TRIGO SOLUTION
EXERCISE - 2B

1. (A)

$$45^{\circ} = \frac{45}{180}\times 200 = 50$$

2. (D)

$$\frac{23\pi^{C}}{4} = \frac{23\times 180^{\circ}}{4} = 1035^{\circ}$$

3. $200^{g} = \frac{200}{300}\times 180^{\circ} = 180^{\circ}$

$$= \frac{200}{200}\times \pi^{C} = \pi^{C}$$

4. (B)

Sum of angles of a hexagon $= 180(n-2)$

$$= 180\times 4 = 720^{\circ} = 800^{g} = 4\pi^{C}$$

5. $\sin\alpha + \sin\beta = \frac{1}{3}$ $\qquad$ $\cos\alpha + \cos\beta = \frac{1}{4}$

6. $\Rightarrow 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{3}$ $\qquad$ ______(1)

7. (A,B,D)

$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{4}$ $\qquad$ ________(2)

$$\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{4}{3}$$

Hence, $\sin(\alpha+\beta) = \frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^{2}\left(\frac{\alpha+\beta}{2}\right)} = \frac{2\times\frac{4}{3}}{1+\frac{16}{9}} = \frac{8\times 3}{25} = \frac{24}{25}$

$$\cos(\alpha+\beta) = \frac{1-\tan^{2}\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^{2}\left(\frac{\alpha+\beta}{2}\right)} = \frac{1-\frac{16}{9}}{1+\frac{169}{}} = \frac{-7}{25}$$

$$\tan(\alpha+\beta) = \frac{24}{25}\times\frac{25}{7} = -\frac{24}{7}$$

8. (B)

$P_n - P_{n-2}$

$= \sin^{n}\theta + \cos^{n}\theta - \sin^{n}\theta - \cos^{\,n-2}\theta$

$= \sin^{n-2}\theta\left(\sin^{2}\theta - 1\right) + \cos^{n-2}\left(\cos^{2}\theta - 1\right)$

$= -\sin^{2}\theta\cos^{2}\theta\left(\sin^{n-4}\theta + \cos^{n-4}\theta\right)$

$= -\sin^{2}\theta.\cos^{2}\theta.P_{n-4}$

$\Rightarrow k = -\sin^{2}\theta.\cos^{2}\theta$

9. (B)

$\sin\theta+\cos\theta=m \Rightarrow \sin\theta.\cos\theta=\left(m^2-1\right)/2$

$4\left(1-P_6\right)=4\left(1-\left(\sin^6\theta+\cos^6\theta\right)\right)$

$=4\left(1-\left(1-3\sin^2\theta.\cos^3\theta\right)\right)$

$=12\sin^2\theta.\cos^2\theta=12\left(\frac{m^2-1}{2}\right)^2$

$=3\left(m^2-1\right)^2$

10. (D)

$P_{n-2}-P_n=\sin^2\theta.\cos^2\theta P_\lambda$

$\lambda=n-4$

11. (C)

$2P_6-3P_4+10=2\left(\sin^6\theta+\cos^6\theta\right)-3\left(\sin^4\theta+\cos^4\theta\right)+10$

$=2\left(1-3\sin^2\theta\cos^3\theta\right)-3\left(1-25\sin^2\theta\cos^2\theta\right)+10$

$=2-3+10=9$

12. (B)

$\sin\frac{\pi}{7}+\sin\frac{2\pi}{7}+\sin\frac{3\pi}{7}$

$=\frac{2\sin\frac{2\pi}{7}.\sin\frac{3\pi}{14}}{2.\sin\frac{\pi}{14}}=\frac{\cos\left(\frac{\pi}{14}\right)-\cos\left(\frac{\pi}{2}\right)}{2\cos\frac{\pi}{14}}$

$\frac{1}{2}\cot\left(\frac{\pi}{14}\right)$

13. (B)

$\sin\frac{\pi}{4}+\sin\frac{3\pi}{4}+\sin\frac{5\pi}{4}+-----n\text{ terms}$

$=\frac{\sin\left(\frac{\pi}{4}+\frac{(n-1)}{2}\frac{2\pi}{n}\right).\sin\left(\frac{2\pi.n}{2n}\right)}{\sin\left(\frac{2\pi}{2n}\right)}$

$=\frac{\sin\pi.\sin\pi}{\sin\frac{\pi}{4}}=0$

14. (C)

$\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$

$$= \frac{\cos\left(\frac{\pi}{11} + \frac{(5-1)}{2}\frac{2\pi}{11}\right).\sin\left(\frac{5}{2}\frac{2\pi}{11}\right)}{\sin\left(\frac{2\pi}{11}/2\right)}$$

$$= \frac{2\cos\frac{5\pi}{11}.\sin\frac{5\pi}{11}}{2\sin\frac{\pi}{11}} = \frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}$$

$$= \frac{\sin\frac{\pi}{11}}{2\sin\frac{\pi}{11}} = \frac{1}{2}$$

15. (C)

$$\sum_{r=0}^{n}\sin^2\frac{r\pi}{n} = \frac{1}{2}\sum_{r=0}^{n}\left(1-\cos\frac{2r\pi}{n}\right)$$

$$= \frac{n+1}{2} - \frac{1}{2}\left(\cos 2 + \cos\frac{2\pi}{n} + \cos\frac{9\pi}{n} + \text{........}\cos 2\pi\right)$$

$$= \frac{n+1}{2} - \frac{\cos\left(0+\frac{2\pi}{2}\right).\sin\left(\frac{2\pi}{n}.\frac{(n+1)}{2}\right)}{2\sin\left(\frac{2\pi}{n.2}\right)}$$

$$= \frac{n+1}{2} - \frac{1}{2}\frac{\sin\left(\pi+\frac{\pi}{n}\right)(-1)}{\sin\frac{\pi}{n}} = \frac{n+1}{2} - \frac{1}{2}$$

$$= n/2$$

16. (B)

$\cos 1 \approx \cos 57^\circ$

$\cos 57 \approx \cos(6.28 + 0.72)$

$\approx \cos(0.72)$

So, $\cos 7 > \cos 1$

(but not a correct explanation)

17. (A)

$\left(27^{\sin 2x}.81^{\sin 2x}\right)_{max}$

$= \left(2^{3\cos 2x + 4\sin 2x}\right)_{min}$

$= 3^{-5} = \frac{1}{3^5}$

18. (B)

$\sin B \simeq \sin 9^\circ$

$\sin 1 \simeq \sin 57$

$\sin 2 \simeq \sin 66^\circ$

Sin x is possible but and in clearly as decreasing

(Not a correct explanation)

19. (A)

$\sin^2 A+\sin^2 B+\sin^2 C=2$

$\Rightarrow \sin^2 A+2\sin^2 B+2\sin^2 C=4$

$\Rightarrow 1-\cos 2A+1-\cos 2B+1-\cos 2C=4$

$\Rightarrow \cos 2A+\cos 2B+\cos 2C=-1$

$\Rightarrow -1-4\cos A\cos B\cos C=-1$

$\Rightarrow \cos A\cos B\cos C=0$

Hence are of them is 90°

$\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$

20. (B)

$xy+yz+zx=1$

Let so y $x=\tan\frac{A}{2}, y=\tan\frac{B}{2}, z=\tan\frac{C}{2}$

$xy+yz+zx=\tan\frac{A}{2}.\tan\frac{B}{2}+\tan\frac{B}{2}.\tan\frac{C}{2}+\tan\frac{C}{2}.\tan\frac{A}{2}=1$

Hence, $\left(\frac{A}{2}+\frac{B}{2}+\frac{C}{2}\right)=n\pi+\frac{\pi}{2}\Rightarrow A+B+C=2n\pi+\pi$

We know $\frac{2x}{1+x^2}=\sin A, \frac{2y}{1+y^2}=\sin B, \frac{2z}{1+z^2}=\sin C$

$\sin A+\sin B+\sin C=4\cos\frac{A}{2}.\cos\frac{B}{2}\cos\frac{C}{2}$

$\Rightarrow \frac{2x}{1+x^2}+\frac{2y}{1+y^2}+\frac{2z}{1+z^2}=4\frac{1}{\sqrt{1+x^2}}.\frac{1}{\sqrt{1+y^2}}.\frac{1}{\sqrt{1+z^2}}$

The identify $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$ is true but not used here

21. (A) $A=\sin^2\theta+\cos^4\theta$

$=\sin^2\theta+\left(1-\sin^2\theta\right)^2$

$=\sin^4\theta-\sin^2\theta+1$

$A\in\left[\frac{3}{4},3\right]$ (minimum at $\sin^2\theta=\frac{1}{2}$,maximum of $\sin^2\theta=1$)

Ans: (Q)

(B) $A=3\cos^2\theta+\sin^4\theta$

$=\sin^4\theta-3\sin^2\theta+3$

$A\in[1,3]$

(min of $\sin^2\theta=0,$ max of $\sin^2\theta=1$)

Ans: (S)

(C) $A=\sin^2\theta-\cos^4\theta$

$=\left(1-\cos^2\theta\right)-\cos^4\theta$

$=-\left(\cos^4\theta+\cos^2\theta\right)+1$

$A\in[-1,1]$

Ans: (P)

(D) $A=\tan^2\theta+2\cot^2\theta$

By $A.M\geq G.M.$, $A\geq 2\sqrt{2}$

$A \in \left[2\sqrt{2}, \infty\right)$

22. $(A-R);(B-P);(C-Q),(D-S)$

$\cos\alpha + \cos\beta = \frac{1}{2}$; $\sin\alpha + \sin\beta = \frac{1}{3}$

$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{2}$ ……..(i)

$2\text{sdin}\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{3}$ …….(ii)

(C) $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{1/3}{1/2} = \frac{2}{3}$

(A) $\cos\left(\frac{\alpha+\beta}{2}\right) = \pm\frac{3}{\sqrt{13}}$

$(i)^2 + (ii)^2$

$$4\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{4} + \frac{1}{9} = \frac{13}{36}$$

$$\cos\left(\frac{\alpha-\beta}{2}\right) = \pm\frac{\sqrt{13}}{12}$$

$$\tan\left(\frac{\alpha-\beta}{2}\right) = \pm\frac{\sqrt{131}}{\sqrt{13}}$$

23. (A) $\cos 20^\circ + \cos 80^\circ - \sqrt{3}\cos 50^\circ$

$= 2\cos 50^\circ - \cos 30^\circ - \sqrt{3}\cos 50^\circ$

$= 0$

(B) $1 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \ldots\ldots. + \cos\frac{6\pi}{7}$

$$= 1 + \frac{\cos\left(\frac{\pi}{7} + \frac{5}{2}.\frac{\pi}{7}\right).\sin\left(\frac{6}{2}.\frac{\pi}{7}\right)}{\sin\left(\frac{\pi}{14}\right)}$$

$$= 1 + \frac{\cos\left(\frac{\pi}{2}\right).\sin\left(\frac{3\pi}{7}\right)}{\sin\frac{\pi}{14}} = 1 + 0 = 1$$

(C) $\cos 20^\circ + \cos 40^\circ + \cos 60^\circ - 4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$= 2\cos 30^\circ.\cos 10^\circ + 2\cos^2 30^\circ - 1 - 4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$= 2\cos 30^\circ\left(\cos 10^\circ + \cos 30^\circ\right) - 1 - 4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$= 4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ - 1 - 4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$= -1$

(D) $\frac{1}{2}\left[2\cos 20^\circ.\cos 100^\circ + 2\cos 100^\circ.\cos 140^\circ - 2\cos 140^\circ.\cos 200^\circ\right]$

$= \frac{1}{2}\left[\cos 120^\circ + \cos 80^\circ + \cos 240^\circ + \cos 40^\circ - \cos 340^\circ - \cos 60^\circ\right]$

$$=\frac{1}{2}\left[-\frac{1}{2}-\frac{1}{2}-\frac{1}{2}+\cos 80^{\circ}+\cos 40^{\circ}-\cos 20^{\circ}\right]$$

$$=-\frac{3}{4}+\frac{1}{2}\left(2\cos 60^{\circ}.\cos 20^{\circ}-\cos 20^{\circ}\right)$$

$$=-\frac{3}{4}$$

Ans: $(A)-(S);(B)-(R),(C)-(P),(D)-(Q)$

24. (A) $\tan\theta\left(\cot\theta.\cos\theta+\sin\theta\right)$

$$=\tan\theta.\cot\theta.\cos+\frac{\sin\theta}{\cos\theta}-\sin\theta$$

$$=\frac{\cos^2\theta+\sin^2\theta}{\cos\theta}=\sec\theta$$

Ans: (R,T)

(B) $\frac{\tan\theta.\cos ec^2\theta}{1+\tan^2\theta}$

$$=\frac{\sin\theta}{\cos\theta}.\frac{1}{\sin^2\theta}.\cos^2\theta=\cot\theta$$

Ans: (P)

(C) $\frac{\cos ec^2\theta-1}{\cos ec\theta.\cos^2\theta}=\frac{\cot^2\theta}{\frac{1}{\sin\theta}.\cos^2\theta}$

$$=\frac{\cos^2\theta}{\sin^2\theta.\cos^2\theta}.\sin\theta$$

$$=\cos ec\theta$$

Ans: (S)

(D) $\frac{\cot\theta.\sec^2\theta}{1+\cot^2\theta}=\frac{\frac{\cos\theta}{\sin\theta}.\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}}$

$$=\frac{\sin\theta}{\cos\theta}=\tan\theta$$

Ans: (Q)

25. (A)

$3\sin 2\theta+4\cos 2\theta+3$

$\in\left[-5+3,5+3\right]$ $\qquad\qquad \lambda+k=6$ $\qquad$ (R)

$\in\left[-2,8\right]$ $\qquad\qquad \lambda-k=10$ $\qquad$ (S)

(B) $5\cos\theta+3\cos\theta.\cos\frac{\pi}{3}-2\sin\theta.\sin\frac{\pi}{3}+3$

$$=\left(\frac{13}{2}\cos\theta-\frac{3\sqrt{3}}{2}\sin\theta\right)+3$$

$$\in\left[\sqrt{\frac{169}{4}+\frac{27}{4}}+3,\sqrt{\frac{169}{4}+\frac{27}{4}}+3\right]$$

$\in\left[-4,10\right]$ $\qquad\qquad \lambda+k=6$ $\qquad$ (R)

$\lambda-k=14$ $\qquad$ (T)

(C) $1+\sin\theta.\frac{1}{\sqrt{2}}+\cos.\frac{1}{\sqrt{2}}+2.\frac{\cos\theta}{\sqrt{2}}+\frac{2\sin\theta}{\sqrt{2}}$

$$=1+\frac{3}{\sqrt{2}}\cos\theta+\frac{3}{\sqrt{2}}\sin\theta$$

$$=\in\left[1-\frac{3\sqrt{2}}{\sqrt{2}},1+\frac{3\sqrt{2}}{\sqrt{2}}\right]$$

$\in[-2,4]$ $\lambda+k=2$ (P)

$\lambda-k=6$ (Q)

26. $\cos A=\frac{1}{3}$

$A\in\left(1350^\circ,1440^\circ\right)$

$\Rightarrow\in\left(270^\circ,360^\circ\right)$ IV quad

$\sin A=-25\frac{2}{3}$

$\frac{A}{2}\in\left(675^\circ,720^\circ\right)$ IV quad

$$\sin\frac{A}{2}=\sqrt{\frac{1-\cos A}{2}}=\sqrt{\frac{1-\frac{1}{3}}{2}}=-\frac{1}{\sqrt{3}}$$

$$\cos\frac{A}{2}=\sqrt{\frac{1+\cos A}{2}}=\sqrt{\frac{2}{3}} \qquad \tan\frac{A}{2}=-\frac{1}{\sqrt{2}}$$

$(A)-(R);(B)-(S);(C)-(Q);(D)-(P)$

27. (A) $\sqrt{3}\sin x-\cos x \quad \in[-2,2]$ (R)

(B) $4\cos^2 x-4\cos x+3$

$=\left(2\cos\alpha-1\right)^2+2$

$\in[0,9]+2$

$\in[2,11]$ (P)

(C) $\frac{2\tan\alpha}{\tan^2\alpha+1}=\sin 2x\in[-1,1]$ (S)

(D) $\sin^4+\cos^4 x=1-2\sin^2 x.\cos^2 x$

$=1-\frac{1}{2}\sin^2 2x$

$\in[0,1]$

So, $\in\left[\frac{1}{2},1\right]$ (Q)

28. Standard solutions

$\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$

$\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$

$\cos 2A+\cos 2B+cps2C=-1-4\cos 2A\cos B\cos C$

$\cos^2 A+\cos^2 B+\cos^2 c=1-2\cos A\cos B\cos C$

$(A)-(Q);(B)-(P),(C)-(R);(D)-(S)$

29. $\sin 2\theta = k$

(A) $\cos ec\, 20 + \cot 2\theta - \cos 2\theta$

$$= \frac{1}{\sin 2\theta} + \frac{1 - \sin\theta}{\sin 2\theta} = \frac{1}{k} + \frac{1-k^2}{k} = \frac{2-k^2}{k} \qquad \text{(s)}$$

(B) $\left(\frac{1+\tan\theta}{1-\tan\theta}\right)^2 = \frac{1+\sin 2\theta}{1-\sin 2\theta}$

$$= \frac{1+k}{1-k} \qquad \text{(p)}$$

(C) $\sin 2\theta - \frac{1}{2}\left(2\cos^2 2\theta\right)$

$$= k - \frac{b}{2}\left(1-k^2\right)$$

$$= k^2 + k - 1 \qquad \text{(Q)}$$

(D) $\sin 60 = 3\sin\theta - 4\sin^3 2\theta$

$$= 3k - 4k^3 \qquad \text{(R)}$$

30. (A) $\sin B = \frac{4}{5}$, $\tan\left(\frac{A}{2}\right) = 1 \Rightarrow A = 90^\circ$

$$B + C = 90^\circ$$

$$\cos C = \frac{4}{5}$$

(A) $\frac{a}{b} = \frac{\sin A}{\sin B} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$ ________(R)

(B) $\frac{b}{c} = \frac{\sin B}{\sin C} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$ ______(S)

(C) $\frac{c+b}{a} = \frac{\sin C + \sin B}{\sin A} = \frac{3}{5} + \frac{45}{} = \frac{7}{5}$ ________ (P)

(D) $\sqrt{\frac{a^2+c^2-b^2}{20c}} = \sqrt{\frac{\sin^2 A + \sin^2 C - \sin^2 B}{2\sin A \sin C}} = \sqrt{\frac{3}{5}}$ _______ (Q)