Parabola

Exercise – 1

Q.1 (A)

S(0, 1) directric x + 2y = 0

$\Rightarrow$ PS = PM

$$\Rightarrow \sqrt{x^2+(y-1)^2} = \left|\frac{x+2y}{\sqrt{5}}\right|$$

$$\Rightarrow 5\left[x^2+y^2-2y+1\right] = (x+2y)^2$$

Q.2 (D)

$y^2 = 18\ x$ $\quad\quad\quad\quad$ $(at^2, 2at)$

$$\Rightarrow \therefore 2at = 3\left[at^2\right]$$

$\Rightarrow$ 2 = 3t

$$\Rightarrow t = \frac{2}{3}$$

$$\Rightarrow \therefore \left(at^2, 2at\right) \equiv \left(\frac{18}{4}\left(\frac{4}{9}\right), 2\left(\frac{18}{4}\right)\left(\frac{2}{3}\right)\right]$$

$$\equiv (2, 6)$$

Q.3 (C)

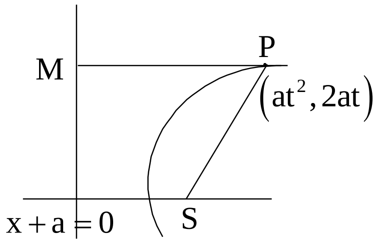
$$y^2 = \left(\frac{4}{5}\right)x$$

$\Rightarrow \therefore$ Equation of directrix is x + a = 0

$$\Rightarrow x + \left(\frac{1}{5}\right) = 0$$

Q.4

$y^2 = 8x$



$\Rightarrow SP = PM = 8$

$\Rightarrow a + at^2 = 8$

$\Rightarrow t = \pm\sqrt{3}$

$\Rightarrow \therefore$ x coordinate = $at^2 \equiv (2)(3) = 6$

Q.5

$y^2 = 4ax$; as parabola passes through (-3,2) So

$\therefore 4 = 4a(-3)$

$\Rightarrow 4a = -\dfrac{4}{3}$

$\Rightarrow \ell(\text{L.R.}) = |4a| = \dfrac{4}{3}$

Q.6 (A)

$\left.\begin{array}{l} x^2 = -8ay \\ Y^2 = 4AX \end{array}\right\}$ compare both of them

$\Rightarrow 4A = 8a;\ A = 2a$

$\Rightarrow Y = x,\ X = -y$

For focus $X = A \Rightarrow -y = 2a$

$\Rightarrow Y = 0 \Rightarrow x = 0$

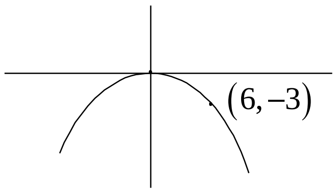
$\Rightarrow S(0, -2a)$

Directrix is $X + A = 0$

$\Rightarrow$ (– y) + 2a = 0

$\Rightarrow$ y = 2a

Q.7 (C)



Equation of parabola

$\Rightarrow x^2 = -4by$; as parabola passes through (6,-3) so

$\Rightarrow 36 = -4b(-3)$

$\Rightarrow b = 3$

$\therefore$ Equation of parabola

$\Rightarrow x^2 = -12y$

Q.8 (A)

$x^2 + 4x + 2y - 7 = 0$

$\Rightarrow (x+2)^2 = 2\left(-y+\frac{11}{2}\right)$ ……..(1)

Compare (1) with $Y^2 = 4AX$ ……..(2)

$\Rightarrow Y = x + 2,\ X = -y+\frac{11}{2}, A = \frac{1}{2}$

$\Rightarrow$ Vertex $X = 0 \Rightarrow -y+\frac{11}{2} = 0$

$\Rightarrow Y = 0 \Rightarrow x + 2 = 0$

$\Rightarrow$ Vertex $A\left(-2, \frac{11}{2}\right)$

Q.9

$4y^2 - 6x - 4y = 5$

$$\Rightarrow y^2 - \frac{3}{2}x - y = \frac{5}{4}$$

$$\Rightarrow \left(y - \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{3}{2}x = \frac{5}{4}$$

$$\Rightarrow \left(y - \frac{1}{2}\right)^2 = \frac{3}{2}(x+1)$$

compare (1) with $y^2 = 4AX$

$$\Rightarrow \therefore \text{focus is } X = A \Rightarrow (x+1) = \frac{3}{8}$$

$$\Rightarrow Y = 0 \Rightarrow y - \frac{1}{2} = 0$$

$$\Rightarrow S\left(-\frac{5}{8}, \frac{1}{2}\right)$$

Q.10 (C)

$y^2 + 8y – 8x = 0$

$\Rightarrow$ $(y + 4)^2 = 8x + 16$

$\Rightarrow$ $(y + 4)^2 = 8 (x + 2)$ ………..(1)

Compare with $Y^2 = 4AX$

Eq^n. of directrix $X + A = 0$

$\Rightarrow$ $(x + 2) + (2) = 0$

$\Rightarrow x + 4 = 0$

Q.11 (A)

$y^2 – 4kx + 8 = 0$

$$\Rightarrow y^2 = 4k\left(x - \frac{2}{k}\right)$$

$$\Rightarrow \text{vertex } A = \left(\frac{2}{k}, 0\right)$$

Perpendicular distance of vertex from directrix x – 3 = 0 is equal to $= \frac{\ell(LR)}{4}$

$$\Rightarrow \left|\frac{\frac{2}{k}-3}{1}\right| = \left|\frac{4k}{4}\right|$$

$$\Rightarrow \frac{|2-3k|}{|k|} = |k|$$

$\Rightarrow k^2 = 2 - 3k$ or $k^2 = 3k - 2$

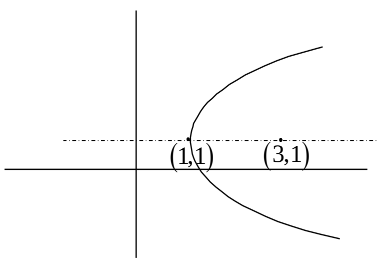
$\Rightarrow k^2 + 3k - 2 = 0$ or $k^2 - 3k + 2 = 0$

$\Rightarrow k = \frac{-3 \pm \sqrt{9+8}}{2}$ or $k = 1, 2$

Q.12 (C)

A (1, 1), S (3, 1)

$\Rightarrow$ AS = 2



$\Rightarrow \therefore$ Eq^n of parabola

$$\Rightarrow (y-1)^2 = 4\times 2(x-1)$$

Q.13 (D)

$x - 2 = t^2$, $y = 2t$

eliminate "t"

$$\Rightarrow \therefore x - 2 = \left(\frac{y}{2}\right)^2$$

$\Rightarrow y^2 = 4(x - 2)$

Q.14

$y^2 = 4x$

$\Rightarrow y^2 - 4x = 0$

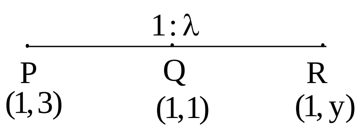
$\Rightarrow (S)_{(1,3)} = 9 - 4 > 0$

$\Rightarrow (S)_{(1,1)} = 1 - 4 < 0$

$\Rightarrow \therefore$ P(1, 3) is outside the Parabola.

$\Rightarrow$ Q (1, 1) is inside the parabola.

Q.15 (D)



$$\Rightarrow \frac{PQ}{QR} = \frac{1}{\lambda}$$

$$\Rightarrow \frac{1 \times y_1 + \lambda \times 3}{\lambda + 1} = 1$$

$$\Rightarrow y_1 = (\lambda + 1) - 3\lambda = 1 - 2\lambda$$

$$\Rightarrow \therefore R(1, 1 - 2\lambda)$$

R is interior point of $y^2 - 4x = 0$

So, $(1 - 2\lambda)^2 - 4 < 0$

$$\Rightarrow 4\lambda^2 - 4\lambda - 3 < 0$$

$$\Rightarrow \lambda \in \left(-\frac{1}{2}, \frac{3}{2}\right)$$

Q.16 (B)

$y = 2x + \lambda;\ y^2 = 2x$

$$\Rightarrow L(\text{chord}) = \frac{4}{m^2}\sqrt{a(a - mc)(1 + m^2)}$$

For line which does not intersect a – mc < 0

$$\Rightarrow \left(\frac{2}{4}\right) - (2)(\lambda) < 0$$

$$\Rightarrow \lambda > \frac{1}{4}$$

Q.17 (C)

$y^2 = 4ax$

$\Rightarrow 4 = 4a(1)$

$\Rightarrow a = 1$

tangent at (1, – 2) is

$\Rightarrow yy_1 = 2a(x + x_1)$

$\Rightarrow y(-2) = 2 \times 1(x + 1)$

$\Rightarrow x + y + 1 = 0$

Q.18 (D)

$y^2=16x$

$\Rightarrow$ tangent in slope form $y = mx + \frac{4}{m}$

$\Rightarrow$ slope of tangent $m = -\frac{1}{3}$

$$\Rightarrow \therefore y = -\frac{1}{3}x - 12$$

Q.19 (D)

$y^2 = x$; slope of tangent m = tan 45°.

$\Rightarrow m = 1$

$\Rightarrow$ point of contact $\equiv \left(\frac{a}{m^2}, \frac{2a}{m}\right)$ as $a = \frac{1}{4}$

$\Rightarrow \equiv \left(\frac{1}{4}, \frac{1}{2}\right)$

Q.20

y = 2x + 2 touches $y^2 = 16x$

$\Rightarrow m = 2, \frac{a}{m} = \frac{4}{2} = 2$

Point of contact of tangent $\equiv \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$\Rightarrow \equiv \left(\frac{4}{4}, \frac{2 \times 4}{2}\right)$

$\Rightarrow \equiv (1, 4)$

Q.21 (A)

Equation of tangent at $\left(\frac{a}{t^2}, \frac{2a}{t}\right)$

$\Rightarrow x = \frac{a}{t^2}$

$\Rightarrow y = \frac{2a}{t}$ put $t = \frac{1}{u}$

$\Rightarrow$ So, $x = au^2$, $y = 2au$ which gives parabola $y^2 = 4ax$

Equation of tangent $uy = x + au^2$

$\Rightarrow \frac{1}{t} y = x + \frac{a}{t^2}$

$\Rightarrow ty = t^2x + a$

Q.22 (A)

$y^2 = 4x$

$\Rightarrow y = mx + \frac{1}{m}$; eq^n of tangent in slope form & it passes through (-2,-1) then

$$\Rightarrow -1 = -2m + \frac{1}{m}$$

$\Rightarrow 2m^2 - 2m - 1 = 0$ $\quad (m_1 = 1,\ m_2 = -\frac{1}{2})$

$$\Rightarrow \tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \tan\alpha = \left|\frac{1 + \frac{1}{2}}{1 - \frac{1}{2}}\right| = 3$$

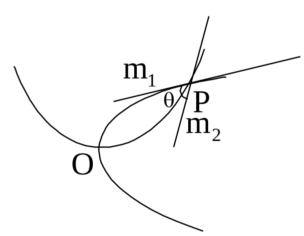
Q.23 (B)

$y^2 = 4ax$ ....(1)

$\Rightarrow x^2 = 4by$ ....(2)

for point of intersection solve (1) & (2)

$$\Rightarrow \left(\frac{x^2}{4b}\right)^2 = 4ax$$



$$\Rightarrow x = 0,\ x = \left(64ab^2\right)^{\frac{1}{3}}$$

$$\Rightarrow x = 4a^{\frac{1}{3}}b^{\frac{2}{3}}$$

$$\Rightarrow \therefore \text{ Point of intersection } P\left(4a^{\frac{1}{3}}b^{\frac{2}{3}}, 4a^{\frac{2}{3}}, b^{\frac{2}{3}}\right)$$

Angle between curves = angle between the in tangents at intersection point

$$\Rightarrow \text{slope}\, m_1 = \frac{2a}{y_1} = \frac{2a}{4b^{\frac{1}{3}}a^{\frac{2}{3}}}$$

$$\Rightarrow m_1 = \frac{1}{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}$$

$$\Rightarrow m_2 = \frac{x_1}{2b} = \frac{4\left(a^{\frac{1}{2}}\right)\left(b^{\frac{2}{3}}\right)}{2b}$$

$$\Rightarrow m_2 = 2\left(\frac{a}{b}\right)^{\frac{1}{3}}$$

$$\Rightarrow \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\left(\frac{a}{b}\right)^{\frac{1}{3}\times\frac{3}{2}}}{1+\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right|$$

$$\Rightarrow \left|\frac{\frac{3}{2}}{\left(\frac{b}{a}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right| = \left|\frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}}\right| = \sqrt{3}$$

$$\Rightarrow \theta = 60^o$$

Q.24 (C)

$(x + 2)^2 + y^2 = 4$ ……(1)

Eq^n of parabola $y^2 = 8x$ in parametric form is $ty = x + 2t^2$ ……. (2)

If (2) touches eq^n (1)

$$\Rightarrow \therefore \left|\frac{-2 + t(0) + 2t^2}{\sqrt{1+t^2}}\right| = 2$$

$$\Rightarrow \left|t^2 - 1\right| = \sqrt{t^2 + 1}$$

$\Rightarrow t^4 + 1 - 2t^2 = t^2 + 1$

$\Rightarrow t^2 = 0, t^2 = 3$

$\Rightarrow t = \pm\sqrt{3}$

$\therefore eq^n$ of tangents are

$\Rightarrow x = 0,$

$\Rightarrow \pm\sqrt{3}y = x + 6$

to be above x – axis slope of tangent is positive.

Q.25 (B)

Intersection point of $y^2 = 4ax$ & $x^2 = 4ay$ are A (0, 0), B (4a, 4a)

$\therefore eq^n$ of line passing through A & B is

$\Rightarrow y = x$ ……(1)

$\Rightarrow 6x + cy + d = 0$ …….(2)

(1) & (2) are same, so

$\Rightarrow d = 0, b = -c$

$\Rightarrow (b + c) = 0$

Q.26

x + y = a is normal.

$\Rightarrow y = -x + a;\ m = -1$

$\Rightarrow y^2 = 12x$

$\Rightarrow \therefore a = -2a'm - a'm^3$

$= -2(3)(-1) - (3)(-1)^3$

$\Rightarrow a = 9$

Q.27 (B)

Point $\left(\frac{a}{4}, a\right) \equiv \left(at^2, 2at\right)$ to $y^2 = 4ax$

$\Rightarrow \therefore t = \frac{1}{2}$

Eq^n of normal is

$\Rightarrow y + tx = 2at + at^3$

$\Rightarrow y + \frac{1}{2}x = 2(a)\left(\frac{1}{2}\right) + a\left(\frac{1}{8}\right)$

$\Rightarrow 8y + 4x = 8a + a$

$\Rightarrow 8y + 4x = 9a$

Q.28 (B)

$y^2 = 8x$

normal is parallel to 2y = x + 5

$\Rightarrow \therefore$ slope of normal $m = \frac{1}{2}$

$\Rightarrow \therefore$ foot of normal = $(am^2, -2am)$

$\equiv \left(2 \times \frac{1}{4}, -2 \times 2 \times \frac{1}{2}\right)$

$\equiv \left(\frac{1}{2}, -2\right)$

Q.29 (A)

$y^2 = 4ax$

end point of L.R.

$\Rightarrow$ L (a, 2a) , L' (a,– 2a)

$\Rightarrow t_1 = 1, \quad t_2 = -1$

$\Rightarrow \therefore eq^n$ of normals are

$\Rightarrow y + t\,x = 2at + at^3$

$\Rightarrow y + x = 3a$

$\Rightarrow y - x = -3a$

combined equation of normal's are

$\Rightarrow$ [y + (x – 3a)] [y – (x – 3a)] = 0

$\Rightarrow y^2 - (x - 3a)^2 = 0$

Q.30 (A)

$x^2 = 8y$

eq^n of normal in parametric form

$\Rightarrow x + ty = 2at + at^3$

$\Rightarrow x + ty = 4t + 2t^3$ ………(1) $(\because a = 2)$

slope of normal $-\frac{1}{t} = m(\text{say})$

so put $t = -\frac{1}{m}$ in eq^n (1)

$\Rightarrow x - \frac{1}{m}y = -\frac{y}{m} - \frac{2}{m^3}$

$\Rightarrow m^3x - m^2y = -4m^2 - 2$ equation of normal in slope form if this normal passes through (h, k) so

$\Rightarrow \therefore m^3h + (4 - k)m^2 + 2 = 0$ (m_1, m_2, m_3) ………(2)

$\Rightarrow m_1m_2m_3 = \frac{-2}{h}$

But $m_1m_2 = -1$ (given) ($\because$ two of the normal's are perpendicular)

$\Rightarrow \therefore m_3 = \frac{2}{h}$

The value of m_3 has to satisfy eq^n (2)

$\Rightarrow \therefore \left(\frac{8}{h^3}\right)h + (4 - k)\left(\frac{4}{h^2}\right) + 2 = 0$

So locus (h, k) is $x^2 = 2(y - 6)$

Q.31

$y^2 = 4ax$

let (h, k) is centroid of P (at_1^2, $2at_1$), Q (at_2^2, $2at_2$), R (at_3^2, $2at_3$) is $k = 2a\frac{(t_1 + t_2 + t_3)}{3} = 0$

so locus of centroid is y = 0

Q.32 (C)

Two of the foot of normal's are (1, 2) & (1, – 2) for $y^2 = 4x$

as we can see that foot of normal's given such that $t_1 = 1$, $t_2 = -1$

$\Rightarrow t_1 + t_2 + t_3 = 0$

$\Rightarrow 1 + (-1) + t_3 = 0$

$\Rightarrow t_3 = 0$

$\Rightarrow$ so 3rd foot is (0, 0)

Q.33 (B)

(au^2, 2au), (av^2, 2av) are extremities of focal chord $y^2 = 4ax$

$\Rightarrow$ So uv = – 1 (by property)

Q.34 (C)

$y^2 = 8x$

Eq^n of chord of contact of (2, 5) is T = 0

$\Rightarrow y(5) - 2\times 2(x+2) = 0$

$\Rightarrow y = \frac{4}{5}x + \frac{8}{5}$

Compare with y = mx + c

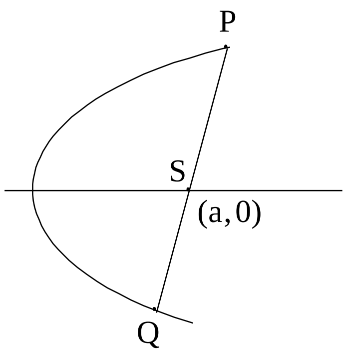
$\Rightarrow m = \frac{4}{5}, c = \frac{8}{5}$

$\Rightarrow \therefore$ Length of chord y = mx + c for $y^2 = 8x$ is

$\Rightarrow \frac{4}{m^2}\sqrt{a(a-mc)(1+m^2)}$ put the values now.

Q.35 (A)

$y^2 = 4ax$



$\Rightarrow \frac{1}{QS} + \frac{1}{PS} = \frac{1}{a}$ (by property for focal chord)

$\Rightarrow \frac{1}{k} + \frac{1}{b} = \frac{1}{a}$ (given QS = k, PS = b)

$\Rightarrow k = \frac{ab}{b-a}$

Q.36 (B)

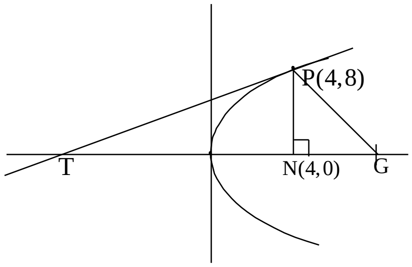
$y^2 = x$ diameter of system of parallel chords which are parallel to $y = x + 1$ (m = 1) is $y = \frac{2a}{m}$

$\Rightarrow y = \frac{2\left(\frac{1}{4}\right)}{1} = \frac{1}{2}$

Q.37 (C)

$y^2 = 16x$

$\Rightarrow$ abscissa is = 4; means the point is (4, 8) or (4, – 8)



$\Rightarrow$ L (sub tangent) = NT

$\Rightarrow$ eq^n of tangent at (4, 8) is

$\Rightarrow$ y (8) 2 × 8(x + 4)

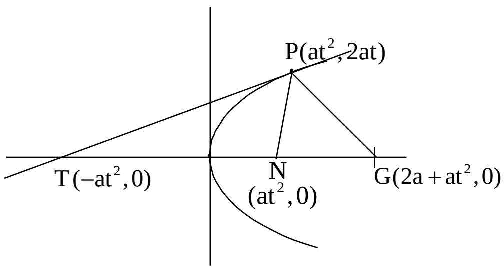
$\Rightarrow \therefore$ y = 2x + 8

$\Rightarrow$ T (– 4, 0)

$\Rightarrow \therefore$ NT = 8

Q.38 (A)

sub-tangent = sub-normal



$\Rightarrow$ NT = NG

$\Rightarrow$ (2at^2) = 2a

$\Rightarrow t = \pm 1$

$\Rightarrow \therefore P\left(at^2, 2at\right) \equiv \left(a, \pm 2a\right)$

Q.39 (A)

y = 2x ....(1)

let (α, β) is pole of (1) w.r.t. y^2 = 2x

$\Rightarrow \therefore y\beta = 2\left(\frac{1}{2}\right)\left(x + \alpha\right)$

$\Rightarrow y\beta = x + \alpha$ ....(2)

compare (1) & (2)

$\Rightarrow \frac{\beta}{1} = \frac{1}{2} = \frac{\alpha}{0}$

$\Rightarrow \alpha = 0, \beta = \frac{1}{2}$

Q.40 (A)

Let pole (h, k) so polar w.r.t circle $x^2 + y^2 = r^2$ is $xh + yk = r^2$ …….(1)

Eq^n of tangent $y^2 = 4ax$ is $ty = x + at^2$ …….(2)

(1) & (2) are same for same value of t so.

$$\Rightarrow \frac{h}{1} = \frac{k}{-t} = \frac{-r^2}{at^2}$$

$$\Rightarrow t = \frac{-k}{h}, t = \frac{r^2}{ak}$$

$$\Rightarrow \therefore \frac{-k}{h} = \frac{r^2}{ak}$$

$$\Rightarrow \therefore \text{ locus of (h, k) is } y^2 = -\frac{r^2}{a}x$$

Q.41 (A)

Ray moving parallel to axis of the parabola then reflected ray has to pass through focus of the parabola.

$$\Rightarrow \therefore (y-2)^2 = 4(x+1)$$

$\Rightarrow$focus is S (0, 2)

Q.42 (A)

S (a, b), directrix $\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow$Apply PS = PM

$$\Rightarrow \sqrt{(x-a)^2 + (y-b)^2} = \left| \frac{\frac{x}{a} + \frac{y}{b} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right|$$

Q.43 (A)

$y^2 = x$

$\Rightarrow$ after interchanging y to – y, eq^n of curve remain same so curve is symmetric about x aixs.

Q.44 (B)

$y^2 = 16x$

$\Rightarrow P(at^2, 2at)$

$\Rightarrow 2at = 2(at^2) \Rightarrow t = 0, 1$

$\Rightarrow \therefore$ Point is (4, 8)

$\Rightarrow \therefore SP = PM = (4 + 4) = 8$

Q.46 (B)

$y^2 = \frac{4x}{5}$

$\Rightarrow$ here $4a = \frac{4}{5}$

$\Rightarrow$ so extremities of latus rectum are $(a, \pm 2a) \equiv \left(\frac{1}{5}, \pm\frac{2}{5}\right)$

Q.47 (C)

vertex A (0, 0)

$\Rightarrow$ directricx x + 5 = 0

$\Rightarrow$ so a = 5

$\Rightarrow \therefore$ latus rectum = 4a = 20

Q.48 (C)

$y^2 = 6x$

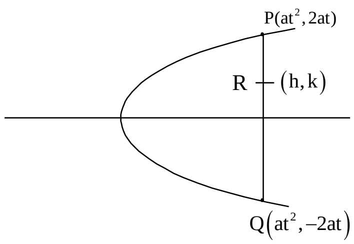
$\Rightarrow$ vertex A (0, 0)

the point, whose abscissa is 24, are $(24, \pm 12)$

$\Rightarrow \therefore$ eq^n of line is $y = \pm\frac{1}{2}x$

Q.49 (A)

$y^2 = 4ax$



$\Rightarrow \frac{PR}{RQ} = \frac{1}{2}$

$\Rightarrow \therefore h = at^2, k = \frac{2(2at)+1(-2at)}{3} = \frac{2at}{3}$

$\Rightarrow \therefore h = a\left[\frac{3k}{2a}\right)^2$

$\Rightarrow 4ah = 9k^2$

So locus is $9y^2 = 4ax$

Q.50 (B)

$\sqrt{(x-2)^2+(y+5)^2} = \left|\frac{3x+4y-1}{5}\right|$

$\Rightarrow \therefore$ PS = PM where S is focus & M is perpendicular on directrix.

so locus of P is parabola where 5 (2, – 5)

Q.51 (A)

$y^2 = 8x$

$\Rightarrow$ S (2, 0) if P(at^2, 2at) the SP = a+ at^2

$\Rightarrow 4 = 2\ (1+t^2)$

$\Rightarrow t = \pm 1$

$\Rightarrow \therefore P(2, \pm 4)$

Q.52 (C)

S (– 3, 0) directric is x + 5 = 0

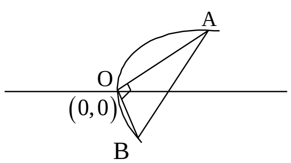
$\Rightarrow \therefore$ eq^n of parabola is

$$\Rightarrow \sqrt{(x+3)^2 + y^2} = \left|\frac{x+5}{1}\right|$$

$\Rightarrow y^2 = 4ax$

Q.53 (A)

$y^2 = 4ax$



eq^n of AB is lx + my + n = 0

$\Rightarrow \therefore$ eq^n of pair of straight line OA & OB is given by homogenization

$$\Rightarrow \text{So, } \left(y^2 - 4ax\left(\frac{lx+my}{-n}\right)\right) = 0$$

Angle between lines is 90^o so

$$\Rightarrow \left(\frac{4al}{n}\right) + 1 = 0$$

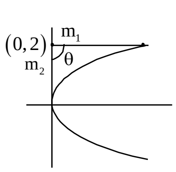
Q.55 (A)

$y^2 = 4ax$

eq^n of tangent m slope form is

$\Rightarrow y = mx + \frac{a}{m}$ & this tangent passes through (0,2) then

$\Rightarrow 2 = \frac{a}{m}$ ……(1)



$\Rightarrow m_2 = \infty, \theta = \frac{3\pi}{4}$

$\Rightarrow$ So, $m_1 = \tan\left(\frac{\pi}{4}\right) = 1$; put this value of m in eq.(1)

$\Rightarrow \therefore a = 2$

Q.56 (C)

Point of intersection of tangents to $y^2 = 4x$ is $(at_1t_2, a(t_1 + t_2))$

$\Rightarrow$ (1 (1) (2), 1 (1 + 2)) = (2, 3)

Q.57 (D)

$y^2 = 4(x - 1)$

$\Rightarrow$ eq^n tangent is $y = m(x-1) + \frac{1}{m}$ & it passes through (0, 0)

So $0 = m(-1) + \frac{1}{m}$

$\Rightarrow 0 = -m^2 + 1$

$\Rightarrow m = \pm 1$

So angle between tangents = $\frac{\pi}{2}$

Q.58 (B)

$y^2 - 4x = 0$

$\Rightarrow (S)_{(0,-2)} = 4 - 0 > 0$

So point lies outside so 2 real & distinct tangents can be drawn.

Q.59 (B)

$y^2 = 4ax$

$\Rightarrow$ eq^n tangent in slop form $y = mx + \frac{a}{m}$ …….(1)

tangents are drawn from (α, β) so

$\Rightarrow \beta = m\alpha + \frac{a}{m}$

$\Rightarrow \alpha m^2 - \beta m + a = 0$ …….(2)

$\Rightarrow m_1 m_2 = \frac{a}{\alpha}, m_1 + m_2 = \frac{\beta}{\alpha}$

given that $m_1 = 2m_2$

So, $3m_2 = \frac{\beta}{\alpha}$

$\Rightarrow m_2 = \frac{\beta}{3\alpha}$

So, m_2 satisfy eq^n (2)

$\Rightarrow \alpha\left(\frac{\beta}{3\alpha}\right)^2 - \beta\left(\frac{\beta}{3\alpha}\right) + a = 0$

$\Rightarrow \beta^2 - 3\beta^2 + 9a\alpha = 0$

$\Rightarrow 2\beta^2 = 9a\alpha$

Q.60 (C)

$y + b = m_1 (x + a)$ ……….(1)

$\Rightarrow y + b = m_2 (x + a)$ ……….(2)

eq^n (1) & (2) are tangents to the parabola $y^2 = 4ax$

both tangents are drawn from (– a, – b)

which lies on x = – a, so tangents drawn from (-a, -b) has to be perpendicular so $m_1m_2 – 1$

Q.61

$y^2 = 4a(x + a)$

eq^n of tangent in slope form is

$\Rightarrow y = m(x+a) + \frac{a}{m}$ ……..(1)

$\Rightarrow y = mx + c$ ………(2)

$\Rightarrow \therefore c = am + \frac{a}{m}$

Q.62 (B)

$y^2 = 4ax$

$\Rightarrow$ (– a, 2a) lies on directrix x = – a of the parabola.

So, tangents drawn from it are perpendicular

Q.63 (B)

Intersection point of tangents $\left(at_1t_2, a\left(t_1+t_2\right)\right]$, which lies on axis y = 0 if $a(t_1 + t_2) = 0$

$\Rightarrow$ So, $t_1 = -t_2$

Q.64 (D)

$y^2 = 4ax$ ……….(1)

eq^n of tangents $ty = x + at^2$

$\Rightarrow x - ty + at^2 = 0$

$\Rightarrow p_1 = \left|\frac{(a+k) - 0 + at^2}{\sqrt{1+t^2}}\right|$

$\Rightarrow p_2 = \left|\frac{(a-k) - 0 + at^2}{\sqrt{1+t^2}}\right|$

$\Rightarrow \therefore p_1^2 - p_2^2 = 4ak$

Q.65 (A)

$kx + y = 4$ ……..(1)

$\Rightarrow y = x - x^2$ ……..(2)

(1) touches parabola (2) then

$\Rightarrow$ (4 – kx) = x – x^2

$\Rightarrow x^2$ – x (k + 1) + 4 = 0

$\Rightarrow \because x_1 = x_2$

$\Rightarrow \therefore D = 0$

$\Rightarrow (k + 1)^2 - 16 = 0$

$\Rightarrow$ k = 3, – 5

Q.66 (A)

$y^2 = 4ax$ ……..(1)

$\Rightarrow x^2 = 4by$ ……..(2)

eq^n of tangent for (1)

$\Rightarrow ty = x + at^2$ ……..(3)

$$\Rightarrow \therefore x^2 = 4b\left(\frac{x + at^2}{t}\right)$$

$\Rightarrow tx^2 - 4bx - 4abt^2 = 0$

$\Rightarrow$ D = 0

$\Rightarrow 16b^2 + 4\,(4ab)\,t^3 = 0$

$$\Rightarrow t^3 = -\frac{b}{a}$$

$$\Rightarrow t = -\frac{b^{\frac{1}{3}}}{a^{\frac{1}{3}}}$$

So eq^n of tangent is

$$\Rightarrow x + \frac{b^{\frac{1}{3}}}{a^{\frac{1}{3}}}y + a\left(\frac{b^{\frac{2}{3}}}{a^{\frac{2}{3}}}\right) = 0$$

$$\Rightarrow a^{\frac{1}{3}}x + b^{\frac{1}{3}}y + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$$

Q.67 (A)

$x\cos\alpha + y\sin\alpha = p$

$\Rightarrow y = -\cot\alpha x + p\,\mathrm{cosec}\,\alpha$ ....(1)

$\Rightarrow y^2 = 4a(x + a)$

tangent in slope form is $y = m(x+a) + \frac{a}{m}$

$\Rightarrow \therefore p\,\mathrm{cosec}\,\alpha = am + \frac{a}{m}$

$\Rightarrow p\,\mathrm{cosec}\,\alpha = a(-\cot\alpha) - a\tan\alpha$

$\Rightarrow p\cot\alpha + a = 0$

Q.68 (C)

$x + y = 1$ ....(1)

$\Rightarrow$ line (1) touches $y^2 - y + x = 0$

$\Rightarrow y^2 - y + (1 - y) = 0$

$\Rightarrow (y - 1)^2 = 0$

$\Rightarrow y = 1$

$\Rightarrow$ So, $x = 0$

Point of contact is (0, 1)

Q.69 (B)

$x^2 + y^2 = 2$ ....(1)

$\Rightarrow y^2 = 8x$ ....(2)

eq^n of tangent for (2) is

$\Rightarrow ty = x + 2t^2$

$\Rightarrow x - ty + 2t^2 = 0$ ....(3)

line (3) touches circle (1) then

$$\Rightarrow \left|\frac{0-0+2t^2}{\sqrt{1+t^2}}\right| = \sqrt{2}$$

$$\Rightarrow (2t^2)^2 = 2 + 2t^2$$

$$\Rightarrow 2t^4 - t^2 - 1 = 0$$

$$\Rightarrow t^2 = 1$$

$$\Rightarrow t = \pm 1$$

so eq^n of tangents are

$$\Rightarrow 2 \pm y + 2 = 0$$