TRIGO – I (SOLUTION)

EXERCISE – 1(B)

1. (A)

$$\frac{-\tan\left(\frac{\pi}{2}-n\right)x-\cos\left(\frac{\pi}{2}+n\right)-\sin^3\left(\frac{\pi}{2}+n\right)}{\cos\left(\frac{\pi}{2}-n\right)+\left(\frac{\pi}{2}+n\right)}$$

$$=\frac{-\cot x\sin n-\cos^3 x}{-\sin n\times\cot x}$$

$$=\frac{\cos x+\cos^3 x}{\cos x}=1+\cos^2 x$$

2. (A)

$$\frac{\sin\theta}{\cos\theta}\left(\sec^2\theta\right)^3+\frac{\cos\theta}{\sin\theta}\left(1+\cot^2\theta\right)^3$$

$$=\tan\theta\left(1+\tan^2\theta\right)^3+\frac{1}{\tan\theta}\left(\frac{1+\tan^2\theta}{\tan^2\theta}\right)^3$$

$$=\sqrt{\frac{a}{b}}\left(1+\frac{a}{b}\right)^3+\sqrt{\frac{b}{a}}\left(1+\frac{b}{a}\right)^3$$

$$=\left(a+b\right)^3\left(\frac{a^{\frac{1}{2}}}{b^{\frac{7}{2}}}+\frac{b^{\frac{1}{2}}}{a^{\frac{7}{2}}}\right)$$

$$=\frac{\left(a+b\right)^3\left(a^4+b^4\right)}{\left(ab\right)^{\frac{7}{2}}}$$

3. (C)

$$f\left(x\right)=3\left[\sin^4\left(\frac{\pi}{2}-n\right)+\sin^4 x\right]-2\left[\cos^6 n+\sin^6 n\right]$$

$$=3\left[\left(\cos^2 n\right)^2+\left(\sin^2 n\right)^2\right]-2\left[\left(\sin^2 n\right)^3+\left(\cos^2 n\right)^3\right]$$

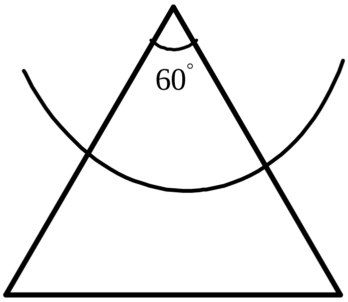
$$=3\left[1-2\sin^2 n\cos^2 n\right]-2\left[1-3\sin^2 n\cos^2 n\right]$$

$$=1$$

4. (D)

$$\frac{\sin^2 20\,\frac{\left(1-\cos^2 20\right)}{\cos^2 20}}{\frac{\sin^2 20.\sin^2 20}{\cos^2 20}}=1$$

5. (A)



Area of $\Delta = \frac{\sqrt{3}}{a}.4 = 4\sqrt{3}$

$2z \_\_\_\_\_\_ zr^2$

$1 \_\_\_\_\_\_ \frac{zr^2}{2\pi}$

$\frac{\pi}{3} \_\_\_\_\_ \frac{zr^2}{2r}.\frac{z}{3}$

Given $\frac{zr^2}{0} = \frac{4\sqrt{3}}{z}$

$zr^2 = 12\sqrt{3}$

$r = \sqrt{\frac{12\sqrt{3}}{z}}$

6. (A)

$2\cos 10 + \sin(90+10) + \sin(3\times 300 - 80) + \sin(27\times 360 + 280)$

$= 2\cos 10 + \cos 10 - 2\sin 80$

$= \cos 10 + 2(\cos 10 - \sin(90-10))$

$= \cos 10 + 2(\cos 10 - \cos 10) = \cos 10$

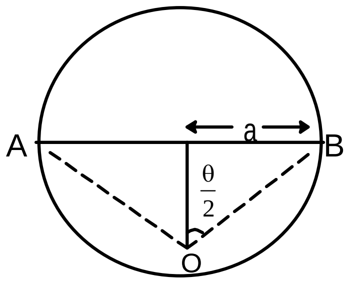
7. (B)

$\sin^2 n + \cos ec^2 x + 2 + \cos^2 n + \sec^2 n + 2 - t^2 n - \cot^2 n - 2$

$= 3 + (\cos ec^2 n - \cot^2 n) + (\sec^2 n - t^2 n)$

$= 5$

8. (C)



$\sin\frac{\theta}{2} = \frac{a}{0B}$

$OB = \frac{a}{\sin\frac{\theta}{2}}$

Required area $= zr^2 = \pi a^2 \cos ec^2 \frac{\theta}{2}$

$$= za^2\left(1+\cot^2\frac{\theta}{2}\right)$$

9. (C)

$$\cos^2(90-17)+\cos^2 47-\sin^2(90-47)+\sin^2(90+17)$$
$$=\sin^2 17+\cos^2 47-\cos^2 47+\cos^2 17$$
$$=1$$

10. (D)

(i) $\sin(2\times 360+4s)=\sin 45=\frac{1}{\sqrt{2}}$

(ii) $\frac{1}{-\sin(3\times 360+330)}=\frac{1}{-\sin(360-30)}=\frac{1}{\sin 30}=2$

(iii) $\tan\left(4z+\frac{\pi}{3}\right)=\tan\left(\frac{z}{3}\right)=\sqrt{3}$

(iv) $\frac{1}{-t\left(4z-\frac{3}{4}\right)}=\frac{1}{t_{\frac{z}{a}}}=1$

11. (C)

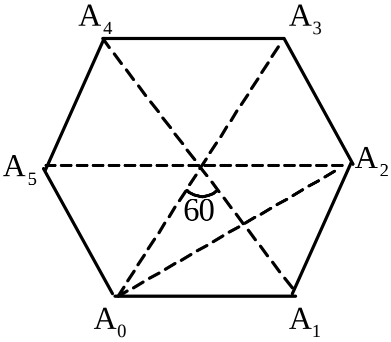
$$\ell^2=\left[\sin\theta-\sin\left(\frac{\pi}{2}-\theta\right)\right]^2+\left[\cos\theta+\cos\left(\frac{\pi}{2}-\theta\right)\right]$$
$$=(\sin\theta-\cos\theta)^2+(\cos\theta+\sin\theta)^2$$
$$=1+1=2$$
$$\ell=\sqrt{2}$$

12. (B)

Given $y=\frac{2\sin\alpha}{(1+\cos\alpha+\sin\alpha)}$

$$\frac{(1+\sin\alpha-\cos\alpha)(1+\sin\alpha+\cos\alpha)}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$
$$=\frac{(1+\sin\alpha)^2-\cos^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}=\frac{(1+2\sin\alpha+\sin^2\alpha+\sin^2\alpha)}{(1+\sin\alpha)(1+\sin x+\cos)}$$
$$=\frac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}=y$$

13. (C)



$A_0A_1 = 1$

$\cos 120^\circ = \dfrac{1^2 + 1^2 - A_0A_2{}^2}{2.1.1.}$

$-\dfrac{2}{2} = 2 - A_0A_2{}^2$

$A_0A_2{}^2 = 3$

$A_0A_2 = \sqrt{3}$

$A_0A_4 = \sqrt{3}$

$A_0A_1 \times A_0A_2 + A_0A_4 = 1.\sqrt{3}.\sqrt{3} = 3$

14. (A)

$$\tan^2 30 + 4\sin^2 45 + \frac{1}{3}\cos^2 30 = \frac{1}{3} + \frac{4}{2} + \frac{1}{3} \times \frac{3}{4}$$
$$= \frac{1}{3} + \frac{1}{4} + 2$$
$$= 2.\frac{7}{12}$$

15. (D)

16. (D)

$$\frac{\tan^2 60 - 2\tan^2 45 + \sec^2 30}{3\sin^2 45 \sin 90 + \cos^2 60.\cos^3 0} = \frac{3 - 2 + \frac{4}{3}}{3.\frac{1}{2} + \frac{1}{4}}$$

$$= \frac{1 + \frac{4}{3}}{\frac{1}{2}\left(3 + \frac{1}{2}\right)} = \frac{\frac{7}{3}}{\frac{1}{2}.\frac{7}{2}} = \frac{4}{3}$$

17. (D)

$$\frac{\tan\theta}{\left(\tan^3\theta + t\theta\right)\cos^2\theta} = \frac{1}{\left(1 + t^2\theta\right)\cos^2\theta}$$

$$= \frac{1}{\frac{\left(\sin^2\theta + \cos^2\theta\right)}{\cos^2\theta}.\cos^2\theta}$$
$$= 1$$

18. (D)

$$\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} = \cos ec^2\theta \sec^2\theta$$

19. (b)

$\sin\left(180 + 20\right) + \cos\left(180 + 20\right)$

$= -\sin 20 - \cos 20 < 0$

20. (A)

$$\frac{1}{2}\left[2\cos\alpha\sin(\beta-r)+2\cos\beta\sin(r-\alpha)+2\cos r\sin(\alpha-\beta)\right]$$

$$=\frac{1}{2}\left[\sin(\alpha+\beta r)-\sin(\alpha+r-\beta)+\sin(\beta+r-\alpha)-\sin(\alpha+\beta-r)+\sin(\alpha+r-\beta)-\sin(r+\beta-\alpha)\right]$$

$$=0$$

21. (D)

$$\frac{\sin\alpha}{\sin\beta}=\frac{5}{3}$$

$$\frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}=\frac{8}{2}=4\ (\text{C \& D})$$

$$\frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}=4$$

$$\frac{t\left(\frac{\alpha+\beta}{2}\right)}{t\left(\frac{\alpha-\beta}{2}\right)}=4$$

22. (B)

$$(\sin 2\theta+\sin 2\phi)^2+(\cos 2\theta+\cos 2\phi)^2=\frac{1}{\alpha}+\frac{9}{4}$$

$$1+1+2\cos(2\theta-2\phi)=\frac{10}{4}=\frac{5}{2}$$

$$2\cos(2\theta-2\phi)=\frac{5}{2}-2$$

$$\cos 2(\theta-\phi)=\frac{1}{4}$$

$$2\cos^2(\theta-\phi)=\frac{1}{4}+1$$

$$\cos^2(\theta-\phi)=\frac{5}{8}$$

23. (A)

$$(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)=1$$

$$(\sec\theta+\tan\theta)-(\sec\theta-\tan\theta)=2a-\frac{1}{2a}$$

Let $\sec\theta-\tan\theta=t$

$$\frac{1}{t}-t=\left(2a-\frac{1}{2a}\right)$$

$$1-t^2=\left(2a-\frac{1}{2a}\right)t$$

$$t^2+\left(2a-\frac{1}{2a}\right)t-1=0$$

$$t=\frac{\left(\frac{1}{2a}-2a\right)\pm\sqrt{\left(2a-\frac{1}{2a}\right)^2+4}}{2}$$

$$t=\frac{\left(\frac{1}{2a}-2a\right)\pm\left(2a+\frac{1}{2a}\right)}{2}$$

$$t=\frac{1}{2a},-2a$$

24. (B)

$$\sec^2\theta\ge 1$$

$$\frac{4xy}{(x+y)^2}\ge 1$$

$$(x+y)^2-4xy\le 0$$

$$(x-y)^2\le 0$$

$$x=y\quad \text{only}\begin{pmatrix}x\ne 0\\ y\ne 0\end{pmatrix}$$

25. (C)

$$f(\theta)=\sin\theta\left(\sin\theta+3\sin\theta-4\sin^3\theta\right)$$

$$f(\theta)=4\sin^2-4\sin^4\theta$$

$$f(\theta)=4\sin^2\theta\left(1-\sin^2\theta\right)\ge 0\quad\forall\theta$$

26. (C)

$$A+B+C=\pi$$

$$\tan A+\tan B+\tan C=\tan A+B+C$$

27. (B)

$$A+B+C=\frac{3\pi}{2}$$

$$\cos 2A+\cos 2B+\cos 2C+4\sin A\sin B\sin C$$

$$=2\cos(A+B)\cos(A-B)+2\cos^2 C-1+a\sin A\sin B\sin C$$

$$=2\cos\left(3\frac{\pi}{2}-c\right)\cos(A-B)+2\cos^2 C-1+4\sin A.\sin B.\sin C$$

$$=-2\sin C\cos(A-B)+1-2\sin^2 C+4\sin A\sin B\sin C$$

$$=-2\sin C\left[\cos(A-B)-\cos(A+B)\right]+4\sin A\sin B\sin C$$

$$=-4\sin A\sin B\sin C+4\sin A\sin B\sin C+1$$

28. (A)

$$\frac{A+B+C}{2}=\frac{\pi}{2}$$

$$\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{c}{2}\right)$$

$$\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\frac{1}{\tan\frac{c}{2}}$$

$$\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{A}{2}\tan\frac{C}{2}=1$$

$$\tan\frac{C}{2}=1-\frac{2}{9}=\frac{7}{9}$$

29. (D)

30. (D)

$$\frac{1}{4}\left(4\cos\theta\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)\right)$$

$$=\frac{\cos 3\theta}{4}$$

31. (B)

$$\frac{\cos 20+4(\cos 40-\cos 60)\sin 70}{\cos^2 10}$$

$$=\frac{\cos 20+4\cos 40\cos 20-2\cos 20}{\cos^2 10}$$

$$=\frac{2(\cos 20+2\cos 60+2\cos 20-2\cos 20)}{2\cos^2 10}$$

$$=\frac{2(\cos 20+1)}{(1+\cos 20)}=2$$

32. (B)

$$\cot\theta-2\cot 2\theta$$

$$\frac{1}{\tan\theta}-\frac{2\times(1-\tan^2\theta)}{2\tan\theta}$$

$$=\frac{\tan^2\theta}{\tan\theta}=\tan\theta$$

33. (D)

$$f(\theta)=5\cos\theta+3\cos\left(\theta+\frac{\pi}{3}\right)+3$$

$$=5\cos\theta+3\cos\theta.\cos\frac{\pi}{3}-3\sin\theta\sin\frac{\pi}{3}+3$$

$$=\frac{13}{2}\cos\theta-\frac{3\sqrt{3}}{2}\sin\theta+3$$

$$-\sqrt{\frac{13^2}{4}+\frac{27}{4}}+3\le f(\theta)\le\sqrt{\frac{13^2}{4}+\frac{27}{4}}+3$$

$$-4\le f(\theta)\le 10$$

34. (D)

$$A+B+C=\pi$$

$$\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=\frac{\pi}{2}$$

$$\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)$$

$$\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\frac{1}{\tan\frac{C}{2}}$$

$$\Rightarrow\sum\tan\frac{A}{2}\tan\frac{B}{2}=1$$

35. (B)

$$a=\sin 170^\circ+\cos 170^\circ$$

$$=\sin(180-10)+\cos(180-10)$$

$$=\sin 10-\cos 10$$

$$\Rightarrow a=-\text{ve}$$

$$\{\because \cos 10^\circ>\sin 10^\circ\}$$

36. (B)

$$\sin^2 A+\sin^2(A-B)+2\sin A\cos B\sin(B-A)$$

$$=\sin^2 A+\sin^2(A-B)+[\sin(A+B)+\sin(A-B)]\sin(B-A)$$

$$=\sin^2 A+\sin^2(A-B)+\sin(A+B)\sin(B-A)-\sin^2(A-B)$$

$$=\sin^2 A+\sin^2 B-\sin^2 A$$

$$=\sin^2 B$$

37. (C)

$$\sin\theta+\cos\beta$$

Maximum Value $=2$

38. (A)

$$\sin\alpha-\cos\alpha=\frac{1}{5}$$

$$\sin^2\alpha+\cos^2\alpha-2\sin\alpha\cos\alpha=\frac{1}{25}$$

$$\sin 2\alpha = \frac{24}{25}$$

$$\frac{2\tan\alpha}{1+\tan^2\alpha} = \frac{24}{25}$$

$$\Rightarrow \tan\theta = \frac{4}{3} \text{ or } \frac{3}{4}$$

Now, $$\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{4}{3} \text{ or } \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{3}{4}$$

$$\Rightarrow \tan\alpha = \frac{1}{2} \text{ or } \tan\alpha = -3$$

39. (C)

$$y = \frac{12}{9+5\left(\frac{3}{5}\cos n + \frac{4}{5}\sin x\right)}$$

$$= \frac{12}{9+5\sin(\alpha+n)} \text{ let} \left(\sin\alpha = \frac{3}{5}\right)$$

$$y_{\text{maximum}} = \frac{12}{4} = 3$$

40. (A)

Let $\lambda = 2\cos\beta - 5\sin\beta$

Let $S = 3\sin\beta + 5\cos\beta$

$\lambda^2 + S^2 = 9 + 25$

$\therefore \lambda^2 = 9$

41. (C)

$$m^2 - n^2 = \frac{\sin^2\alpha}{\sin^2\beta} - \frac{\cos^2\alpha}{\cos^2\beta}$$

$$= \frac{(\sin\alpha\cos\beta)^2 - (\sin\beta\cos\alpha)^2}{\sin^2\beta\cos^2\beta}$$

$$\therefore (m^2 - n^2)\sin^2\beta = \frac{\sin(\alpha+\beta)\sin(\alpha-\beta)}{\cos^2\beta}$$

$$(m^2 - n^2)(\sin^2 B) = \sin^2\alpha - \sin^2\beta$$

$$= \frac{\cos^2\beta - \cos^2\alpha}{\cos^2\beta}$$

$$= 1 - \left(\frac{\cos\alpha}{\cos\beta}\right)^2$$

$$= 1 - n^2$$

42. (C)

$$\frac{b}{y}=\cot\theta \ \ \& \ \ \frac{a}{x}=\operatorname{cosec}\theta$$

$$\operatorname{cosec}^2\theta-\cot^2\theta=\frac{a^2}{n^2}-\frac{b^2}{y^2}$$

$$\frac{a^2}{n^2}-\frac{b^2}{y^2}=1$$

43. (C)

$$\sin\beta=\frac{12}{13},\cos\beta=\frac{5}{13}$$

$$\therefore\frac{13\sin\beta+\frac{5}{\cos\beta}}{5\tan\beta+\frac{6}{\sin\beta}}=\frac{13\times\frac{12}{13}+\frac{5}{5}\times 13}{5.\frac{12}{5}+\frac{6}{12}\times 13}$$

$$=\frac{25\times 2}{37}=\frac{50}{37}$$

44. (C)

If $A+B+C=n\pi$ then

$$\tan A+\tan B+\tan C=\tan A\tan B\tan C$$

45. (D)

$$y_{\text{minimum}}=\frac{1}{9}$$

46. (D)

$$4\sin\frac{4}{a}\cos\frac{4}{a}\cos\frac{4}{2}$$

$$=2\sin\frac{4}{2}\cos\frac{4}{2}=\sin 4$$

47. (B)

$$\left(\frac{\cos B}{\cos B}+\frac{\cos C}{\sin C}\right)\left(\frac{\cos C}{\sin C}+\frac{\cos A}{\sin A}\right)\left(\frac{\cos A}{\sin A}+\frac{\cos B}{\sin B}\right)$$

$$=\frac{\sin(B+C).\sin(A+C).\sin(A+B)}{\sin^2 A.\sin^2 B\sin^2 C}$$

$$=\operatorname{cosec}A.\operatorname{cosec}B.\operatorname{cosec}C$$

48. (A)

$$\tan^2\alpha+\cot^2\alpha+2=m^2$$

$$\left(\tan^2\alpha+\cot^2\alpha\right)^2=\left(m^2-2\right)^2$$

$$\tan^4\alpha+\cot^4\alpha+2=m^4+4-4m^2$$

$$\tan^4\alpha+\cot^4\alpha=m^4-4m^2+2$$

49. (A)

$$\frac{\frac{\sin\left(\frac{3\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}\sin\left(\theta+\frac{2\theta}{2}\right)}{\frac{\sin\left(\frac{3\theta}{2}\right)}{\sin\frac{\theta}{2}}\cos\left(\theta+\frac{2\theta}{2}\right)}=\frac{\sin\alpha}{\cos\alpha}$$

$$\tan 2\theta=\tan\alpha$$

$$\theta=\frac{\alpha}{2}$$

50. (D)

$$\frac{\cos 10-\sqrt{3}\sin 10}{\sin 10.\cos 10}$$

$$=2.2\frac{\left(\frac{1}{2}\cos 10-\frac{\sqrt{3}}{2}\sin 10\right)}{2\sin 10\cos 10}$$

$$=\frac{4\sin(30-10)}{\sin 20}=4$$

51. (B)

$$B+C=\pi-A$$

$$\cos B\cos C-\sin B\sin C=-\cos A$$

$$\cos B\cos C-\sin B\sin C=-\cos B\cos C$$

$$2\cos B\cos C=\sin B\sin C$$

$$\tan B\tan C=2$$

52. (D)

$$\cos 3\theta=4\cos^3\theta-3\cos\theta$$

$$\cos^3\theta=\left(\frac{\cos 3\theta+3\cos\theta}{4}\right)$$

$$\frac{1}{4}\sum_{r=0}^{10}\left[\cos(\pi r)+3\cos\left(\frac{\pi r}{3}\right)\right]$$

$$=\frac{1}{4}\left[(\cos\theta+\cos\pi+\cos\lambda-\cos 10\pi)+3\left(\cos\theta+\cos\frac{\pi}{3}+----+\cos 10\frac{\pi}{2}\right)\right]$$

$$=\frac{1}{4}\left(\frac{\sin\left(11.\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)}\left(10.\frac{\pi}{2}\right)+3\frac{\sin\left(11.\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)}\cos\left(10.\frac{\pi}{36}\right)\right)$$

$$=\frac{1}{4}\left(\tan 1+3\frac{\sin\left(2\pi-\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)}\cos\left(2\pi-\frac{\pi}{3}\right)\right)$$

$$=\frac{1}{4}\left(\tan+1-3\frac{\sin\frac{\pi}{6}}{\sin\frac{\pi}{6}}\times\cos\frac{\pi}{3}\right)$$

$$=+\frac{1}{4}\left(+1-\frac{3.1}{2}\right)=\frac{-1}{8}$$

53. (B)

$$4_1 4_n - 4_{n-1}$$

$$=2\cos\theta 2\cos n\theta-2\cos(n-1)\theta$$

$$=2[2\cos\theta.\cos n\theta-\cos n\theta\cos\theta-\sin n\theta\sin\theta]$$

$$=2\cos(n+1)\theta=4_{n+1}$$

54. (C)

$$\cos(5\theta)=\cos(2\theta+3\theta)$$

$$=\cos 2\theta.\cos 3\theta-\sin 2\theta\sin 3\theta$$

$$=(2\cos^2\theta-1)(4\cos^3\theta-3\cos\theta)-2\sin\theta\cos\theta(3\sin\theta-4\sin^3\theta)$$

$$=8\cos^5\theta-6\cos^3\theta-4\cos^3\theta+3\cos\theta\ -2\sin^2\theta.\cos\theta(3-4\sin^2\theta)$$

$$=8\cos^5\theta-6\cos^3\theta-4\cos^3\theta+3\cos\theta-2(1-\cos^2\theta).\cos\theta(4\cos^2\theta-1)$$

$$=16\cos^5\theta-20\cos^3\theta+5\cos\theta$$

55. (D)

$$y=\frac{\sin\theta(3-4\sin^2\theta)}{\sin\theta}$$

$$\sin^2\theta=\left(\frac{3-y}{4}\right)$$

$$0<\sin^2\theta\le 1$$

$$0<\frac{3-y}{4}\le 1$$

$$0<3-y\le 4$$

$$-3<-y\le 1$$

$$-1\le y<3$$