TRIGO – I (SOLUTION)

EXERCISE – 1(A)

1. (B)

π radian = 180 degree $\Rightarrow$ 1 radian = $\frac{180}{\pi}$ degree $\simeq$ 57.3 degree, hence sin $1^c > 1^0$.

2. (B)

Length of arc of circle of radius r subtending θ at the center = $r\theta$.

Hence $15 = r \times \frac{3}{4}$ or r = 20 cm

3. (D)

In second quadrant sin A > 0, cos A < 0 & tan A < 0.

Given $\tan A = -\frac{4}{3}$, hence $\sin A = \frac{4}{5}$ & $\cos A = -\frac{3}{5}$

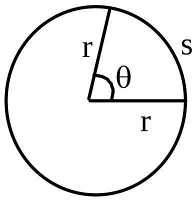
Now $2\cot A - 5\cos A + \sin A = -2 \times \frac{3}{4} \times \left(-\frac{3}{5}\right) + \frac{4}{5} = \frac{23}{10}$

4. (D)

S + 2r = mr

$\Rightarrow$ S = (m – 2) r

$\therefore \theta = \frac{S}{r} = (m-2)^c$



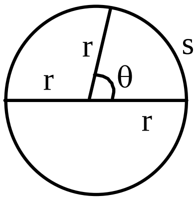
5. (C)

$\sin 15^0 \cos 15^0 = \frac{1}{2}\sin 30^0 = \frac{1}{4}$

6. (C)

$\pi r = S + 2r$

$\Rightarrow S = (\pi - 2)r$



7. (B)

$\frac{\cot\theta + \text{cosec}\theta - 1}{\cot\theta - \text{cosec}\theta + 1}$

$= \frac{\frac{1}{\text{cosec}\theta - \cot\theta} - 1}{\cot\theta - \text{cosec}\theta + 1} \quad \left(\because (\text{cosec}\theta + \cot\theta)(\text{cosec}\theta - \cot\theta) = 1\right)$

$= \frac{1 - \text{cosec}\theta + \cot\theta}{(\text{cosec}\theta - \cot\theta)(\cot\theta - \text{cosec}\theta + 1)}$

$= \frac{1}{\text{cosec}\theta - \cot\theta} = \text{cosec}\theta + \cot\theta$

$= \frac{1 + \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 - \cos\theta}$

8. (A)

$\tan(90^0 - \theta) = \cot\theta \Rightarrow \tan 89^0 = \cot 1^0$, tan 88^0 = cot 2^0, tan87^0 = cot3^0,…etc

Also $\tan\theta \times \cot\theta = 1$, hence tan$1^0$ tan2^0 tan3^0….tan89^0 = 1

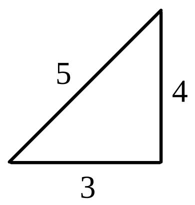
9. (C)

$\tan A = -\frac{4}{3}$

$\cot A = -\frac{3}{4}$

$\cos A = -\frac{3}{5}$

$\sin A = \frac{4}{5}$



($\because$ In 2^{nd} quadrant sin A > 0, cos A < 0, tan A < 0, cot A < 0)

$\therefore 2\cot A - 5\cos A + \sin A = -\frac{6}{4} + 3 + \frac{4}{5}$

10. (A)

$\sin\theta - \cos\theta = 1 \Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = 1$

$\Rightarrow \sin\theta\cos\theta = 0$

11. (C)

$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5\tan\theta - 3}{5\tan\theta + 2}$

$5\tan\theta = 4 \Rightarrow \frac{5\tan\theta - 3}{5\tan\theta + 2} = \frac{4-3}{4+2} = \frac{1}{6}$

12. (A)

Sin x + cosec x = 2 $\Rightarrow$ $\sin^2 x - 2\sin x + 1 = 0$

$\Rightarrow$ sin x = 1 $\Rightarrow$ $\sin^n x + \text{cosec}^n x = 2$

13. (C)

$$x\sin 45^0 \cos^2 60^0 = \frac{\tan^2 60^0 \operatorname{cosec} 30^0}{\sec 45^0 \cot^2 30^0} \Rightarrow x\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)^2 = \frac{\left(\sqrt{3}\right)^2 (2)}{\left(\sqrt{2}\right)\left(\sqrt{3}\right)^2} \Rightarrow x = 8$$

14. (A)

$90^0 < 130^0 < 135^0$ hence sin A > 0 & cos A < 0 & $|\sin A| > |\cos A|$

$\Rightarrow \sin A + \cos A > 0$

15. (A)

$\frac{\cos 17^0 + \sin 17^0}{\cos 17^0 - \sin 17^0} = \frac{1 + \tan 17^0}{1 - \tan 17^0}$

$= \tan\left(45^0 + 17^0\right) = \tan 62^0$

16. (A)

$\tan 75^0 - \cot 75^0 = -2\cot 150^0$

$= 2\tan 60^0 = 2\sqrt{3}$

17. (A)

$\cos^2\alpha + \cos^2\left(\alpha + 120^0\right) + \cos^2\left(\alpha - 120^0\right)$

$= \frac{1 + \cos\alpha}{2} + \frac{1 + \cos 2\left(\alpha + 120^0\right)}{2} + \frac{1 + \cos 2\left(\alpha - 120^0\right)}{2}$

$$= \frac{3+\cos 2\alpha + 2\cos 2\alpha \cos 120^0}{2} = \frac{3+\cos 2\alpha - \cos 2\alpha}{2} = \frac{3}{2}$$

18. (A)

$$\cos 24^0 + \cos 5^0 + \cos 175^0 + \cos 204^0 + \cos 300^0$$
$$= \cos 24^0 + \cos 5^0 + \cos\left(180^0 - 5^0\right) + \cos\left(180^0 + 24^0\right) + \cos\left(360^0 - 60^0\right)$$
$$= \cos 24^0 + \cos 5^0 - \cos 5^0 - \cos 24^0 + \cos 60^0$$
$$= \cos 60^0 = \frac{1}{2}$$

19. (B)

$$\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$$
$$\Rightarrow \cos^2 48^0 - \sin^2 12^0 = \cos 60^0 \cos 36^0 = \frac{\sqrt{5}+1}{8}$$

20. (B)

$$\sin\alpha + \sin\beta = a \Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = a \qquad \ldots(i)$$
$$\& \ \cos\alpha - \cos\beta = b \Rightarrow 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} = -b \qquad \ldots(ii)$$

From (i) & (ii) $\tan\frac{\alpha-\beta}{2} = -\frac{b}{a}$

21. (D)

$$\frac{1}{2}\sin^2\left(\frac{\pi}{4}+\theta\right) = \cos 2\left(\frac{\pi}{4}+\theta\right)$$
$$= \cos\left(\frac{\pi}{2}+2\theta\right) = -\sin 2\theta$$

22. (D)

$$\cos\alpha + \cos\beta = 0 \Rightarrow \cos^2\alpha + \cos^2\beta = -2\cos\alpha\cos\beta$$
$$\& \sin\alpha + \sin\beta = 0 \Rightarrow \sin^2\alpha + \sin^2\beta = -2\sin\alpha\sin\beta$$

Now $\cos 2\alpha + \cos 2\beta = 2\left(\cos^2\alpha + \cos^2\beta - 1\right) = 2\left(1 - \sin^2\alpha - \sin^2\beta\right)$

$$\Rightarrow \cos\alpha + \cos\beta = 2\left(-2\cos\alpha\cos\beta - 1\right) \qquad ...(i)$$
$$\& \ \cos 2\alpha + \cos 2\beta = 2\left(1 + 2\sin\alpha\sin\beta\right) \qquad \ldots(ii)$$

(i) + (ii) $\Rightarrow \cos\alpha + \cos 2\beta = 2\cos(\alpha+\beta)$

23. (B)

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \Rightarrow \tan(A+B) = \frac{-\frac{1}{2}-\frac{1}{3}}{1-\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)}$$

$\Rightarrow$ tan (A + B) = –1

$$\Rightarrow A + B = \frac{3\pi}{4}$$

24. (A)

$$\frac{\sin 3\theta - \cos 3\theta}{\sin\theta + \cos\theta} + 1 = \frac{2\sin\theta - 4\sin^2\theta - 4\cos^3\theta + 3\cos\theta}{\sin\theta + \cos\theta} + 1$$

$$= 4\frac{\sin\theta + \cos\theta - \sin^3\theta - \cos^3\theta}{\sin\theta + \cos\theta}$$

$$= 4\frac{\sin\theta + \cos\theta - \left(\sin^3\theta + \cos^3\theta\right)\left(1 - \sin\theta\cos\theta\right)}{\sin\theta + \cos\theta}$$

$$= 4\sin\theta\cos\theta = 2\sin\theta$$

25. (A)

$$\cos A = m\cos B \Rightarrow \frac{\cos A}{\cos B} = m$$

Apply component to get $\frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$

$$\Rightarrow -\frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\sin\frac{A+B}{2}\sin\frac{A-B}{2}} = \frac{m+1}{m-1}$$

$$\Rightarrow \cot\frac{A+B}{2} = \frac{m+1}{m-1}\tan\frac{B-A}{2}$$

26. (D)

$$\tan 45^0 = \tan\left(180^0 + 45^0\right) \Rightarrow \tan 225^0 = \tan\left(100^0 + 125^0\right)$$

$$\Rightarrow \frac{\tan 100^0 + \tan 125^0}{1 - \tan 100^0 \tan 125^0} = 1$$

$$\Rightarrow \tan 100^0 + \tan 125^0 + \tan 100^0 \tan 125^0 = 1$$

27. (D)

As A & B are the acute angles hence,

$$\sin A = \frac{1}{\sqrt{10}} \Rightarrow \cos A = \frac{3}{\sqrt{10}} \;\&\; \sin B = \frac{1}{\sqrt{5}} \Rightarrow \cos B = \frac{2}{\sqrt{5}}$$

Now cos (A + B) = cos A cos B – sin A sin B

$$\Rightarrow \cos\left(A + B\right) = \frac{3}{\sqrt{10}} \times \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{10}} \times \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow A + B = \frac{\pi}{4}$$

28. (B)

$$\sin 50^0 - \sin 70^0 + \sin 10^0 = -2\cos 60^0 \sin 10^0 + \sin 10^0 = 0$$

29. (C)

$$\sin 12^0 \sin 48^0 \sin 54^0 = \sin\left(60^0 - 12\right)\sin 12^0 \sin\left(60^0 + 12^0\right)\frac{\sin 54^0}{\sin 72^0}$$

$$= \frac{\sin 36^0 \sin 54^0}{4\sin 72^0} = \frac{\sin 36^0 \sin 54^0}{8\sin 36^0 \cos 36^0}$$

$= \frac{\sin 54^0}{8\cos 36^0}$, but $\sin 54^0 = \sin\left(90^0 - 36^0\right) = \cos 36^0$

$\therefore \sin 12^0 \sin 48^0 \sin 54^0 = \frac{1}{8}$

30. (C)

$$\frac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta} = \frac{(\sin 3\theta + \sin 9\theta) + (\sin 5\theta + \sin 7\theta)}{(\cos 3\theta + \cos 9\theta) + (\cos 5\theta + \cos 7\theta)}$$

$$= \frac{2\sin 6\theta \cos 3\theta + 2\sin 6\theta \cos\theta}{2\cos 6\theta \cos 3\theta + 2\cos 6\theta \cos\theta} = \tan 6\theta$$

31. (B)

$$\frac{\sin(B+A)+\cos(B-A)}{\sin(B-A)+\cos(B+A)} = \frac{\sin(B+A)+\sin\left(\frac{\pi}{2}-(B-A)\right)}{\cos\left(\frac{\pi}{2}-(B-A)\right)+\cos(B+A)}$$

$$= \frac{2\sin\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-B\right)}{2\cos\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-B\right)} = \tan\left(\frac{\pi}{4}+A\right)$$

$$= \frac{1+\tan A}{1-\tan A} = \frac{\cos A + \sin A}{\cos A - \sin A}$$

32. (A)

$\frac{\sin 2x}{\sin 2y} = n \Rightarrow \frac{\sin 2x + \sin 2y}{\sin 2x - \sin 2y} = \frac{n+1}{n-1}$ {By componendo & dividendo}

$$\Rightarrow \frac{2\sin(x+y)\cos(x-y)}{2\cos(x+y)\sin(x-y)} = \frac{n+1}{n-1}$$

$$\Rightarrow \frac{\tan(x+y)}{\tan(x-y)} = \frac{n+1}{n-1}$$

33. (D)

$$2\cos^2\theta - 2\sin^2\theta = 1 \Rightarrow \cos 2\theta = \frac{1}{2} \therefore \theta = 60^0$$

34. (C)

$\cos^2 A(3 - 4\cos^2 A)^2 + \sin^2 A\ (3 - 4\sin^2 A)^2 = (3\cos A - 4\cos^3 A)^2 + (3\sin A - 4\sin^3 A)^2$
$= \cos^2 3A + \sin^2 3A = 1$

35. (C)

$$2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta)$$

$$= 2\sin^2\beta + 2\cos(\alpha+\beta)\left(\cos(\alpha-\beta) - \cos(\alpha-\beta)\right) + \cos 2(\alpha+\beta)$$

$$= 2\sin^2\beta + 2\cos(\alpha+\beta)\cos(\alpha-\beta) - 2\cos^2(\alpha+\beta) + \cos 2(\alpha+\beta)$$

$$2\sin^2\beta + 2\cos^2\alpha - 2\sin^2\beta - 2\cos^2(\alpha+\beta) + 2\cos^2(\alpha+\beta) - 1$$

$$= 2\cos^2\alpha - 1 = \cos 2\alpha$$

36. (C)

$$\frac{3\cos\theta+\cos3\theta}{3\sin\theta-\sin3\theta}=\frac{3\cos\theta+\left(4\cos^3\theta-3\cos\theta\right)}{3-\sin\theta-\left(3\sin\theta-4\sin^3\theta\right)}=\cot^3\theta$$

37. (B)

$$2\sin A\cos^3 A-2\sin^3 A\cos A=2\sin A\cos A\left(\cos^2 A-\sin^2 A\right)$$
$$=\sin2A\cos2A=\frac{\sin4A}{2}$$

38. (C)

$$32\sin\left(\frac{A}{2}\right)\sin\left(\frac{5A}{2}\right)=16\left(\cos2A-\cos3A\right)$$
$$=16\left(2\cos^2 A-1-4\cos^3 A+3\cos A\right)$$
$$=16\left(2\times\frac{9}{16}-1-4\times\frac{27}{64}+3\times\frac{3}{4}\right)=11$$

39. (D)

$$-\sqrt{3^2+4^2}+8\le3\cos x+4\sin x+8\le\sqrt{3^2+4^2}+8$$
$$\Rightarrow3\le3\cos x+4\sin x+8\le13$$

40. (B)

$$\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3},\cos\frac{4\pi}{3}=\cos\left(\pi+\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}$$
$$\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3},\cos\frac{4\pi}{3}=\cos\left(\pi+\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}$$
$$\Rightarrow x=y\cos\frac{2\pi}{3}=z\cos\frac{4\pi}{3}\Rightarrow x=-\frac{y}{1}=-\frac{z}{2}\Rightarrow y=z=-2x$$

Now $xy + yz + zx = x(-2x) + (-2x)(-2x) + (-2x)x = 0$

41. (C)

$\sin\theta<0\ \&\ \tan\theta>0\Rightarrow\theta$ lies in 3rd quadrant.

42. (A)

$$8x^2-26x+15=0\Rightarrow\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}=\frac{13}{4}\ \&\ \tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{15}{8}$$

Now $\tan\dfrac{\alpha+\beta}{2}=\dfrac{\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}\Rightarrow\tan\dfrac{\alpha+\beta}{2}=\dfrac{\frac{13}{4}}{1-\frac{15}{8}}=-\dfrac{26}{7}$

Further $\cos(\alpha+\beta)=\dfrac{1-\tan^2\frac{\alpha+\beta}{2}}{1+\tan^2\frac{\alpha+\beta}{2}}\Rightarrow\cos(\alpha+\beta)=-\dfrac{627}{725}$

43. (C)

$$\tan9^0-\tan27^0-\tan63^0+\tan81^0=\tan9^0-\tan27^0-\tan\left(90^0-27^0\right)+\tan\left(90^0-9^0\right)$$
$$=\tan9^0+\cot9^0-\tan27^0-\cot27^0=2\left(\operatorname{cosec}18^0-\operatorname{cosec}54^0\right)$$

$$=2\frac{\sin 54^0-\sin 18^0}{\sin 54^0\sin 18^0}=\frac{4\sin 18^0\cos 36^0}{\sin 54^0\sin 18^0}$$
$$=\frac{4\cos 36^0}{\sin\left(90^0-36^0\right)}=4$$

44. (A)

$$\left(1+\cos\alpha\right)\left(1+\cos 3\alpha\right)\left(1+\cos 5\alpha\right)\left(1+\cos 7\alpha\right)=16\cos^2\frac{\alpha}{2}\cos^2\frac{3\alpha}{2}\cos^2\frac{5\alpha}{2}\cos^2\frac{7\alpha}{2}$$
$$=\left(2\cos\frac{\pi}{16}\cos\frac{7\pi}{16}\right)^2\left(2\cos\frac{3\pi}{16}\cos\frac{5\pi}{16}\right)^2$$
$$=\left(\cos\frac{\pi}{2}+\cos\frac{3\pi}{8}\right)^2\left(2\cos\frac{\pi}{2}+\cos\frac{\pi}{8}\right)^2$$
$$=\left(\cos\frac{\pi}{8}\cos\frac{3\pi}{8}\right)^2=\left(\cos\frac{\pi}{8}\sin\frac{\pi}{8}\right)^2$$
$$=\frac{1}{4}\sin^2\frac{\pi}{4}=\frac{1}{8}$$

45. (C)

$$\sqrt{3}\cos ec20^0-\sec 20^0=\frac{\frac{\sqrt{3}}{2}\cos 20^0-\frac{1}{2}\sin 20^0}{\frac{1}{2}\cos 20^0\sin 20^0}$$
$$=\frac{4\sin\left(60^0-20^0\right)}{2\cos 20^0\sin 20^0}=\frac{4\sin 40^0}{\sin 40^0}=4$$

46. (C)

$\sin 20^0\sin 40^0\ \sin 60^0\sin 80^0=(\sin(60^0-80^0)\sin 20^0\sin(60^0+20^0))\sin 60^0$
$$=\frac{1}{4}\sin\left(3\times 20^0\right)\sin 60^0=\frac{1}{4}\sin^2 60^0=\frac{3}{16}$$

47. (D)

$$\cos 20^0\cos 40^0\cos 80^0=\frac{2\sin 20^0\cos 20^0\cos 40^0\cos 80^0}{2\sin 20^0}$$
$$=\frac{2\sin 40^0\cos 40^0\cos 80^0}{4\sin 20^0}=\frac{2\sin 80^0\cos 80^0}{8\sin 20^0}$$
$=\frac{\sin 160^0}{8\sin 20^0}$, but $\sin 160^0=\sin(180^0-20^0)=\sin 20^0$, hence
$$\cos 20^0\cos 40^0\cos 80^0=\frac{1}{8}$$
Alternately
$\cos 20^0\cos 40^0\ \cos 80^0=\cos(60^0-20^0)\cos 20^0\cos(60^0+20^0)$
$$\frac{1}{4}\cos\left(3\times 20^0\right)=\frac{1}{8}$$

48. (A)

$\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{6}\right)$ acquires maximum at $\alpha+\frac{\pi}{6}=\frac{\pi}{4}$ i.e. $\theta=\frac{\pi}{12}$

49. (D)

$5\sin^2\theta + 4\cos^2\theta = 4 + \sin^2\theta \geq 4$

50. (B)

$x + \frac{1}{x} = 2\cos\theta \; \& \; x^3 + \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 - 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

$\Rightarrow x^3 + \frac{1}{x^3} = 8\cos^3\theta - 6\cos\theta = 2\cos 3\theta$

51. (B)

$0 < 2 < \pi, \pi < 3 < 2\pi \; \& \; \pi < 5\pi$

$\Rightarrow$ sin 2 > 0, sin 3 < 0, sin 5 < 0

$\Rightarrow$ sin 2 sin 3 sin 5 > 0

52. (A)

(sec A – tan A) (sec B – tan B) (sec C – tan C) = X &

(sec A – tan A) (sec B – tan B) (sec C – tan C) = X

$\Rightarrow X^2 = (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C) = 1$

$\Rightarrow X = \pm 1$