(21) Hint: Common tangent Concept.

(22) Hint: Apply condition of tangency and check options.

(23) $y^2 = 2px$ — (1)

focus $\equiv \left(\frac{p}{2}, 0\right)$, directrix $x = -\frac{p}{2}$

$\Rightarrow 2x + p = 0$

so circle is

$$\left(x - \frac{p}{2}\right)^2 + y^2 = r^2 \quad \text{— (2)}$$

As circle touches the directrix $2x + p = 0$, so

Applying condn of tangency

$$\left|\frac{2 \cdot \frac{p}{2} + p}{2}\right| = r$$

$$|p| = r$$

so eqn (2) becomes

$$\left(x - \frac{p}{2}\right)^2 + y^2 = p^2 \quad \text{— (3)}$$

solving eqn (1) and (3) simultaneously

options

(24) Hint: direct property.

(25) Hint ÷ Use concept of perpendicular chords.

(26) Hint ÷ make the diagram using given information.

(27) Hint: Use concept of Normals and property.

(28) let $(x_1, y_1) \equiv (at^2, 2at)$

Tangent at this point is $ty = x + at^2$

Any point on this tangent is $\left(h, \frac{h+at^2}{t}\right)$

Chord of contact of this point w.r.to. the circle

$x^2+y^2=a^2$ is

$$hx + \left(\frac{h+at^2}{t}\right)y = a^2$$

or $(aty - a^2) + h\left(x + \frac{y}{t}\right) = 0$

which is a family of st. lines passing through the point of intersection of

$$\left.\begin{array}{l} aty - a^2 = 0 \\ \text{and} \\ x + \frac{y}{t} = 0 \end{array}\right\} \text{fixed point is } \left(-\frac{a}{t^2}, \frac{a}{t}\right)$$

$\therefore \quad x_2 = -\frac{a}{t^2}, \quad y_2 = \frac{a}{t}$

clearly $x_1 x_2 = -a^2, \; y_1 y_2 = 2a^2$

Also $\frac{x_1}{x_2} = -t^4$

$$\frac{y_1}{y_2} = 2t^2$$

$$\Rightarrow 4\frac{x_1}{x_2} + \left(\frac{y_1}{y_2}\right)^2 = 0$$

(29) $t_2 = -t_1 - \frac{2}{t_1}$

Sol

Also $\frac{2at_1}{at_1^2} \times \frac{2at_2}{at_2^2} = -1$

$\Rightarrow t_1 \cdot t_2 = -4$

$\therefore -\frac{4}{t_1} = -t_1 - \frac{2}{t_1}$

$\Rightarrow t_1^2 + 2 = 4 \Rightarrow t_1 = \pm\sqrt{2}$

So point can be $(2a, \pm 2a\sqrt{2})$

(30) As a Circle Can intersect a parabola in 4 points

AB so, quadrilateral may be cyclic.

The diagonals of the quadrilateral may be equal as the quadrilateral may be isosceles trapezium

A rectangle cannot be inscribed in a parabola, so (C) is wrong.

(31)
(32)
(33)
(34)

~~st~~ Concept Question

(31.) Any point on the line $x+y=1$ is $(h, 1-h)$.
eqn of chord whose mid point is known w.r.t. to $y^2 = 4ax$

is $T = S_1$

$$(1-h)y - 2a(x+h) = (1-h)^2 - 4ah \quad -(1)$$

eqn (1) is passing through $(a, 2a)$, so.

$$(1-h)2a - 2a(a+h) = (1-h)^2 - 4ah$$

$$\Rightarrow 2a - \cancel{2ah} - 2a^2 \cancel{-2ah} = h^2 - 2h + 1 \cancel{-4ah}$$

$$\Rightarrow h^2 - 2h + 1 + 2a^2 - 2a = 0$$

$$D > 0 \Rightarrow B^2 - 4AC > 0$$

$$\cancel{4} - \cancel{4}(1)(1 + 2a^2 - 2a) > 0$$

$$\cancel{1} - \cancel{1} - 2a^2 + 2a > 0$$

$$-2a(a-1) > 0$$

$$\Rightarrow a(a-1) < 0$$

$$\Rightarrow 0 < a < 1$$

$$\Rightarrow \underline{0 < 4a < 4}$$

so L.R. can be (A, B, D).

(32) direct (Solve both eqⁿs and use condⁿ of common tangency.)

(33) $y^2 = 4x$

Normal to the given parabola with slope m is

$y = mx - 2m - m^3$ — (1)

eqⁿ (1) is passing through $(9,6)$, so

$6 = 9m - 2m - m^3$

$\Rightarrow$ ~~$m^3 - 7m$~~ $m^3 - 7m + 6 = 0$

$\Rightarrow (m-1)(m^2+m-6) = 0$

$\Rightarrow (m-1)(m+3)(m-2) = 0$

$\Rightarrow m = 1, -3, 2$

options A, B, D.

(34) $A \equiv (at_1^2, 2at_1)$

$B \equiv (at_2^2, 2at_2)$

$OA \perp^r OB \Rightarrow t_1 t_2 = -4$

find OA and OB using distance formula.

Substitute $OA^2 = r_1^2$

$OB^2 = r_2^2$

then simplify the expression.

Passages

Any parabola whose axes is parallel to x-axis will be of the form

$$(y-a)^2 = 4b(x-c) \quad \text{---} (1)$$

Now $lx+my=1$ can be rewritten as

$$y-a = -\frac{l}{m}(x-c) + \frac{1-lc-am}{m} \quad \text{---} (2)$$

eqn (2) will touch (1) if

$$\frac{1-am-lc}{m} = -\frac{b}{l/m}$$

$$-\frac{l}{m} = \frac{bm}{1-am-lc}$$

$$\Rightarrow cl^2 - bm^2 + alm - l = 0 \quad \text{---} (3)$$

But given that $5l^2 + 6m^2 - 4lm + 3l = 0$ — (4)

combining (3) and (4), we get,

$$\frac{c}{5} = \frac{-b}{6} = \frac{a}{-4} = \frac{-1}{3}$$

$$\Rightarrow c = -5/3,\ b = 2 \text{ and } a = \frac{4}{3}$$

so parabola is

$\left(y-\frac{4}{3}\right)^2 = 8\left(x+\frac{5}{3}\right)$ whose focus is $\left(\frac{1}{3}, \frac{4}{3}\right)$ and directrix is $3x+11=0$

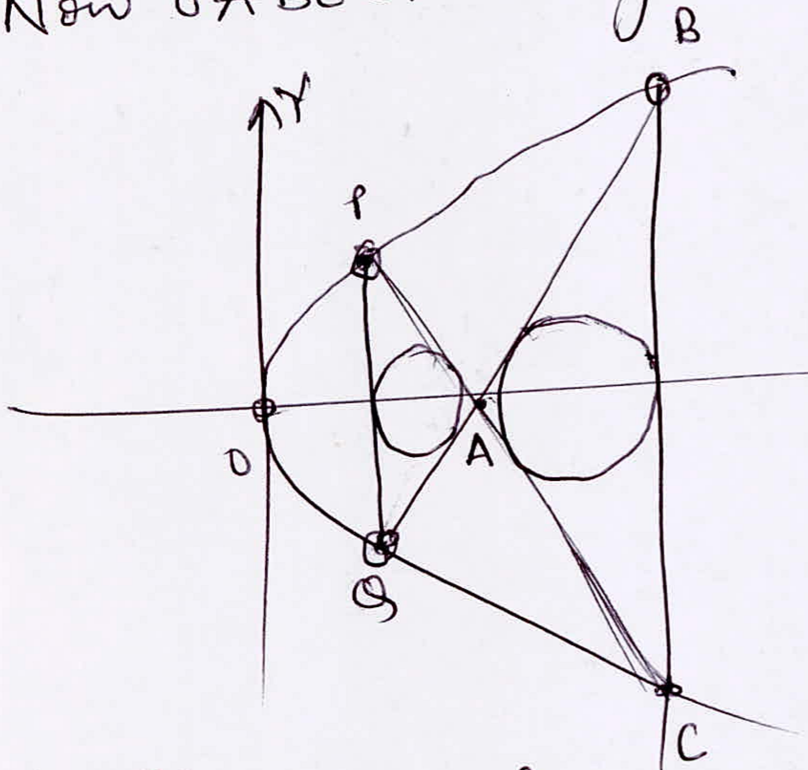
Passage-6 (b, c, d)

Solving given parabolas

$$-8(x-a) = 4x \Rightarrow x = \frac{2a}{3}$$

$\Rightarrow$ points of intersection are $\left(\frac{2a}{3}, \pm\sqrt{\frac{8a}{3}}\right)$

Now OABC is concyclic.



ΔAPQ is isosceles right angled. Therefore, slope of PA is -1 and its equation is

$$y-2 = -(x-1) \text{ or } x+y=3$$

Similarly eq^n of line QB is $x-y-3=0$

solving $x+y=3$ with the parabola $y^2=4x$, we have $(3-x)^2 = 4x$ or $x^2-10x+9=0 \Rightarrow x=1, 9$

Therefore co-ord. of B and C are $(9,-6)$ and $(9,6)$ resp.

$\circledast$ 3

$\therefore$ Area of trapezium $PBCQ = \frac{1}{2}(12+4)\times 8$

$= 64$ sq. units

let the circumcentre of trapezium $PBCQ$ is $T(h,0)$. $PT = TB$

$\sqrt{(h-1)^2 + 4} = \sqrt{(h-9)^2 + 36}$

$\Rightarrow h = 7$

Hence radius is $\sqrt{40} = 2\sqrt{10}$

let the inradius of ΔPAQ be r_1, then $r_1 = \frac{\Delta_1}{s_1}$

$$r_1 = \frac{\frac{1}{2}\times 4\times 2}{4+2\sqrt{4+4}} = \sqrt{2}-1$$

let inradius of ΔABC be r_2, then

$$r_2 = \frac{\Delta_2}{s_2} = \frac{\frac{1}{2}\times 12\times 6}{12+2\sqrt{36+36}}$$

$$= \frac{3}{1+\sqrt{2}} = 3(\sqrt{2}-1)$$

so $\frac{r_2}{r_1} = 3$

Passage-7

$$|z-5-3i| = \frac{z+\bar{z}-6}{2}$$

Use $z = x+iy$

On simplification it gives

$$(y-3)^2 = 4(x-4)$$

with L.R. $= 4$, Vertex $(4,3)$

focus $=(5,3)$

~~so~~ Based on the above information options are

53 C, 54 A 55 B.

Passage-8

A line through $P(6,5)$ having slope m is

$y-5 = m(x-6)$

$\Rightarrow y = mx + 5-6m$ — (1)

eq^n (1) is a tangent to $y^2=4x$ so,

$5-6m = \frac{1}{m} \Rightarrow 6m^2-5m+1=0$

$m = 1/2, 1/3$

so tangents PQ and PR are,

PQ: $2y = x+4$

PR: $3y = x+9$

so solving ~~PR with~~ tangent PR with $y^2 = 4x$

point R is $(9, 6)$.

Now eqn of a family of circles touching a given ~~the PR and~~ line PR at a given point R is

$$(x-9)^2 + (y-6)^2 + \lambda(3y - x - 9) = 0 \quad \text{—} (2)$$

eqn (2) is passing through focus $(1,0)$ so,

$$64 + 36 + \lambda(-10) = 0$$

$$\lambda = 10$$

eqn (2) becomes

$$x^2 + y^2 - 28x + 18y + 27 = 0$$

whose radius is $5\sqrt{10}$.

Based on above information

Q.56 — (A)

Q.57 — (B)

Q.58 — (C)