TRIGO – 1

EX – 2A

1. **(ABC)**

$$a = \cos x + \cos\left(x + \frac{2\pi}{3}\right) + \cos\left(x \frac{4\pi}{3}\right)$$
$$= \cos x + 2\cos x \cdot \cos(2\pi/3)$$
$$= \cos x - \cos x = 0$$
$$b = \sin x + \sin\left(x + \frac{2\pi}{3}\right) + \sin\left(x + \frac{4\pi}{3}\right)$$
$$= \sin x + 2\sin x \cdot \cos\left(\frac{2\pi}{3}\right)$$
$$= \sin x - \sin x = 0$$

2. **(BD)**

$$\left.\begin{array}{l}\sin 1 \approx \sin 57^o \\ \sin 2 \approx \sin 114^o \approx \sin 66^o \\ \sin 3 \approx \sin 171^o \approx \sin 9^o\end{array}\right\} \sin 2 > \sin 1 > \sin 3$$
$$\left.\begin{array}{l}\cos 1 \approx \cos 57^o \\ \cos 2 \approx \cos 114^o \approx -\cos 66^o \\ \cos 3 \approx \cos 171^o \approx -\cos 9^o\end{array}\right\} \cos 1 > \cos 2 > \cos 3$$

3. **(AB)**

$$x = r\sin A\cos B$$
$$y = r\sin A\sin B$$
$$z = r\cos A$$
$$x^2 + y^2 + z^2 = r^2\sin^2 A + r^2\cos^2 A = r^2$$

4. **(BCD)**

Quadrilateral is cyclic hence $\left.\begin{array}{l}P + R = \pi \\ Q + S = \pi\end{array}\right\}$

5. **(ABD)**

$$\tan\frac{x}{2} \in \theta$$

(i) $\sin x = \dfrac{2\tan x/2}{1 + \tan^2 x/2} \in \theta$

(ii) $\cos x = \dfrac{1 - \tan^2 x/2}{1 + \tan^2 x/2} \in \theta$

(iii) $\sec\dfrac{x}{2} = \sqrt{1 + \tan^2\dfrac{x}{2}}$ (not necessarily)

(iv) $\tan x = \dfrac{2\tan x/2}{1 - \tan^2 x/2} \in \theta$

6. **(ABCD)**

$$2\sin \frac{A}{2} = \pm\sqrt{1+\sin A} \pm \sqrt{1-\sin A}$$

7. **(AD)**

$$\left(4\cos^2 9^o - 3\right)\left(4\cos^2 27^0 - 3\right)$$

$$= \frac{\left(4\cos^3 9^o - 3\cos 9^o\right)\left(4\cos^3 27^o - 3\cos 27^0\right)}{\cos 9^o \cdot \cos 27^o}$$

$$= \frac{\cancel{\cos 27^o} \cdot \cos 80^o}{\cos 9^o \cdot \cancel{\cos 27^o}} = \frac{\sin 9^o}{\cos 9^o} = \tan 9^o = \cot 81^o$$

8. **(AD)**

$$\cos\beta = \sqrt{\sin\alpha \cdot \cos\alpha}$$

$$\Rightarrow \cos^2\beta = \sin\alpha \cdot \cos\beta$$

$$\Rightarrow \frac{1+\cos 2\beta}{2} = \frac{\sin 2\alpha}{2} \Rightarrow \cos 2\beta = -\left(1-\sin 2\alpha\right)$$

$$= -\left(1-\cos\left(\frac{\pi}{2} - 2\alpha\right)\right)$$

$$= -2\sin^2\left(\frac{\pi}{4} - \alpha\right)$$

$$= -2\cos^2\left(\frac{\pi}{4} + \alpha\right)$$

9. **(BC)**

$$\frac{\sqrt{1+\cos\alpha} + \sqrt{1-\cos\alpha}}{\sqrt{1+\cos\alpha} - \sqrt{1-\cos\alpha}} \quad ; \quad \alpha \in \left(\pi, 2\pi\right)$$

$$= \frac{\cancel{\sqrt{2}}\left(\left|\cos\frac{\alpha}{2}\right| + \left|\sin\frac{\alpha}{2}\right|\right)}{\cancel{\sqrt{2}}\left(\left|\cos\frac{\alpha}{2}\right| - \left|\sin\frac{\alpha}{2}\right|\right)}$$

$$= \frac{-\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}}{-\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}} = \frac{1-\tan\frac{\alpha}{2}}{1+\tan\frac{\alpha}{2}} = \tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$$

$$= \cot\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$$

10. **(CD)**

$$\operatorname{cosec}\theta - \cot\theta = q$$

$$\operatorname{cosec}\theta + \cot\theta = \frac{1}{q}$$

So, $\operatorname{cosec}\theta = \frac{\left(a + \frac{1}{q}\right)}{2}$

$$\cot\theta = \frac{\left(\frac{1}{q} - q\right)}{2}$$

11. **(AD)**

$$\cos A \cdot \cos B + \sin A \sin B = \frac{3}{5} \qquad \ldots(1)$$

$$\sin A \sin B = 2\cos A \cos B \qquad \ldots(2)$$

So, $\cos A \cos = \frac{1}{5}$

$$\sin A \sin B = \frac{2}{5}$$

12. **(BCD)**

13. **(ABD)**

$$\frac{\sin 22^{o} \cos 8^{o} + \cos 158^{o} \cos 98^{o}}{\sin 23^{o} \cos 7^{o} + \cos 157 \cos 97^{o}}$$

$$= \frac{\frac{1}{2}\left(\sin 30^{o} + \sin 14^{o} + \cos 256^{o} + \cos 60^{o}\right)}{\frac{1}{2}\left(\sin 30^{o} + \sin 16^{o} + \cos 254^{o} + \cos 60^{o}\right)}$$

$$\sin 30^{o} = \cos 60^{o} = \frac{1}{2}$$

$$\cos 254^{o} = \cos\left(270^{o} - 16^{o}\right) = -\sin 16^{o}$$

$$\cos 256^{o} = \cos\left(270^{o} - 14^{o}\right) = -\sin 14^{o}$$

$$\therefore = \frac{\frac{1}{2}\left(\frac{1}{2} + \sin 14^{o} - \sin 14^{o} + \frac{1}{2}\right)}{\frac{1}{2}\left(\frac{1}{2} + \sin 16^{o} - \sin 16^{o} + \frac{1}{2}\right)}$$

$$\sec\left(-100\pi\right) = 1$$

$$\operatorname{cosec}\left(\frac{-3\pi}{2}\right) = \operatorname{cosec}\left(\frac{\pi}{2}\right) = 1 \qquad \ldots(B)$$

$$\sin\left(\frac{7\pi}{2}\right) = \sin\left(\frac{3\pi}{2}\right) = -1 \neq 1$$

$$\cot\left(\frac{5\pi}{4}\right) = 1 \qquad \ldots(D)$$

14. **(AD)**

$$\sqrt{\frac{1-\sin A}{1+\sin A}} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$$

$$\sqrt{\frac{1-\sin A}{1+\sin A}}$$

$$\Rightarrow \sqrt{\left(\frac{1-\sin A}{1+\sin A}\right)\left(\frac{1+\sin A}{1+\sin A}\right)}$$

$$\Rightarrow \sqrt{\frac{1-\sin^2 A}{(1+\sin A)^2}} = \sqrt{\frac{\cos^2 A}{(1+\sin A)^2}}$$

$\Rightarrow \left|\frac{\cos A}{1+\sin A}\right|$ now 1 + sin A is always positive

$$\Rightarrow \left|\frac{\cos A}{(1+\sin A)}\frac{(1-\sin A)}{(1-\sin A)}\right|$$

$$\Rightarrow \left|\frac{\cos A(1-\sin A)}{(1-\sin^2 A)}\right| \Rightarrow \left|\frac{1-\sin A}{\cos A}\right|$$

L.H.S

$$\Rightarrow \frac{1-\sin A}{|\cos A|} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$$

$$\frac{1-\sin A}{|\cos A|} = \frac{1-\sin A}{\cos A}$$

$\Rightarrow \sin A = 1$ or $|\cos A| = \cos A$

But $\sin A \neq 1$

$\because$ then $\cos A = 0$

$|\cos A| = \cos A$ means cos A is positive i.e. 1^{st} and 4^{th} Quadrant

15. **(ABCD)**

$\sec A = \frac{17}{8}$ $\quad$ $\operatorname{cosec} B = \frac{5}{4}$

$\cos A = \frac{8}{17}$ $\quad$ $\sin B = \frac{4}{5}$

$\sin A = \pm\frac{15}{17}$ $\quad$ $\cos B = \pm\frac{3}{5}$

$$\therefore \sec(A+B) = \frac{1}{\cos(A+B)} = \frac{1}{\cos A \cos B - \sin A \sin B}$$

$$= \frac{1}{\left(\frac{8}{17}\right)\left(\pm\frac{3}{5}\right) - \left(\pm\frac{15}{17}\right)\left(\frac{4}{5}\right)}$$

$$= \frac{85}{\pm 24 \pm 60}$$

Possible answer are

$$\frac{85}{84}, \frac{85}{-84}, \frac{85}{36}, \frac{85}{-36}$$

16. **(ABCD)**

(A) $$\frac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}-\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$$

$$= \frac{\cos 2\alpha}{2\tan\left(\frac{\pi}{4}-\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$$

$$=\frac{\cos 2\alpha}{2\sin\left(\frac{\pi}{4}-\alpha\right)\cos\left(\frac{\pi}{4}-\alpha\right)}$$

$$=\frac{\cos 2\alpha}{\sin\left(2\left(\frac{\pi}{4}-\alpha\right)\right)}$$

$$=\frac{\cos 2\alpha}{\sin\left(\frac{\pi}{2}-2\alpha\right)}=\frac{\cos 2\alpha}{\cos 2\alpha}=1$$

(B) $$\frac{\sin(\pi-\alpha)}{\sin\alpha-\cos\alpha\tan\frac{\alpha}{2}}+\cos(\pi-\alpha)$$

$$=\frac{\sin\alpha}{\frac{\left(\sin\alpha\cos\frac{\alpha}{2}-\cos\alpha\sin\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}}}-\cos\alpha$$

$$=\frac{\sin\alpha\cos\frac{\alpha}{2}}{\sin\left(\alpha-\frac{\alpha}{2}\right)}-\cos\alpha$$

$$=\frac{\left(2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}-\cos\alpha$$

$$=2\cos^2\frac{\alpha}{2}-\cos\alpha$$

$$=1+\cos\alpha-\cos\alpha=1$$

(C) $$\frac{\left(1-\tan^2\alpha\right)}{4\tan^2\alpha}$$

$$=\left(\frac{1\tan^2\alpha\pi}{2\tan\alpha}\right)^2=\cot^2 2\alpha$$

$$\frac{1}{4\sin^2\alpha\cos^2\alpha}=\cos ec^2 2\alpha$$

$$\therefore \cos ec^2 2\alpha-\cot^2 2\alpha=1$$

(D) $$1+\sin 2\alpha=\left(\sin\alpha+\cos\alpha\right)^2$$

$$\therefore\frac{1+\sin 2\alpha}{\left(\cos\alpha+\sin\alpha\right)^2}=1$$

17. **(AD)**

$$\frac{\cos A}{\sin B\sin C}+\frac{\cos B}{\sin C\sin A}+\frac{\cos C}{\sin A\sin B}$$

$$=\frac{\sin A\cos A+\sin B\cos B+\sin C\cos C}{\sin A\sin B\sin C}$$

$$A+B+C=\pi$$

$$\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$$

$\therefore$ Numerator = 2sin A sin B sin C
L.H.S = 2.

18. **(ABC)**

$$\frac{\sqrt{3}\sin(\alpha+\beta)-\dfrac{1}{\cos\left(\dfrac{\pi}{6}\right)}\cos(\alpha+\beta)}{\sin\alpha}$$

$$=\frac{\sqrt{3}\sin(\alpha+\beta)-\dfrac{4}{\sqrt{3}}\cos(\alpha+\beta)}{\sin\alpha}$$

$\sin\beta=\frac{4}{5}$

If $\beta\in\left(0,\frac{\pi}{2}\right)$ and $\tan\beta>0$

Then $\cos\beta=\frac{3}{5}$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$=\frac{3}{5}\sin\alpha+\frac{4}{5}\cos\alpha$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=\frac{3}{5}\cos\alpha-\frac{4}{5}\sin\alpha$

$=\frac{1}{\sqrt{3}\sin\alpha}\left(3\sin(\alpha+\beta)-4\cos(\alpha+\beta)\right)$

$=\frac{1}{\sqrt{3}\sin\alpha}\left(\frac{9}{5}\sin\alpha+\frac{12}{5}\cos\alpha-\frac{12}{5}\cos\alpha+\frac{16}{5}\sin\alpha\right)$

$=\frac{1}{\sqrt{3}\sin\alpha}\left(\frac{25}{5}\sin\alpha\right)=\frac{5}{\sqrt{3}}$

(A) & (B) true

For $\tan\beta<0 \quad \cos\beta=\frac{-3}{5}$

$\sin(\alpha+\beta)=\frac{-3}{5}\sin\alpha+\frac{4}{5}\cos\alpha$

$\cos(\alpha+\beta)=\frac{-3}{5}\cos\alpha-\frac{4}{5}\sin\alpha$

L.H.S. $=\frac{1}{\sqrt{3}\sin\alpha}\left(3\sin(\alpha+\beta)-4\cos(\alpha+\beta)\right)$

$=\frac{1}{\sqrt{3}\sin\alpha}\left(\frac{-9}{5}\sin\alpha+\frac{12}{5}\cos\alpha+\frac{12}{5}\cos\alpha+\frac{16}{5}\sin\alpha\right)$

$=\frac{1}{\sqrt{3}\sin\alpha}\left(\frac{24}{5}\cos\alpha+\frac{7}{5}\sin\alpha\right)$

$$=\left(\frac{24\cos\alpha+7\sin\alpha}{5\sqrt{3}\sin\alpha}\right)$$

$$=\frac{\sqrt{3}}{15}\left(24\cot\alpha+7\right)$$

$\therefore$ (A) (B) (C)

19. **(ACD)**

$\cos 20^o+\cos 40^o+\cos 60^o+.....+\cos 140^o$

$=\left(\cos 40^o+\cos 140^o\right)+\left(\cos 60^o+\cos 120^o\right)+......+\left(\cos 80^o+\cos 100^o\right)+\cos 20^o$

As $\cos\left(\pi-\theta\right)=-\cos\theta$

$\therefore \cos\left(\pi-\alpha\right)+\cos\theta=0$

L.H.S

$=0+\cos 20^o$ (A)

$=\cos 20^o$

$=\sin 70^o$ (D)

$=\cos 20^o$

$=\cos\left(30^o-10^o\right)$

$=\frac{\sqrt{3}}{2}\cos 10^o+\frac{1}{2}\sin 10^o$ (C)

20. **(ABC)**

$x\cos\alpha+4\sin\alpha=K$ where

$\alpha=A,B$ are roots of this equation

$x\cos\theta=k-y\sin\theta$

Square

$x^2\cos^2\theta=\left(k-y\sin\theta\right)^2$

$x^2\left(1-\sin^2\theta\right)=k^2+y^2\sin^2\theta-2ky\sin\theta$

$x^2-x^2\sin^2\theta=k^2+y^2\sin^2\theta-2ky\sin\theta$

$0=\left(x^2+y^2\right)\sin\theta-2ky\sin\theta+\left(k^2-x^2\right)$

Quadratic in $\sin\theta$ roots are sin t, sin B

$$\sin A\sin B=\frac{k^2-x^2}{x^2+y^2}$$

$$\sin A+\sin B=\frac{2ky}{x^2+y^2}$$

$y\sin\theta=k-x\cos\theta$

Square

$y^2\sin^2\theta=\left(k-x\cos\theta\right)^2$

$y^2\left(1-\cos^2\theta\right)=k^2+x^2\cos^2\theta-2kx\cos\theta$

$\left(x^2+y^2\right)\cos^2\theta-2kx\cos\theta+k^2-y^2=0$

Quadratic in cos B having roots cos A, cos B

$$\cos A+\cos B=\frac{2kx}{x^2+y^2}$$

$$\cos A\cos B=\frac{k^2-y^2}{x^2+y^2}$$

21. **(AC)**

$$\frac{2\sin 2^o+4\sin 4^o+......+180\sin 180^o}{90}=A$$

$$A=\frac{\left(2\sin 2^o+4\sin 4^o+.....+176\sin 176^o+178\sin 178^o\right)}{90}$$

Supplementary angles have the same sign value

$\sin 2^o=\sin 178^o$

$\sin 4^o=\sin 176^o$ etc

$$90A=\left(2\sin 2^o+178\sin 178^o\right)+\left(4\sin 4^o+176\sin 176^o\right)+.....+\left(88\sin 88^o+92\sin 92^o\right)+90\sin 90^o$$

$$=\left(2\sin 2^o+178\sin 2^o\right)+\left(4\sin 4^o+176\sin 176^o\right)+.....+\left(88\sin 88^o+92\sin 88^o\right)+90$$

$$=\left(180\sin 2^o+180\sin 4^o+....+180\sin 88^o\right)+90$$

$$90A=180\left(\sin 2^o+\sin 4^o+....+\sin 88^o\right)+90$$

$$\sin 2^o+\sin 4^o+....+\sin 88^o$$

$$=\frac{\sin\left(44^o\right)\sin\left(45^o\right)}{\sin 1^o}$$

$$90A=180\frac{\sin 44^o\sin 45^o+90^o}{\sin 1^o}$$

$$=90\left(\frac{2\sin 44^o\sin 45^o}{\sin 1^o}+1\right)$$

$$=90\left(\frac{\cot 1^o-\cos 89^o}{\sin 1^o}+1\right)$$

$$=90\left(\cot 1^o-\frac{\cos 89^o}{\sin 1^o}+1\right)$$

$\cos 89^o=\sin 1^o$

$\therefore 90A=90\left(\cot 1^o-1+1\right)$

$=90\cot 1^o$

$A=\cot 1^o$ (A)

(B) is $\tan 1^o$

(C) $=\dfrac{\cos\left(31^o\right)\cos 1^o}{\sin 51^o\sin 1^o}$

(D) is $\tan 1^o$

22. **(ABCD)**

$$\sin\theta+\cos\alpha=-\frac{1}{5}$$

$$\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}+\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}=\frac{-1}{5}$$ Let $\tan\frac{\theta}{2}=x$

$$\frac{2x+1-x^2}{1+x^2}=\frac{-1}{5}$$

$5\left(2x+1-x^2\right)+1\left(1+x^2\right)=0$

$10x+5-5x^2+1+x^2=0$

$6+10x-4x^2=0$

$2x^2-5x-3=0$ (C)

$x=\frac{5\pm\sqrt{25+4(2)(3)}}{2}$

$=\frac{5\pm 7}{2}=6-1$

Now question is value of $\tan\frac{\theta}{2}$ is a root of which equation. If we see the other equations putting $x=-1$, or 6 will satisfy them. So All (A) (B) (C) (D) Have roots either -1 or 6

23. **(ABC)**

$\cos(A-B)=\frac{3}{5}$

$\cos A\cos B+\sin A\sin B=\frac{3}{5}$

$\tan A\tan B=2$

$\sin A\sin B=2\cos A\cos B$

$\therefore 3\cos A\cos B=\frac{3}{5}$

$\cos A\cos B=\frac{1}{5}$ (A)

$\sin A\sin B=\frac{2}{5}$ (B)

$\cos(A+B)=\cos A\cos B-\sin A\sin B$

$=\frac{1}{5}-\frac{2}{5}=\frac{-1}{5}$ (C)

$\cos(A+B)=\frac{-1}{5}$

$\sec^2(A+B)=25$

$\tan^2(A+B)=24$ (D) is wrong

24. **(ABCD)**

$\sin a+\sin b=\frac{1}{\sqrt{2}}$ …(1)

$\cos a+\cos b=\frac{\sqrt{3}}{\sqrt{2}}$ …(2)

Square and add the equations we get

$2+2\cos a\cos b+2\sin a\sin b=\frac{1}{2}+\frac{3}{2}$

$\therefore 2\cos(a-b)=0$

$\cos(a-b)=0$

$2\cos^2\left(\frac{a-b}{2}\right)-1=0$

$$\cos^2\left(\frac{a-b}{2}\right)=\frac{1}{2}$$
$$\sec^2\left(\frac{a-b}{2}\right)=2$$
$$\tan^2\left(\frac{a-b}{2}\right)=1$$
$$\therefore \cot^2\left(\frac{a-b}{2}\right)=1 \qquad \text{(C)}$$
$$\sin a+\sin b=\frac{1}{\sqrt{2}}$$
$$2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)=\frac{1}{\sqrt{2}}$$
$$\cos a+\cos b=\frac{\sqrt{3}}{\sqrt{2}}$$
$$2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)=\frac{\sqrt{3}}{\sqrt{2}}$$
Hence $\tan\left(\frac{a+b}{2}\right)=\frac{1}{\sqrt{3}}$ (D)
$$\sin(a+b)=\frac{2\tan\left(\frac{a+b}{2}\right)}{1+\tan^2\left(\frac{a+b}{2}\right)}=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{\sqrt{3}}{2}$$
$$\sin(a+b)=\frac{\sqrt{3}}{2} \qquad \text{(A)}$$

25. **(AC)**

$$\frac{\sin A}{\sin B}=p,\frac{\cos A}{\cos B}=q$$
Dividing we get $\frac{\tan A}{\tan B}=\frac{p}{q}$

Now $\sin A=p\sin B$ …(1)

$\cos A=q\cos B$ …(2)

$$\sin^2 A+\cos^2 A=1$$
$$p^2\sin^2 B+q^2\cos^2 B=1$$
$$p^2\sin^2 B+q^2\left(1-\sin^2 B\right)=1$$
$$\left(p^2-q^2\right)\sin^2 B=1-q^2$$
$$\sin^2 B=\left(\frac{1-q^2}{p^2-q^2}\right)$$
$$\cos^2 B=1-\sin^2 B$$
$$=\left(\frac{p^2-1}{p^2-q^2}\right)$$
$$\tan^2 B=\left(\frac{1-q^2}{p^2-1}\right)=\frac{q^2-1}{1-p^2} \qquad \text{(C)}$$
From (1) & (2)

$\sin^2 A = p^2 \sin^2 B$

$= p^2\left(\dfrac{1-q^2}{p^2-q^2}\right)$

$\cos^2 A = q^2\left(\dfrac{p^2-1}{p^2-q^2}\right)$

$\tan^2 A = \dfrac{p^2\left(1-q^2\right)}{q^2\left(p^2-1\right)}$ (A)

26. **(ABCD)**

$0 \le \theta \le \pi$

$81^{\sin^2\theta} + 81^{\cos^2\theta} = 30$

$81^{\sin^2\theta} + 81^{-\sin^2\theta} = 30$

Let $81^{\sin^2\theta} = x$

$x + \dfrac{81}{x} = 30$

$x^2 + 81 = 30x$

$x^2 - 30x + 81 = 0$

$x = 27$ or 3

$\therefore\ 81^{\sin^2\theta} = 27, 3$

$3^{4\sin^2\theta} = 3^3, 3^1$

$\therefore\ 4\sin^2\theta = 3, 1$

$\sin^2\theta = \dfrac{3}{4}, \dfrac{1}{4}$

$\sin\theta = \pm\dfrac{\sqrt{3}}{2}, \pm\dfrac{1}{2}$

For $0 \le \theta \le \pi$ $\theta = 30^o, 60^o, 120^o, 150^o$

27. **(BCD)**

For a cyclic Quadrilateral B + D = π or B = π – D

sin B = sin D $\Rightarrow$ cosec B = cosec D

cot B = – cot D or tan B = – tan D

similarly A = π – C

cot A = – cot C

cot A + cot C = 0

sec B = –sec D

28. **(ABCD)**

$\sin\left(\dfrac{11\pi}{12}\right)\sin\left(\dfrac{5\pi}{12}\right)$

$= \dfrac{1}{2}\left(\cos\left(\dfrac{6\pi}{12}\right) - \cos\left(\dfrac{16\pi}{12}\right)\right)$

$= \dfrac{1}{2}\left(0 - \cos\left(\dfrac{4\pi}{3}\right)\right)$

$=\frac{1}{2}\left(+\frac{1}{2}\right)=\frac{1}{4}$ … (A)

$\operatorname{cosec}\left(\frac{9\pi}{10}\right)\sec\left(\frac{4\pi}{5}\right)$

$=\frac{1}{\sin\frac{\pi}{10}\cos\left(\frac{\pi}{5}\right)}=\frac{1}{(-1)\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)}$

= (–1) (4) … (B)

$\sec^4\theta+\cos^4\theta$

$=1-2\sin^2\theta\cos^2\theta$

$=1-\frac{\sin^2 2\theta}{2}\quad \theta=\frac{\pi}{8}$

$=1-\frac{\sin^2\frac{\pi}{4}}{2}$

$=1-\frac{\frac{1}{2}}{2}$

$=\frac{3}{4}$ …(C)

$\left(1+\cos\frac{2\pi}{9}\right)\left(1+\cos\frac{4\pi}{9}\right)\left(1+\cos\frac{8\pi}{9}\right)$

$=2\cos^2\frac{\pi}{9}\times 2\cos^2\frac{2\pi}{9}\times 2\cos^2\frac{4\pi}{9}$

$=8\left(\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\right)$

$=8\left(\frac{1}{8}\frac{\sin\left(\frac{8\pi}{9}\right)}{\sin\frac{\pi}{9}}\right)^2$

$=\frac{8}{64}=\frac{1}{8}$ …(D)

29.

(A) $\tan\alpha\tan 2\alpha\tan 3\alpha$

$\tan 3\alpha-\tan 2\alpha-\tan\alpha$

Is true for all angles

Hint: $2\alpha+\alpha-3\alpha$

$\therefore$ take tan on both sides to get the answer

(B) $\operatorname{cosec}2\alpha+\operatorname{cosec}4\alpha-\operatorname{cosec}\alpha$

$=\frac{1}{\sin\frac{2\pi}{7}}+\frac{1}{\sin\frac{4\pi}{7}}-\frac{1}{\sin\frac{\pi}{7}}$

$$=\frac{2\sin\left(\frac{3\pi}{7}\right)\cos\left(\frac{\pi}{7}\right)}{\left[2\sin\left(\frac{\pi}{7}\right)\cos\left(\frac{\pi}{7}\right)\right]\sin\frac{4\pi}{7}}-\frac{1}{\sin\frac{\pi}{7}}$$

$$=\frac{1}{\sin\frac{\pi}{7}}=\frac{1}{\sin\frac{\pi}{7}}=0$$

(D) $8\cos\alpha\cos 2\alpha\cos 4\alpha$

$$=\frac{\sin 8\alpha}{\sin\alpha}=\frac{\sin\frac{8\pi}{7}}{\sin\frac{\pi}{7}}=-1$$

(C) $\cos\alpha+\cos 3\alpha-\cos 2\alpha$

$= 2\cos 2\alpha\cos\alpha-\cos 2\alpha$

$=\cos 2\alpha\left(2\cos\alpha-1\right)$

$$=\cos\left(\frac{2\pi}{7}\right)\left(2\cos\frac{\pi}{7}-1\right)$$

$$=\cos\left(\frac{2\pi}{7}\right)\left(2\cos\frac{\pi}{7}-1\right)\frac{\sin\frac{\pi}{7}}{\sin\frac{\pi}{7}}$$

$$=\cos\left(\frac{2\pi}{7}\right)\left(\frac{2\sin\frac{\pi}{7}\cos\frac{\pi}{7}-\sin\frac{\pi}{7}}{\sin\frac{\pi}{7}}\right)$$

$$=\frac{\cos\left(\frac{2\pi}{7}\right)}{\sin\frac{\pi}{7}}\left(\sin\frac{2\pi}{7}-\sin\frac{\pi}{7}\right)$$

$$=\frac{\cos\left(\frac{2\pi}{7}\right)\left(2\sin\frac{\pi}{14}\right)\left(\cos\frac{3\pi}{14}\right)}{2\sin\frac{\pi}{14}\cos\frac{\pi}{14}}$$

$$=\frac{\cos\frac{3\pi}{14}\cos\frac{2\pi}{7}}{\left(\cos\frac{\pi}{14}\right)}$$

$$=\frac{1}{2}\frac{\left(2\cos\left(\frac{3\pi}{14}\right)\sin\frac{3\pi}{14}\right)}{\cos\frac{\pi}{14}}=\frac{1}{2}\frac{\sin\left(\frac{6\pi}{14}\right)}{\cos\frac{\pi}{14}}$$

$\frac{6\pi}{14},\frac{\pi}{14}$ are complementary

$\therefore \text{L.H.S.} = \frac{1}{2} \times 1 = \frac{1}{2}$

30. **(A)**

$\cot\alpha - \tan\alpha = 2\cot 2\alpha$ …(A)

(A) is true

$\tan\alpha + \cot\alpha = 2\operatorname{cosec} 2\alpha$ …(D)

(D) is wrong

$\tan\left(45^0 + \alpha\right) - \tan\left(45^0 + 1 - \alpha\right)$

$= \cot\left(45^0 - \alpha\right) - \tan\left(45^0 - \alpha\right)$

$= 2\cot\left(2\left(45^0 - \alpha\right)\right)$

$2\cot\left(90^0 - 2\alpha\right)$

$= \tan 2\alpha$

(B) (C) wrong