# ELLIPSE

## EXERCISE 1(B)

**Q.1 (B)**

Given ae = 2 & e = 1/2.

Hence a = 4.

Now $b^2 = a^2 - a^2e^2 \Rightarrow b^2 = 12$.

Required equation is $\frac{x^2}{16} + \frac{y^2}{12} = 1$.

**Q.2 (B)**

$9x^2 + 5y^2 - 30y = 0 \Rightarrow \frac{x^2}{5} + \frac{(y-3)^2}{9} = 1$.

Hence $a^2 = 5, b^2 = 9$

$e = \sqrt{\frac{b^2 - a^2}{b^2}} \Rightarrow e = \frac{2}{3}$

**Q.3 (B)**

Given that $\frac{2b^2}{a} = b$, hence $\frac{b}{a} = \frac{1}{2}$.

Now $e = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow e = \frac{\sqrt{3}}{2}$.

**Q.4 (C)**

Given that $\frac{2a}{e} = 3 \times 2ae$, hence $e = \frac{1}{\sqrt{3}}$.

**Q.5 (B)**

Standard Fact.

**Q.6 (B)**

By definition of ellipse i.e. distance from focus = e(distance from directrics),

$$\sqrt{(x-6)^2 + (y-7)^2} = \frac{1}{\sqrt{3}} \frac{|x+y+2|}{\sqrt{2}}$$

$\Rightarrow 6(x-6)^2+6(y-7)^2=(x+y+2)^2$

$\Rightarrow 5x^2-2xy+5y^2-76x-88y+506=0.$

**Q.7 (A)**

Given that $\frac{2b^2}{a}=a$, hence $\frac{b^2}{a^2}=\frac{1}{2}$.

Now $e=\sqrt{1-\frac{b^2}{a^2}} \Rightarrow e=\frac{1}{\sqrt{2}}$.

**Q.8 (B)**

Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

As it passes through (–3, 1) & (2, –2) hence

$\frac{9}{a^2}+\frac{1}{b^2}=1$ & $\frac{4}{a^2}+\frac{4}{b^2}=1$.

Solving these equations simultaneously gives $a^2=\frac{32}{3}$ & $b^2=\frac{32}{5}$.

Hence required ellipse is $3x^2+5y^2=32$.

**Q.9 (C)**

Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

As it passes through (–3, 1) hence $\frac{9}{a^2}+\frac{1}{b^2}=1$.

Also given that $e=\sqrt{\frac{2}{5}}$ hence $\frac{b^2}{a^2}=\frac{3}{5}$.

Hence $a^2=\frac{32}{3}$ & $b^2=\frac{32}{5}$.

Required ellipse is $3x^2+5y^2=32$.

**Q.10 (C)**

$4x^2+9y^2-8x-36y+4=0 \Rightarrow \frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1$

Now $e=\sqrt{\frac{a^2-b^2}{a^2}}\Rightarrow e=\sqrt{\frac{9-4}{9}}=\frac{\sqrt{5}}{3}$.

**Q.11 (A)**

Given that $\frac{2b^2}{a}=10$ & $2b=2ae$.

Now $b^2=a^2e^2\Rightarrow 2b^2=a^2$

Further $b^2=5a\Rightarrow a^2=10$ & $b^2=50$.

Required ellipse is $\frac{x^2}{100}+\frac{y^2}{50}=1$ or $x^2+2y^2=100$.

**Q.12 (B)**

$2b=2ae\Rightarrow\frac{b^2}{a^2}=e^2$ or $1-e^2=e^2$.

Hence $e=\frac{1}{\sqrt{2}}$.

**Q.13 (B)**

Given equation is standard form of equation of an ellipse.

**Q.14 (B)**

Given : $C\equiv(2,-3)$, $S\equiv(3,-3)$, $A\equiv(4,-3)$

Now $a$ = CA = 2 & $ae$ = CS = 1.

Hence $b=a\sqrt{1-e^2}=\sqrt{3}$.

Equation of ellipse : $\frac{(x-2)^2}{4}+\frac{(y+3)^2}{3}=1$.

**Q.15 (B)**

$4x^2+y^2-8x+2y+1=0$

$\Rightarrow\frac{(x-1)^2}{3/2}+\frac{(y+1)^2}{6}=1$

Hence $e=\sqrt{1-\frac{3/2}{6}}=\frac{\sqrt{3}}{2}$

**Q.16 (D)**

$2ae = 8 \text{ \& } \frac{2a}{e} = 18 \Rightarrow a^2 = 36.$

Also $b^2 = a^2 - a^2e^2$ gives $b^2 = 36 - 16 = 20.$

Required ellipse is $\frac{x^2}{36} + \frac{y^2}{20} = 1$ or $5x^2 + 9y^2 = 180.$

**Q.17 (C)**

Given that $2ae = 6 \text{ \& } 2b = 8.$

Now $a^2 = b^2 + a^2e^2 \Rightarrow a^2 = 16 + 9 = 25.$

Hence $e = \frac{3}{5}.$

**Q.18 (B)**

Given that $e = \frac{2}{3} \text{ \& } \frac{2b^2}{a} = 5.$

Now $\frac{b^2}{a^2} = 1 - e^2$ gives $\frac{b^2}{a^2} = \frac{5}{9}.$

Further $\frac{b^2}{a} = \frac{5}{2} \text{ \& } \frac{b^2}{a^2} = \frac{5}{9}$ gives $a^2 = \frac{81}{4} \text{ \& } b^2 = \frac{45}{4}.$

Required equation is $\frac{4x^2}{81} + \frac{4y^2}{45} = 1.$

**Q.19 (B)**

$\frac{x^2}{36} + \frac{y^2}{49} = 1 \to a = 6, b = 7.$

$L.R. = \frac{2a^2}{b}(a < b) \Rightarrow L.R. = \frac{72}{7}.$

**Q.20 (A)**

$\frac{x^2}{64} + \frac{y^2}{28} = 1 \to a^2 = 64, b^2 = 28.$

$e = \sqrt{\frac{a^2 - b^2}{a^2}}(a > b) \Rightarrow e = \frac{3}{4}.$

**Q.21 (B)**

Vertex : $(0,7) \to b = 7$ & Directrics : $y = 12 \to \frac{b}{e} = 12 \Rightarrow e = \frac{7}{12}.$

Now $a^2 = b^2 - b^2e^2 \Rightarrow a^2 = 49 - \frac{49^2}{144} = \frac{49 \times 95}{144}$.

Required equation is $\frac{144x^2}{49 \times 95} + \frac{y^2}{49} = 1$ or $144x^2 + 95y^2 = 4655$.

**Q.22 (A)**

$\frac{(x+1)^2}{9} + \frac{(y+2)^2}{25} = 1 \Rightarrow a^2 = 9, b^2 = 25$ & $b^2e^2 = 16$.

Foci will be $(-1, -2 \pm 4)$ or $(-1, -6)$ & $(-1, 2)$.

**Q.23 (B)**

$25x^2 + 16y^2 - 150x - 175 = 0 \Rightarrow \frac{(x-3)^2}{16} + \frac{y^2}{25} = 1$.

Now $a^2 = 16$, $b^2 = 25$, $e = \sqrt{\frac{b^2 - a^2}{b^2}}$ give $e = \frac{3}{5}$.

**Q.24 (A)**

$ae = 5, \frac{a}{e} = \frac{36}{5} \Rightarrow a = 6$.

Further $b^2 = a^2 - a^2e^2$ gives $b^2 = 11$.

Required equation is $\frac{x^2}{36} + \frac{y^2}{11} = 1$.

**Q.25 (B)**

SP + S'P = length of major axis i.e. 2b (as b > a).

**Q.26 (A)**

$\frac{x^2}{25} + \frac{y^2}{16} = 1 \rightarrow a = 5, b = 4$ & $ae = 3$.

Let coordinates of P be $(5\cos\theta, 4\sin\theta)$.

Also S & S' are (3, 0) & (–3, 0).

Now area of triangle PSS' = $\frac{1}{2} \times 6 \times 4\sin\theta$ or $12\sin\theta$.

Maximum value of A is 12.

**Q.27 (A)**

Foci : $(-1,0)$ & $(7,0) \rightarrow$ Center : $(3,0)$ & $ae = 4$.

Now $e = \frac{1}{2} \Rightarrow a = 8$ & $b = 4\sqrt{3}$

Parametric coordinates : $\left(3+8\cos\theta, 4\sqrt{3}\sin\theta\right)$

**Q.28 (A)**

Substituting P(4, –3) in $2x^2 + 5y^2 - 20$ gives 57 which is positive hence P lies outside the ellipse.

**Q.29 (C)**

Tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $P(\theta)$ is $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$.

Now $\left\{\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1\right\} \equiv \left\{\frac{x}{a\sqrt{2}} + \frac{y}{b\sqrt{2}} = 1\right\}$ gives $\cos\theta = \sin\theta = \frac{1}{\sqrt{2}}$, hence $\theta = 45^0$.

**Q.30 (A)**

For $x^2 + 2y^2 = 4$, $y = 1 \rightarrow y = \pm\sqrt{2}$.

Now tangents at $\left(\pm\sqrt{2}, 1\right)$ will be $x.\left(\pm\sqrt{2}\right) + 2y.1 = 4$

or $\pm x + \sqrt{2}\, y = 2\sqrt{2}$.

**Q.31 (A)**

If intercepts on the coordinate axes are equal, then slope must be 1 or –1.

By $y = mx \pm \sqrt{a^2m^2 + b^2}$ equations of tangents : $y = \pm x \pm \sqrt{a^2 + b^2}$.

**Q.32 (A)**

Let the tangent be $y = mx + \sqrt{16m^2 + 9}$.

As it pass through (2, 3) hence $3 = 2m \pm \sqrt{16m^2 + 9}$ or $-12m = 12m^2$.

Hence slopes of tangents are 0 & –1.

Equations of tangents will be y = 3 & x + y = 5.

**Q.33 (C)**

Any tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$: $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$.

x – intercept, h = $\frac{a}{\cos\theta}$ & y – intercept, k = $\frac{b}{\sin\theta}$.

Eliminating θ gives $\frac{a^2}{h^2}+\frac{b^2}{k^2}=1$.

**Q.34 (A)**

Tangent to at (1/4, 1/4) : $\frac{x}{16}+\frac{y}{48}=1$ or $3x+y=48$ $\{\text{By } T=0\}$.

**Q.35 (C)**

$x\cos\alpha+y\sin\alpha=p \rightarrow y=-(\cot\alpha)x+p\,\text{cosec}\,\alpha$

Now by $c^2=a^2m^2+b^2$, $p^2\,\text{cosec}^2\alpha=a^2\cot^2\alpha+b^2$

or $a^2\cos^2\alpha+b^2\sin^2\alpha=p^2$.

**Q.36 (C)**

By $c^2=a^2m^2+b^2$, $c=\pm\sqrt{9m^2+4}$.

**Q.37 (C)**

For y = mx + c & $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to intersect in real points $c^2\le a^2m^2+b^2$.

**Q.38 (D)**

By $c^2=a^2m^2+b^2$, $c=\pm\sqrt{8\times16+4}$ i.e. $c=\pm2\sqrt{33}$.

**Q.39 (B)**

By $c^2=a^2m^2+b^2$, $c=\pm\sqrt{\frac{5}{4}\times9+\frac{5}{3}}$ i.e. $c=\pm\frac{\sqrt{155}}{12}$.

Required tangents are $y=3x\pm\frac{\sqrt{155}}{12}$.

**Q.40 (A)(B)**

By $c^2 = a^2m^2 + b^2$, $c = \pm\sqrt{3\times16+1}$ i.e. $c = \pm7$.

Required tangents are $y = -4x \pm 7$.