**TRIGO SOLUTION**

**EXERCISE - 1(C)**

**No solution: Q No 39**

1. (B)

$\tan A = \sqrt{2} \qquad \tan^2 A = 2$

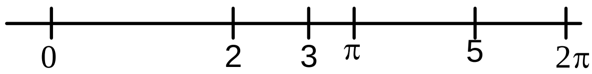
$$\frac{\sin^4 A - 3\sin^2 A\cos^2 A + 7\cos^4 A}{1+\sin^2 A\cos^2 A + 5\cos^4 A}$$

$$= \frac{\sin^4 A - 3\sin^2 A\cos^2 A + 7\cos^4 A}{\left(\sin^2 A + \cos^2 A\right)^2 + \sin^2 A\cos^2 A + 5\cos^4 A}$$

$$= \frac{\tan^4 A - 3\tan^2 A + 7}{\left(\tan^2 A + 1\right)^2 + \tan^2 A + 5}$$

$$= \frac{4-6+7}{9+2+5} = \frac{5}{16}$$

2. (A)




$\sin 2 > 0$

$\sin 3 > 0$

$\sin 5 < 0$

$\sin 2 \sin 3 \sin 5 < 0$

3. (A)

$f(x) = \sec x - \tan x$

$g(x) = \sec x + \tan x$

$f(x).g(x) = 1$

$f(A).f(B).f(C) = g(A).g(B).g(C)$

$f^2(A).f^2(B).f^2(C) = f(A)g(A)f(B).g(B).f(C)g(C)$

$f(A)f(B)f(C) = \pm 1$

4. $\cos(A) = \cos B\cos C$

$\cos(\pi - (B+C)) = \cos B\cos C$

$-[\cos B\cos C - \sin B\sin\ ] = \cos B\cos C$

$\sin B\sin C = 2\cos B\cos C$

$\tan B\tan C = 2$

5. (D)

$$\sum_{r=0}^{10}\cos^3\left(\frac{\pi r}{3}\right)$$

$$1+\left(\frac{1}{2}\right)^3+\left(-\frac{1}{2}\right)^3+(-1)^3+\left(-\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)+(1)^3+\left(\frac{1}{2}\right)^3+\left(-\frac{1}{2}\right)^3+(-1)^3+\left(-\frac{1}{2}\right)^3+(-1)^3+\left(-\frac{1}{2}\right)^3=-\frac{1}{8}$$

6. (B)

$U_n = 2\cos n\theta$

$U_1U_n - U_{n-1}$

$= 2\left(2\cos n\theta.\cos\theta\right) - 2\cos\left(n-1\right)\theta$

$= 2\left(\cos\left(n+1\right)\theta + \cos\left(n-1\right)\theta\right) - 2\cos\left(n-1\right)\theta$

$= 2\cos\left(n+1\right)\theta$

$= U_{n+1}$

7. (C)

$\cos 5\theta = a\cos^5\theta + b\cos^3\theta + c\cos\theta$

$\frac{\cos 5\theta}{\cos\theta} = a\cos^4\theta + b\cos^2\theta + c$

$\lim_{\theta\to\frac{\pi}{2}} \frac{\cos 5\theta}{\cos\theta} = c$

$c = \lim_{\theta\to\frac{\pi}{2}} + \frac{5\left(\sin 5\theta\right)}{\sin\theta}$

$c = +5$

8. (D)

$f\left(x\right) = \frac{\sin 3x}{\sin x}$ $\qquad\qquad \sin x \neq 0$

$3 - 4\sin^2 x$

Range in $\left[-1, 3\right)$

9. (C)

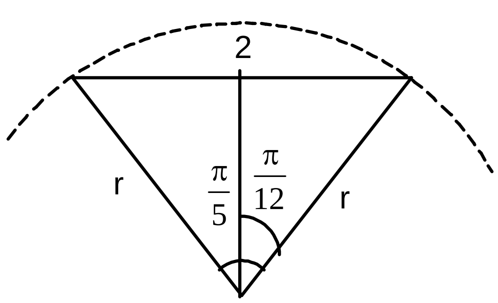
$\cos\left(\theta+\phi\right) = m\left(\cos\left(\theta-\phi\right)\right)$

$\frac{\cos\theta\cos\phi - \sin\theta\sin\phi}{\cos\theta\cos\phi + \sin\theta\sin\phi} = \frac{m}{1}$

$\frac{\sin\theta\sin\phi}{\cos\theta\cos\phi} = \frac{1-m}{1+m}$

$\tan\theta = \left(\frac{1-m}{1+m}\right)\cot\phi$

10. (B)




$\sin\frac{\pi}{12} = \frac{1}{r}$

$r = \frac{1}{\sin\frac{\pi}{12}} = \sqrt{6} + \sqrt{2}$

11. (B)

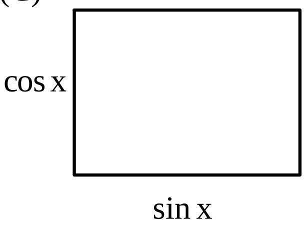
$$\frac{1-2\sin^2\frac{\alpha}{2}}{1+\sin\alpha}=\frac{\cos\alpha}{1+\sin\alpha}$$

$$=\frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)^2}$$

$$=\frac{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{3}}{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}$$

$$=\frac{1-m}{1+m}$$

12. (C)




$A=\sin x\cos x$

$A=\frac{1}{2}\sin 2x$

$A\le\frac{1}{2}$

13. (D)

$\sin^8 75^\circ-\cos^8 75^\circ$

$=\left(\sin^4 75^\circ+\cos^4 75^\circ\right)\left(\sin^2 75^\circ-\cos^2 75^\circ\right)(1)$

$=\left(1-\frac{1}{2}\sin^2 150\right)\left(-\cos 150^\circ\right)$

$=\left(1-\frac{1}{8}\right)\left(+\frac{53}{2}\right)$

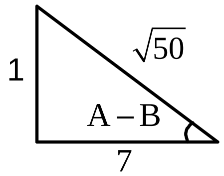
$=\frac{753}{16}$

14. (D)

$\tan A=3$

$\tan B=2$

$\tan(A-B)=\frac{3-2}{1+6}=\frac{1}{7}$



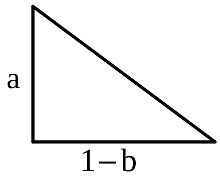

$\sin\left(A-B\right)=\frac{1}{\sqrt{50}}$

$\sin 2\left(A-B\right)=2\times 1\frac{1}{\sqrt{50}}\times\frac{7}{550}$

$=\frac{7}{25}$

15. $\tan A+\tan B=a$
$\tan A.\tan B=b$

$\tan\left(A+B\right)=\frac{a}{1-b}$




$\sin\left(A+B\right)=\frac{a}{\sqrt{a^2+\left(1-b\right)^2}}$

$\sin^2\left(A+B\right)=\frac{a^2}{a^2+\left(1-b\right)^2}$

16. (A)

$\tan B=\frac{n\sin A\cos A}{1-n\cos^2 A}$

$\tan\left(A+B\right)=\frac{\tan A+\tan B}{1-\tan A\tan B}$

$=\frac{\frac{\sin A}{\cos A}+\frac{n\sin A\cos A}{1-n\cos^2 A}}{1-\frac{\sin A}{\cos A}\frac{n\sin A\cos A}{1-n\cos^2 A}}$

$=\frac{\frac{\sin A-n\cos^2 A\sin A+n\cos^2 A\sin A}{\cos A\left(1-n\cos^2 A\right)}}{\frac{1-n\cos^2 A-n\sin^2 A}{1-n\cos^2 A}}=\frac{\sin A}{\cos A\left(1-n\right)}$

17. (B)

$\tan\beta=\frac{1}{3}$

$\tan 2\beta=\frac{\frac{2}{3}}{1-\frac{1}{9}}=\frac{\frac{2}{5}}{\frac{8}{9}}=\frac{3}{4}$

$\tan\alpha = \frac{1}{7}$

$\tan(\alpha + 2\beta) = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{3}{28}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$

$\alpha + 2\beta = \frac{\pi}{24}$

18. (C)

$\sin A = \sin B$

$\cos A = \cos B$

$\Rightarrow A = B + 2n\pi$

$\sin\left(\frac{A-B}{2}\right) = 0$

19. (C)

$1 - \frac{1}{\sin\theta + \cos\theta}\left(\sin^3\theta + \cos^3\theta\right)$

$= 1 - \frac{1}{\sin\theta + \cos\theta}(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$

$= \sin\theta\cos\theta$

20. (B)

$\frac{\cos x\cos y + \sin x\sin y}{\cos x\cos y - \sin x\sin y} = \frac{a}{1}$

$\cot x\cot y = \frac{1+a}{a-1}$

21. $2\sin A = \sqrt{3}\sin B$ ________(1)

$2\cos A = \sqrt{5}\cos B$ _________-(2)

Square and add (1) & (2)

$3\sin^2 B + 5\cos^2 B = 4$

$3\tan^2 B + 5 = 4\sec^2 B \text{ or } \tan B = 1$

$\tan A = \frac{\sqrt{3}}{\sqrt{5}}$

$\tan A + \tan\beta = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5}}$

22. $P_n - P_{n+2} = \cos^n\theta + \sin^n\theta - \cos^{n-2}\theta - \sin^{n-2}\theta$

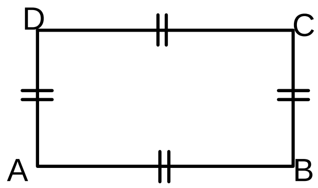
$= \cos^{n-2}\theta\left(-\sin^2\theta\right) + \sin^{n-2}\theta\left(-\cos^2\theta\right)$

$= -\sin^2\theta\cos^2\theta\left(\cos^{n-4}\theta + \sin^{n-4}\theta\right)$

$= k\,P_{n-4}$

$K = -\sin^2\theta\cos^2\theta$

23. Given



$AB^2 = AC \times BD$

$AC^2 + BD^2 = 4AB \,\&\, \tan\theta = \dfrac{BD}{AC}$

$\Rightarrow AC^2 + BD^2 = 4AC\,BD$

$\Rightarrow \left(\dfrac{BD}{AC}\right)^2 + 1 = 4\left(\dfrac{BD}{AC}\right)$

$\tan^2\theta + 1 = 4\tan\theta$

$\sin 2\theta = \dfrac{1}{2} \Rightarrow \theta = 15^\circ$

24. (A)

$\sin A\left(\dfrac{A}{2} + \left(\dfrac{A}{2} + \dfrac{C}{2}\right)\right) = K\sin\left(\dfrac{\pi - (A+B)}{2}\right)$

$\sin\left(\dfrac{A}{2} + \dfrac{\pi - B}{2}\right) = K\cos\left(\dfrac{A+B}{2}\right)$

$\cos\left(\dfrac{A-B}{2}\right) = K\cos\left(\dfrac{A+B}{2}\right)$

$\dfrac{\cos\frac{A}{2}\cos\frac{B}{2} + \sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2} - \sin\frac{A}{2}\sin\frac{B}{2}} = \dfrac{K}{1}$

$\dfrac{K-1}{K+1} = \tan\dfrac{A}{2}\tan\dfrac{B}{2}$

25. $\dfrac{3 + \cot 16^\circ \cot 16^\circ}{\cot 76^\circ + \cot 16^\circ}$

$\cot 17^\circ + \cot 16^\circ = \dfrac{\sin 92^\circ}{\sin 76^\circ \sin 16} = \dfrac{\sin 88}{\sin 16 \sin 16^\circ}$

$1 + \cot 76 \cot 16 = \dfrac{1}{2\sin 76 \sin 16}$

$LHS = \dfrac{3 + \cot 17\cot 16}{\cot 76 + \cot 16} = \dfrac{2 + \dfrac{1}{2\sin 76\sin 16}}{\dfrac{\sin 88}{\sin 76\sin 16}} = \dfrac{2(\cos 60 - \cos 92) + 1}{\sin 88^\circ}$

$= \dfrac{2\sin^2 46}{2\sin 44^\circ \cos 44} = \cot 44^\circ = \tan 46^\circ$

26. $\dfrac{2\left(\sin 1^\circ + \sin 2^\circ - - - - - - - - \sin 89^\circ\right)}{2\left(\cos 1^\circ + \cos 2^\circ - - - - - - - \cos 44h^\circ\right) + 1}$

$$N^r = 2\left(\sin 1^\circ + \sin 2^\circ ------\sin 89^\circ\right) = \frac{2\sin\left(\frac{89}{2}\right)\sin 45}{\sin\left(\frac{1}{2}\right)}$$

$$D^r = 2\left(\cos 1^\circ + \cos 2^\circ -------\cos 89^\circ\right) = \frac{2\sin 22^\circ \cos\frac{45}{2}}{\sin\frac{1^\circ}{2}} + 1$$

$$= \frac{-\sin\frac{1^\circ}{2} + \sin\frac{89^\circ}{2}}{\sin\frac{1^\circ}{2}} + 1$$

$$\frac{N^r}{D^r} = \frac{\frac{2\sin\frac{89}{2}\sin 45}{\sin\frac{1}{2}}}{\frac{\sin\frac{89}{2}}{\sin\frac{1}{2}}} = \sqrt{2} \qquad = \frac{\sin\frac{89}{2}}{\sin\frac{1}{2}}$$

27. (A)

$x \in \left(\frac{5\pi}{4}, \frac{9\pi}{4}\right)$

$\sin x < -\frac{1}{2}$

$\cos x > -\frac{1}{2}$

$\sin x - \cos x < 0$

$\sin x - \cos x = t$

$1 - \sin 2x = t^2$

$t^2 = 1 - \frac{2024}{2025}$

$t^2 = \frac{1}{2025}$

$t = \frac{-1}{45}$

28. (D)

$\left(\tan 4\theta + \tan 2\theta\right)\left(1 - \tan^2 3\theta \tan^2 \theta\right)$

$= \frac{\sin 6\theta}{\cos 4\theta \cos 2\theta}\left(1 - \tan 2\theta \tan\theta\right)\left(1 + \tan 3\theta \tan\theta\right)$

$= \frac{\sin 6\theta}{\cos 4\theta \cos 2\theta}\cos^2 3\theta \cos^2 \theta$

$= 2\tan 3\theta \sec^2 \theta$

29. (D)

$$\frac{1}{\tan x}+\frac{1-\sqrt{3}\tan x}{\sqrt{3}+\tan x}+\frac{1+\sqrt{3}\tan x}{\tan x-\sqrt{3}}$$

$$=\frac{3}{\tan 3x}=\frac{3\left(1-3\tan^2 x\right)}{3\tan x-\tan^3 x}$$

30. $$\frac{a\cos\alpha-b\sin\alpha}{\sin\alpha\cos\alpha}$$

$$=\sqrt[2]{a^2+b^2}\,\frac{\left(\sin\theta\cos\alpha-\cos\alpha\sin\alpha\right)}{\sin 2\alpha}$$

$$=\sqrt[2]{a^2+b^2}\qquad\frac{\sin\left(\theta-\alpha\right)}{\sin 2\alpha}\qquad\left(\theta=3\alpha\right)$$

$$=\sqrt[2]{a^2+b^2}$$

31. (D)

$$2\sin\frac{A}{2}=\sin\frac{A}{2}+\cos\frac{A}{2}-\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right)$$

$$=-\sqrt{1+\sin A}+\sqrt{1-\sin A}$$

32. (A)

$$p^2=1-\sin 40^\circ$$

$$\sin 40^\circ=1-p^2\qquad\left(p<0\right)$$

$$\cos 40^\circ=\sqrt{1-\left(1-p^2\right)^2}$$

$$=-p\sqrt{2-p^2}$$

33. (C)

$$\tan\frac{\alpha}{2}+\cot\frac{\alpha}{2}$$

$$=\frac{2}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$$

$$=\frac{2}{p}$$

$$x^2-\frac{2}{p}+1=0$$

$$px^2-2x+p=0$$

34. (B)

$$\frac{1}{\cos 290^\circ}+\frac{1}{\sqrt{3}\sin 250^\circ}$$

$$\frac{1}{\sin 20^\circ}-\frac{1}{\sqrt{3}\cos 20^\circ}$$

$$\frac{2\left(\sqrt{3}\cos 20^\circ-\sin 20^\circ\right)}{\sqrt{3}\,2\sin 20^\circ\cos 20^\circ}$$

$$=\frac{4}{13}\frac{\sin\left(60^\circ-20^\circ\right)}{\sin 40^\circ}=\frac{4}{\sqrt{3}}$$

35. (B)

$\sin x+\sin y=a$

$\cos x+\cos y=b$

$=2$

$$a=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$b=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\frac{a}{b}+\frac{b}{a}=\frac{1}{\sin\left(\frac{x+y}{2}\right)\cos d\left(\frac{x+y}{2}\right)}$$

$$\frac{a^2+b^2}{ab}=\frac{2}{\sin\left(x+y\right)}$$

$$\sin\left(x+y\right)=\frac{2ab}{a^2+b^2}$$

36. (C)

$$\sqrt{2}\sin\frac{3\pi}{20}+\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\frac{\pi}{10}-\frac{1}{\sqrt{2}}\cos\frac{\pi}{10}\right)$$

$$=\sqrt{2}\sin\frac{3}{20}-\sqrt{2}\left(\sin\left(\frac{\pi}{4}-\frac{\pi}{10}\right)\right)=0$$

37. (A)

If $A+B=\frac{\pi}{4}$

$\tan\left(A+B\right)=1$

$\tan A+\tan B+\tan A\tan B+1=2$

$\left(1+\tan A\right)\left(1+\tan\right)=2$

$A=11^\circ$ $\qquad\qquad$ $B=34^\circ$

$\left(1+\tan A\right)\left(1+\tan 34^\circ\right)=2$

$A=17^\circ$ $\qquad\qquad$ $B=28^\circ$

$\left(1+\tan 17^\circ\right)$ $\qquad\qquad$ $\left(1+\tan 28^\circ\right)=2$

Hence $\dfrac{\left(1+\tan 11^\circ\right)\left(1+\tan 34^\circ\right)}{\left(1+\tan 17^\circ\right)\left(1+\tan 28^\circ\right)}=1$

38. $$\frac{4\sin 9^\circ\sin 21^\circ\sin 39^\circ\sin 51^\circ\sin 69^\circ\sin 81^\circ}{\sin 54^\circ}$$

$$=\frac{4\left(\sin 9^\circ\sin 31^\circ\sin 69^\circ\right)\left(\sin 21^\circ\sin 39^\circ\sin 81^\circ\right)}{\sin 54^\circ}$$

$$=\frac{\frac{4\left(\sin 27\right)}{4}\left(\frac{\sin 63}{4}\right)}{\sin 54}$$

$$=\frac{1}{4}\frac{\sin 27^\circ \cos 27^\circ}{\sin 54}=\frac{1}{8}$$

**39. NO SOLUTION**

40. (A)

$$\sin^2\theta=\frac{1}{2}\left(\frac{x}{y}+\frac{y}{x}\right)$$

Possible only when $x=y\left(\frac{x}{y}+\frac{y}{x}\in(-\infty,-2]\cup[2,\infty)\right)$

41. $$\cos\frac{6\pi}{7}=-\cos\frac{\pi}{7}$$

$$\cos\frac{5\pi}{7}=-\cos\frac{2\pi}{7}$$

$$\cos\frac{3\pi}{7}=-\cos\frac{4\pi}{7}$$

$$\left(\cos\frac{\pi}{7}+\cos\frac{6\pi}{7}\right)+\left(\cos\frac{2\pi}{7}+\cos\frac{5\pi}{7}\right)+\left(\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}\right)+\cos\pi=-1$$

42. (D)

$$\cot(\alpha+\beta)=0$$

$$\alpha+\beta=\frac{\pi}{2}$$

$$\sin(\alpha+2\beta)=\sin\left(\frac{\pi}{2}+\beta\right)=\cos\beta$$

43. (C)

$$\sin x=1-\sin^2 x$$

$$\sin x=\cos^2 x$$

$$=\sin^6+3\sin^5 x+3\sin^4 x+\sin^3 x-2$$

$$=\left(\sin x+\sin^2 x\right)^3-2$$

$$1-2=-1$$

44. (D)

$$\frac{1}{4}\left[\sqrt{3}\cos 30^\circ-\sin 23^\circ\right]$$

$$\frac{1}{2}\left[\sin 60^\circ\cos 23-\cos 60\sin 23\right]$$

$$\frac{1}{2}\sin 37^\circ$$

45. (B)

$$\tan 60^\circ=\sqrt{3}$$

$$\tan\left(20^\circ+40^\circ\right)=\sqrt{3}$$

$$\tan 20^\circ+\tan 40^\circ=\sqrt{3}-\sqrt{3}\tan 20^\circ\tan 40^\circ$$

$$\tan 20^\circ + \tan 40^\circ + \sqrt{3}\tan 20^\circ \tan 40^\circ = \sqrt{3}$$

46. (A)

$$\tan(\alpha+\beta)=1$$
$$\tan\alpha+\tan\beta=1-\tan\alpha\tan\beta$$
$$(1+\tan\alpha)(1+\tan\beta)=2$$
$$f(\theta)=\frac{1}{1+\tan\theta}$$
$$f(\alpha)f(\beta)=\frac{1}{(1+\tan\alpha)(1+\tan\beta)}=\frac{1}{2}$$

47. (A)

If $A+B+C=\pi$ $\quad \tan A+\tan B+\tan C=\tan A+\tan B+\tan C$

$\tan A\tan B\tan C=6$ $\quad \tan A+\tan B=3$

$\tan A\tan B=2$ $\quad \tan A=1$

$\Rightarrow \tan C=3$ $\quad \tan B=2$

48. (D)

$$\frac{(\sin\alpha+\cos\alpha)^2}{\cos 2\alpha\left(\frac{\tan\alpha+1}{\tan\alpha-1}\right)}-\frac{1}{4}\sin 2\alpha\left[\cot\frac{\alpha}{2}-\tan\frac{\alpha}{2}\right]$$

$$\frac{(\sin\alpha+\cos\alpha)^2(\sin\alpha-\cos\alpha)}{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)(\sin\alpha+\cos\alpha)}-\frac{1}{42}\sin\alpha\cos\alpha\frac{\cos\alpha}{\left(\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)}$$

$$1-\cos^2\alpha=\sin^2\alpha$$

49. $x=\sqrt{2+\sqrt{2-\sqrt{2+x}}}$

$$x=2\cos\theta$$
$$2\cos\theta=\sqrt{2+\sqrt{2-2\cos\frac{\theta}{2}}}$$
$$=\sqrt{2+\sqrt{2(1-\cos)\left(\frac{\theta}{2}\right)}}$$
$$=\sqrt{2+2\sin\frac{\theta}{4}}$$
$$=\sqrt{2\left(1+\cos\left(\frac{\pi}{2}-\frac{\theta}{4}\right)\right)}$$
$$2\cos\theta=2\cos\left(\frac{\pi}{4}-\frac{\theta}{8}\right)$$
$$\theta=\frac{\pi}{4}-\frac{\theta}{8}$$
$$\frac{9\theta}{8}=45^\circ$$
$$\theta=40^\circ$$

Hence $x=2\cos 40^\circ$

50. (B)

$$\frac{1}{\sin 1^\circ}\left(\frac{\sin\left(46^\circ-45^\circ\right)}{\sin 46\sin 45}+\frac{\sin\left(48-47\right)}{\sin 48\sin 47}-------\right)$$

$$\frac{1}{\sin 1^\circ}\left(\cot 45-\cot 46+\cot 47-\cot 48------\right)$$

$$=\frac{1}{\sin 1^\circ}=\operatorname{cosec}1^\circ$$

51. $2\sin x+4\cos x+12\sin y+5\cos y+5\cos y=18$

$$5\cos\left(x-\tan^{-1}\frac{3}{4}\right)+13\cos\left(y-\tan^{-1}\frac{12}{5}\right)=8$$

Possible $x=\tan^{-1}\frac{3}{4}$

$y=\tan^{-1}\frac{12}{5}$

$$\tan\left(x+y\right)=\frac{\frac{3}{4}+\frac{12}{5}}{1-\frac{3}{4}\times\frac{12}{5}}=-\frac{63}{16}$$

52. $\sum\tan B\tan C=x$

$$\left(1-\tan B\tan C\right)=\frac{\cos\left(B+C\right)}{\cos B\cos C}=\frac{-\cos A}{\cos B\cos C}$$

$$\tan B\tan C=1+\frac{\cos A}{\cos B\cos C}$$

$$\sum\tan B\tan C=3+\sum\frac{\cos A}{\cos B\cos C}$$

$$=3+\sum\frac{\cos^2 A}{\cos A\cos B\cos C}$$

$$\sum\cos^2 A=\frac{3+\cos 2A+\cos 2B+\cos 2C}{2}$$

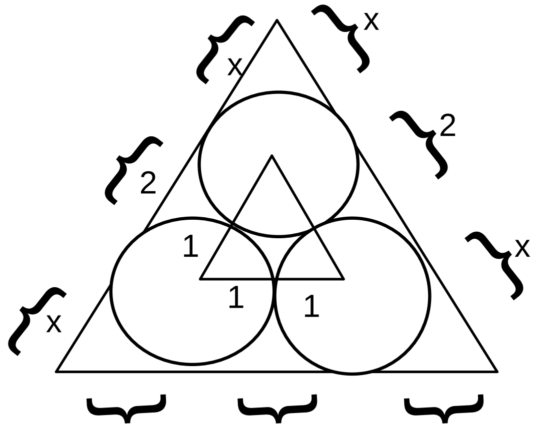
$$\frac{2-4\pi\cos A}{2}=1-2\pi\left(\cos A\right)$$

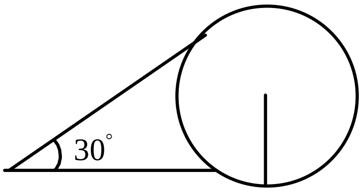
$$\sum\tan B\tan C=3+\frac{1-2\pi\cos A}{\pi\cos A}$$

$$=1+\frac{1}{\pi\cos A}$$

$$=\frac{9+1}{9}$$

53. $\tan 30^\circ=\frac{1}{x}=\frac{1}{\sqrt{3}}$




$x = \sqrt{3}$

Side of equilateral triangle

$= 2 + 2\sqrt{3}$

Area $= \dfrac{\sqrt{3}}{4}\left(2 + 2\sqrt{3}\right)^2$

$= 4\sqrt{3} + 6$

54. $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + (m-1)\dfrac{\pi}{4}\right) = 4\sqrt{2}$

$$\frac{1}{\sin\frac{\pi}{4}} \sum_{m=1}^{6} \frac{\sin\frac{\pi}{4}}{\sin\left(\theta + (m-1)\frac{\pi}{4}\right)\sin\left(\theta + \frac{M\pi}{4}\right)} = 4\sqrt{2}$$

$$= \sqrt{2}\sum_{M=1}^{6} \frac{\sin\left(\theta + m\frac{\pi}{4} - \left(\theta + (m-1)\frac{\pi}{4}\right)\right)}{\sin\left(\theta + (m-1)\frac{\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)}$$

$$\frac{\sin(A-B)}{\sin A \sin B} = \cot B - \cot A$$

$$\sqrt{2}\sum_{m=1}^{6}\left(\cot\left(\theta + (m-1)\frac{\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right)\right)$$

$$= \sqrt{2}\left(\cot\theta - \cot\left(\theta + \frac{3\pi}{2}\right)\right) = 4$$

$$\cot\theta + \tan\theta = 4$$

$$\sin 2\theta = \frac{1}{2}$$

$2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$

55. $3\sin^2 A + 2\sin^2 B = 1$

$\Rightarrow 3\sin^2 A = \cos 2B$ ________(1)

Also $3\sin 2A = 2\sin 2B$ __________(2)

From (1) & (2)

$$\frac{3\sin^2 A}{3(2\sin A\cos A)} = \frac{\cos 2B}{2\sin 2B}$$

$\Rightarrow \tan A = \cot 2B$

$\tan A = \tan\left(\frac{\pi}{2} - 2B\right)$

$A + 2B = \frac{\pi}{2}$

56. $\tan 20^\circ \tan 80^\circ \cot 50^\circ$

$= \tan 20 \tan(60+20)\tan(60-20)$

$= \tan(3\times 20^\circ)$

$= \sqrt{3}$

57. $a\cos x + b\sin x = c$

Put $\cos x = \frac{1-t^2}{1+t^2}$ and $\sin x = \frac{2t}{1+t^2}$

Where $t = \tan\frac{x}{2}$

$a\left(\frac{1-t^2}{1+t^2}\right) + b\left(\frac{2t}{1+t^2}\right) = c$

$(a+c)t^2 - 2bt + c - a = 0$

Roots of this equation are $\tan\frac{x_1}{2}$ and $\tan\frac{x_2}{2}$

$$\tan\frac{x_1}{2} + \tan\frac{x_2}{2} = \frac{2b}{a+c}$$

$\tan\frac{x_1}{2}, \tan\frac{x_2}{2} = \frac{c-a}{a+c}$

$$\tan\left(\frac{x_1+x_2}{2}\right) = \frac{\frac{2b}{a+c}}{1-\frac{c-a}{c+1}} = \frac{2b}{2a} = \frac{b}{a}$$

58. $\sin x + \cos x = \frac{\sqrt{7}}{2}$

$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = \frac{\sqrt{7}}{2}$ Where $t = \tan\frac{x}{2}$

$\left(\sqrt{7}+2\right)t^2-4t+\left(\sqrt{7}-2\right)=0$

$$t=\frac{4\pm\sqrt{16-4(57+2)(57-2)}}{2\left(\sqrt{7}+2\right)}=\frac{1}{3}\left(\sqrt{7}-2\right)\text{ or }\left(\sqrt{7}-2\right)$$

$\tan\frac{x}{2}<\tan\frac{\pi}{8}$

$x\in\left[0,\frac{\pi}{4}\right]\quad \frac{x}{2}\in\left[0,\frac{\pi}{8}\right]$

So $\tan\frac{x}{2}$ will have lower value

$\tan\frac{x}{2}=\frac{\sqrt{7}-2}{3}$

59. $\cos x+\cos y-\cos(x+y)=\frac{3}{2}$

$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)-2\cos^2\left(\frac{x+y}{2}\right)+1=\frac{3}{2}$

$4\cos^2\left(\frac{x+y}{2}\right)-4\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)+1=0$

$D\geq 0$

$16\cos^2\left(\frac{x-y}{2}\right)-16\geq 0$

$-16\sin^2\left(\frac{x-y}{2}\right)\geq 0$

Possible only at $x=y$

60. $\tan x=n\tan y \qquad (\tan y=t)$

$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$

$=\frac{nt-t}{1+nt^2}$

$\tan(x-y)=\frac{t(n-1)}{1+nt^2}=\alpha$

$\sec^2(x-y)$ will be max if $\tan(x-y)$ is max

$\alpha+n\alpha t^2=(n-1)t$

$D\geq 0$

$(n-1)^2-4n\alpha^2\geq 0$

$\alpha^2\leq\frac{(n-1)^2}{4n}$

$\tan^2(x-y)\leq\frac{(n-1)^2}{4n}$

$\sec^2(x-y)\leq\frac{(n-1)^2}{4n}+1$

$$\le \frac{(n+1)^2}{4n}$$