,1) = \frac{4}{9} + \frac{1}{4} - 1 < 0$$

$$S_2(x\ ,\ y) = x^2 + y^2 = 9$$

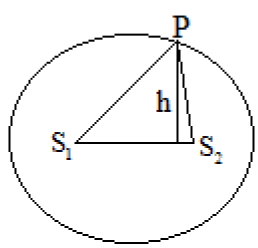
$$S_2(1,\ 2)\ = 1 + 4 - 9\ < 0$$

$$S_2(2\ ,\ 1)\ = 1 + 4 - 9 < 0$$

$\therefore$ P lies inside ‘ C ’ and outside ‘ E ’

Q.21 (A)

$$\text{Area}\ = \frac{1}{2}(2ae)\ \times\ h = aeh$$



Area is maximum when h is maximum and equal to b, so maximum area is (abe)

Q.22 (A)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

End point of latus rectum is $\left(ae\ ,\ \frac{a^2}{b}\right)$

Equation of tangent at $\left(ae\ ,\ \frac{a^2}{b}\right)$ is

$$\frac{x(ae)}{a^2}\ +\ \frac{y}{b^2}\left(\frac{a^2}{b}\right) = 1$$

When y = 0 {as tangent will instant at y axis}

$x = \frac{a}{e}$

Q.23 (B)

$$\frac{x^2}{a^2+b^2} + \frac{y^2}{b^2} = 1 \equiv E_1$$

$$\frac{x^2}{a^2} - \frac{y^2}{a^2+b^2} = 1 \equiv E_2$$

Let the slope of common tangent is M. So equation of tangent to E_1 is

$$y = mx \pm \sqrt{(a^2+b^2)m^2+b^2}$$

And equation of tangent to E_2

$$y = mx \pm \sqrt{a^2m^2+(a^2+b^2)}$$

Comparing both the equation we get

$$(a^2+b^2)m^2+b^2 = a^2m^2+(a^2+b^2)$$

$$m^2 = \frac{a^2}{b^2} \Rightarrow m = \frac{a}{b}$$

$$\therefore y = \frac{a}{b}x \pm \sqrt{a^2\left(\frac{a^2}{b^2}\right) + (a^2+b^2)}$$

$$\Rightarrow by = ax \pm \sqrt{a^4+a^2b^2+b^4}$$

Q.24 (A)

Given equation is $\sqrt{3}bx + ay = 2ab$

Equation of tangent of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at point $P(a\cos\theta\ ,\ b\sin\theta)$ is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1.$$

Comparing the two equations gives

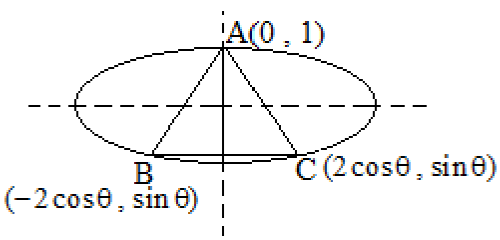
$\cos\theta = \frac{\sqrt{3}}{2}$ & $\sin\theta = \frac{1}{2}$

Hence $\theta = \frac{\pi}{6}$.

Q.25 (A)

$\frac{x^2}{4} + y^2 = 1$

$\tan 30^\circ = \left|\frac{2\cos\theta}{1-\sin\theta}\right|$



$\Rightarrow 1 - \sin\theta = 2\sqrt{3}\cos\theta$

Therefore $\sin\theta = \frac{-11}{13}$ & $\cos\theta = \frac{4\sqrt{3}}{13}$

Which gives $B \equiv \left(\frac{-8\sqrt{3}}{13}, \frac{-11}{13}\right)$ & $C \equiv \left(\frac{8\sqrt{3}}{13}, \frac{-11}{13}\right)$

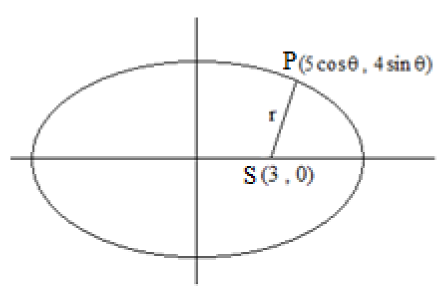
$\therefore BC = \frac{16\sqrt{3}}{13}$

Q.26 (B)

Ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

Focus $\equiv$ (ae , 0) = (3 , 0)



r is the shortest distance from S to P on ellipse

$r^2 = (5\cos\theta - 3)^2 + (4\sin\theta)^2$

$r^2 = 25\cos^2\theta + 16\sin^2\theta - 30\cos\theta + 9$

r^2 is minimum when $\frac{d}{d\theta}(r^2) = 0$

$\Rightarrow \sin\theta = 0 \;\; \cos\theta = 1 \;\Rightarrow\; r = 2$

Q.27 (A)

$\frac{2b^2}{a} = a + b \Rightarrow b = a\sqrt{1-e^2}$

$\Rightarrow 2a^2(1-e^2) = a^2 + a^2\sqrt{1-e^2}$

$\Rightarrow 2 - 2e^2 = 1 + \sqrt{1-e^2}$

$\Rightarrow (1-2\lambda) = \sqrt{1-\lambda} \qquad (\lambda = e^2)$

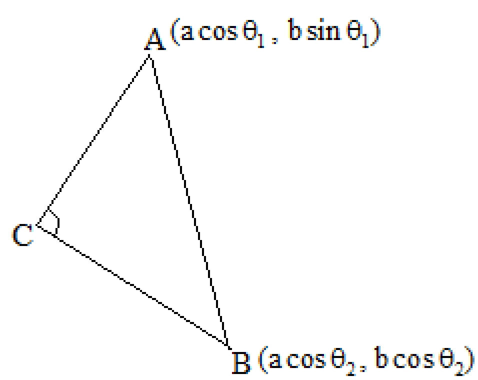
$\Rightarrow 4\lambda^2 - 4\lambda + 1 = 1 - \lambda$

$\Rightarrow \lambda = 0 \qquad$ or $\qquad \lambda = \frac{3}{4}$

$\lambda = 0 \;\Rightarrow a = b$, hence ellipse will limit to a circle.

Q.28 (B)

$\tan\theta_1 \cdot \tan\theta_2 = \frac{-a^2}{b^2}$



Slope of CA $= \frac{b}{a}\tan\theta_1 = M_1$

Slope of CB $= \frac{b}{a}\tan\theta_2 = M_2$

$M_1M_2 = -1$

$\therefore$ AB subtends 90° at center

Q.29 (D)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Let tangents be

$$bx\cos\alpha + at\sin\alpha = ab \qquad \dots (1)$$

$$bx\cos\beta + ay\sin\beta = ab \qquad \dots (2)$$

Where $\alpha + \beta = h$

(1) – (2) gives

$$bx(\cos\alpha - \cos\beta) + ay(\sin\alpha - \sin\beta) = 0$$

$$2bx\sin\frac{\alpha+\beta}{2}\sin\frac{(\beta-\alpha)}{2} + 2ay\sin\frac{\alpha-\beta}{2}\cos\frac{(\alpha+\beta)}{2}$$

$$\sin\frac{\alpha+\beta}{2} = \sin\frac{k}{2} = c_1$$

$$\cos\frac{\alpha+\beta}{2} = \cos\frac{k}{2} = c_2$$

$$\therefore\ 2bxc_1 - 2ayc_2 = 0$$

$$y = \frac{bc_1}{ac_2}x$$

Q.30 (C)

$$\frac{x^2}{a^2}+\frac{y^2}{b} = 1$$

Let point P be $(a\cos\theta\ ,\ b\sin\theta)$

Then, tangents are

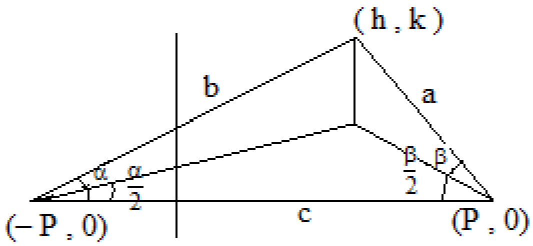
$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b} = 1 \qquad \dots (1)$$

And $x\cos\theta + y\sin\theta = a$ $\qquad \dots (2)$

Solving (1) and (2) we get $y = 0$

Q.31 (D)

$$\frac{\tan\frac{\alpha}{2}}{\tan\frac{\beta}{2}} = h$$



$$\sqrt{\frac{s(s-a)}{(s-b)(s-c)}} \times \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} = h$$

$$\Rightarrow \frac{s-a}{s-b} = k$$

$$\Rightarrow \frac{2s-a-b}{s-b} = k+1$$

Let $k + 1 = k'$, then $\frac{c}{s-b} = k'$

$$\Rightarrow \frac{c}{|c|} = s-b$$

$$\Rightarrow \frac{a+b+c}{2} - b = \frac{c}{|c|} = k''$$

$$\Rightarrow a-b = \text{constant}$$

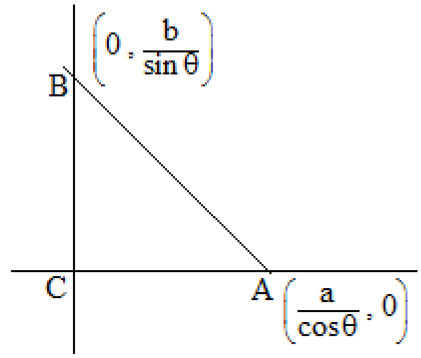
$\Rightarrow$ Locus is a hyperbola

Q.32 (C)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Let the equation of tangent be

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$



$$AB = \sqrt{a^2\sec^2\theta + b^2\operatorname{cosec}^2\theta}$$

$$\frac{d(AB)}{d\theta} = 0$$

$$\Rightarrow a^2(2\sec\theta)\cdot\sec\theta\tan\theta - 2b^2(\operatorname{cosec}\theta)\operatorname{cosec}\theta\cot\theta$$

$$\frac{a^2}{b^2} = \frac{\cos\theta\times\cos^3\theta}{\sin^2\theta\cdot\sin\theta\cdot\sin\theta}$$

$$\Rightarrow \frac{a^2}{b^2} = \frac{\cos^4\theta}{\sin^4\theta}$$

$$\Rightarrow \tan\theta = \sqrt{\frac{b}{a}}$$

$$AB_{minimum} = \sqrt{\frac{a^2\times(a+b)}{a} + \frac{b^2(a+b)}{b}} = (a+b)$$

Q.33 (D)

Given ellipse is $\frac{x^2}{10} + \frac{y^2}{4} = 1$

Chord is bisected at (2 , 1)

Equation of chords bisected at (x_1, y_1) is given by $T = S_1$

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$$

$$\Rightarrow \frac{x(2)}{10} + \frac{y(1)}{4} = \frac{4}{10} + \frac{1}{4}$$

Hence required equation is 4x + 5y = 13

Q.34 (C)

Equation of chord is

$$\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha-\beta}{2}\right)$$

P (0, c) is the point where chord cuts the major axis.

Hence $\frac{c}{a}\cos\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha-\beta}{2}\right)$ Or $\frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}=\frac{a}{c}$

Taking componendo and dividendo

$$\frac{\cos\frac{\alpha}{2}\cos\frac{\beta}{2}}{-\sin\frac{\alpha}{2}\sin\frac{\beta}{2}}=\frac{a+c}{a-c} \quad \text{Or } \tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{c-a}{c+a}$$

Q.35 (A)

Equation of normal through to point $\left(ae\,,\frac{b^2}{a}\right)$

$$\frac{x-ae}{\frac{ae}{a^2}}=\frac{y-\frac{b^2}{a}}{\frac{\frac{b^2}{a}}{b^2}}$$

$$\Rightarrow a\,\frac{(x-ae)}{e}=a\left(y-\frac{b^2}{a}\right)$$

Required equation is $x-ey-e^3a=0$

Q.36 (B)

By property if (a > b), then product of perpendicular drawn from the two foci of an ellipse is b^2

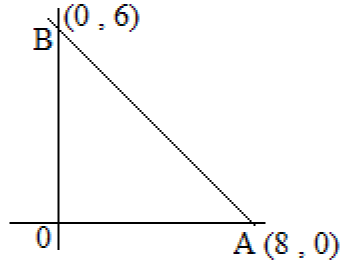
Q.37 (B)

$$\frac{x^2}{18}+\frac{y^2}{32}=1$$

Equation of tangent is $y=mx+\sqrt{a^2m^2+b^2}$

$m = \frac{-4}{3}$

$a^2 = 18 \qquad b^2 = 32$



Substituting there values we get.

$y = -\frac{4}{3}x + \sqrt{\frac{18 \times 16}{9} \times 32}$

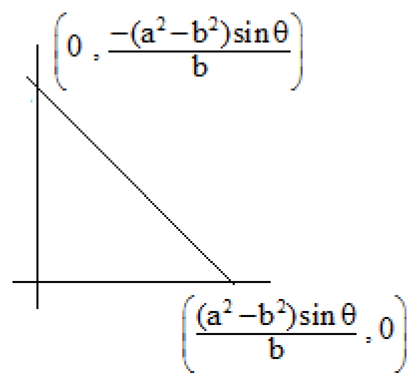
$y = -\frac{4}{3}x + 8$

3y + 4x = 2y

Area of $\Delta OAB = \frac{1}{2} \times 8 \times 6 = 24$

Q.38 (C)

Equation of normal : $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$



Area $= \frac{1}{2}\frac{(a^2-b^2)}{ab}\sin\theta\cos\theta$

Slope of normal

$\frac{a}{b}\tan\theta = 1 \Rightarrow \tan\theta = \frac{b}{a}$

Area $= \frac{1}{2}\frac{(a^2-b^2)^2}{(a^2+b^2)}\frac{a \cdot b}{a \cdot b} = \frac{1}{2}\frac{(a^2-b^2)^2}{(a^2+b^2)}$

Q.39 (B)

y = 2x + k is tangent to $\frac{x^2}{5} + \frac{y^2}{1} = 1$

$a^2 = 5 \ \& \ b^2 = 1$

equation of tangent : $y = mx + \sqrt{a^2m^2 + b^2}$, where m = 2

$\Rightarrow k = \sqrt{a^2m^2 + b^2} = \sqrt{5 \times 4 + 1} = \sqrt{21}$

Q.40 (C)

Equation of tangent to $x^2 + 2y^2 = 3$ at (h, k) will be hx + 2ky = 3

Comparing with x – 2y + 3 = 0 gives

point of contact $\equiv (-1, 1)$

Q.41 (C)

Let the equation of tangent be $y = mx + \sqrt{a^2m^2 + b^2}$

i.e. $y = mx + \sqrt{3m^2 + 2}$

If it pass through (1, 2)

$\Rightarrow 2 = m + \sqrt{3m^2 + 2}$

$\Rightarrow 2m^2 + 4m - 2 = 0$ or $m^2 + 2m - 1 = 0$

Product of slopes is -1.

Q.42 (C)

Let the equation of tangent be $y = mx + \sqrt{a^2m^2 + b^2}$

If it passes through (h, k), then $(k - mh)^2 = (a^2m^2 + b^2)$

$\Rightarrow m^2(h^2 - a^2) - 2mkh + (k^2 - b^2) = 0$

$m_1m_2 = \tan\theta_1 \tan\theta_2 = k'$

$\frac{k^2 - b^2}{k^2 - a^2} = k'$

$\therefore y^2 - b^2 = k(x^2 - a^2)$ is the required locus.

Q.43 (D)

Let the point be $P(a\cos\theta, b\sin\theta)$

Equation of tangent will be $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

Given equation of tangent is $\frac{x}{a} - \frac{y}{b} = \sqrt{2}$

By comparing them

$$\cos\theta = \frac{1}{\sqrt{2}} \quad \& \quad \sin\theta = \frac{-1}{\sqrt{2}} \Rightarrow \theta = \frac{7\pi}{4}$$

Q.44 (D)

Polar of (h, k) w.r.to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ will be $\frac{hx}{9} + \frac{ky}{4} = 1$ (T = 0)

Equation of given polar is $\frac{2x}{-5} + \frac{y}{-5} = 1$

Comparing terms we get

$$h = -\frac{8}{5} \ \& \ k = -\frac{9}{5}$$

Q.45 (B)

Polar of (x_1, y_1) w.r.to $x^2 + 2y^2 - 4x + 12y + 14 = 0$ will be

$$x x_1 + 2y y_1 - \frac{4(x + x_1)}{2} - \frac{12(y + y_1)}{2} + 14 = 0$$

Given equation of polar is y = x + 2

Comparing terms we get $x_1 = \frac{6}{7}$ & $y_1 = \frac{-17}{7}$.

Q.46 (B)

2x + 3y + 1 = 0, x + y + k = 0 are conjugate lines w.r.t. $3x^2 + 4y^2 = 12$,

Hence pole of 2x + 3y + 1 = 0 w.r.t. $3x^2 + 4y^2 = 12$ must lie on x + y + k = 0.

Polar of (h, k) : 3hx + 4ky = 12

Comparing with 2x + 3y + 1 = 0 gives

$$\frac{3h}{2} = \frac{4k}{3} = \frac{12}{-1} \text{ or } h = -8, k = -9$$

Substituting these in x + y + K = 0 gives K = 17.

Q.47 (B)

Let the mid – point be $(x_1 \, , \, y_1)$

Equation of AB : $T = S_1$

$$\Rightarrow \frac{x\,x_1}{2} + \frac{y\,y_1}{3} = \frac{x_1^2}{2} + \frac{y_1^2}{3}$$

Comparing with x + y = 2 gives

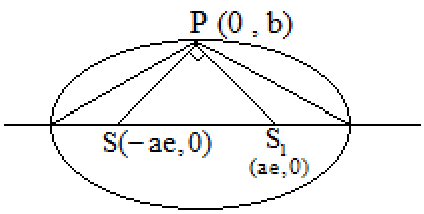
$$\frac{\frac{x_1}{2}}{1} = \frac{\frac{y_1}{3}}{1} = \frac{\frac{x_1^2}{2} + \frac{y_1^2}{3}}{2}$$

i.e. $x_1 = \frac{4}{5}$ & $y_1 = \frac{6}{5}$

Q.48 (B)

$PS + PS_1 = 2a$

$PS^2 + PS_1^2 = S_1^2$



$$a^2e^2 + b^2 + a^2e^2 + b^2 = 4a^2e^2$$

$$\Rightarrow 2b^2 = 2a^2e^2$$

$$\Rightarrow b^2 = a^2 - b^2$$

$$\Rightarrow 2b^2 = a^2$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$$

$$e = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Q.49 (B)

Given $e = \frac{1}{2}$ & $\frac{a}{e} = 4 \Rightarrow a = 2$

Now $\sqrt{1 - \frac{b^2}{4}} = \frac{1}{2} \Rightarrow b = \sqrt{3}$

$\therefore$ equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Q.50 (B)

$$e = \frac{\sqrt{a^2 - b^2}}{a} \Rightarrow \sqrt{a^2 - b^2} = ae$$

Now focus is $(0\ ,\ \sqrt{a^2 - b^2}\ )$ i.e. (0, ae)

And directrics is $y = \frac{a^2}{\sqrt{a^2 - b^2}} \Rightarrow y = \frac{a}{e}$

$\therefore$ Equation of the ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

Q.51 (D)

2a = 8 & 2ae = 4 gives a = 4 & $e = \frac{1}{2}$

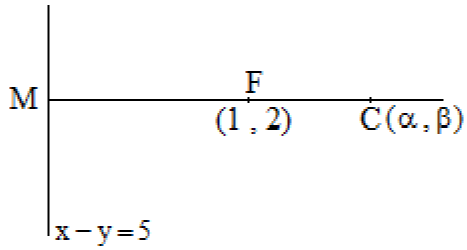
Hence $b = 4\sqrt{1 - \frac{1}{4}} = 2\sqrt{3}$

Required area $= \pi ab = 8\sqrt{3}\pi$

Q.52 (B)

$$e = \frac{1}{2}$$

$FM = \frac{|1-2-5|}{2} = \frac{6}{\sqrt{2}}$



$FM = \left|\frac{a}{e} - ae\right|$

$\frac{6}{\sqrt{2}} = \left|2a - \frac{a}{2}\right|$

$a = 2\sqrt{2}$

Let center be $C(\alpha, \beta)$, then $CF = ae = \sqrt{2}$

Slope of CF × Slope of MN = – 1 gives $\frac{\beta - 2}{\alpha - 1} = -1$

$\Rightarrow \alpha + \beta = 3$

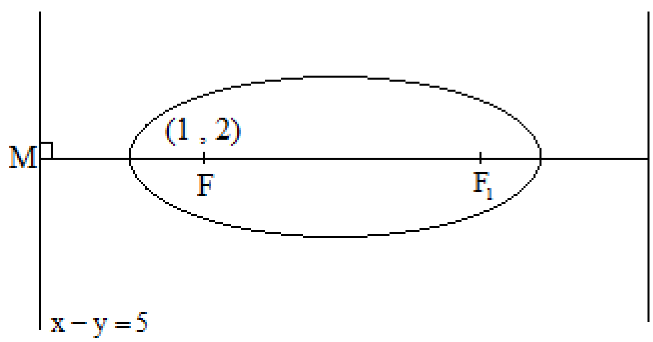
$CF = \sqrt{2} = \sqrt{(\alpha - 1)^2 + (\beta - 2)^2}$

$\Rightarrow \beta = 3$ and $\alpha = 0$

Therefore $C \equiv (0, 3)$

Q.53 (C)

$FM = \left|\frac{1-2-5}{\sqrt{2}}\right| = \frac{6}{\sqrt{2}} = 3\sqrt{2}$



$\left|\frac{a}{e} - ae\right| = 3\sqrt{2}$

$\Rightarrow \left|2a - \frac{a}{2}\right| = 3\sqrt{2}$

$\Rightarrow a = 2\sqrt{2}$

The equation of other directrics is $x - y + \lambda = 0$

$\frac{|\lambda + 5|}{\sqrt{2}} = \frac{2a}{e} = \frac{4\sqrt{2}}{\frac{1}{2}}$

$|\lambda+5|=16$

$\lambda=1$ or $\lambda=-21$

Required equation is x – y + 11 = 0

Q.54 (A)

From above problem. Let F' be the second focus. Let its co – ordinate be (h , k).

slope of FF' = slope of MF

$\Rightarrow \frac{\beta-2}{\alpha-1}=-1$

$\Rightarrow (\beta-2)=-(\alpha-1)$

$\Rightarrow \alpha=-\beta+3$

$FF'=2ae$

$\Rightarrow (\alpha-1)^2+(\beta-2)^2=4\times 8\times \frac{1}{4}$

$\Rightarrow 2(\beta-2)^2=8$

$\Rightarrow \beta=4\ ,\ 0$

$\Rightarrow \alpha=-1\ ,\ 3$

$F'M=\left|ae+\frac{a}{e}\right|$

Co – ordinates of $F'\equiv(-1\ ,\ 4)$

Q.55 (A)

Given ellipse is $\frac{x^2}{\cot^2\alpha}+\frac{y^2}{\cos^2\alpha}=1$

Latus rectum $=\frac{1}{2}=\sqrt{1-\frac{\cos^2\alpha}{\cot^2\alpha}}$

Hence $\cos\alpha=\frac{1}{2}\Rightarrow \alpha=\frac{\pi}{3}$

Q.56 (C)

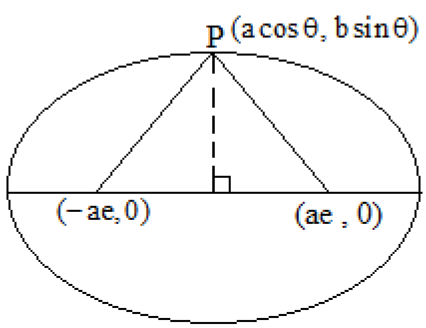
Given ellipse is $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Hence $e = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$

Distance between directrices $= \frac{2 \times 3}{\frac{\sqrt{5}}{3}} = \frac{13}{\sqrt{5}}$

Q.57 (C)

$\frac{x^2}{16} + \frac{y^2}{9} = 1$



Area $\Delta PSS^1 = \frac{1}{2} \times b \sin\theta \times 2ae$

$= \frac{1}{2} 2abe \sin\theta = abe \sin\theta$

Maximum Area = abe $= 4 \times 3\sqrt{1 - \frac{9}{16}} = 3\sqrt{7}$

Q.58 (D)

Center (0, 0)

Let the equation of tangent be

$y = mx + \sqrt{a^2m^2 + b^2}$ … (1)

Equation of circle $x^2 + y^2 = r^2$

The equation of tangent

$$y = mx + r\sqrt{1+m^2} \qquad \ldots (2)$$

Comparing (1) and (2)

$$r\sqrt{1+m^2} = \sqrt{a^2m^2+b^2}$$

$$\Rightarrow r^2 + r^2m^2 = a^2m^2 + b^2$$

$$\Rightarrow m^2(a^2 - r^2) = r^2 - b^2$$

$$m = \sqrt{\frac{r^2-b^2}{a^2-r^2}}$$

Q.59 (A)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Equation of tangent $(\sqrt{3})bx + ay = 2ab$

$$\frac{x}{\frac{2}{\sqrt{3}}a} + \frac{y}{2b} = 1 \qquad \ldots (1)$$

Let the point of contact is $P(a\cos\theta\,,\, b\sin\theta)$

Equation of tangent $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \qquad \ldots (2)$

Comparing (1) and (2)

$$\cos\theta = \frac{\sqrt{3}}{2} \ \& \ \sin\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

Q.60 (C)

$$SS_1 = 2ae = \sqrt{36+64} = 10$$

ae = 5

center = (6 , 8)

cm = a

$\Rightarrow a = \sqrt{25 + 144}$

$e = \frac{5}{13}$